ASTR430: HOMEWORK #3 SOLUTIONS

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Table 1: For Question 1

planet	d (meters)	Energy (megatons of TNT)	how often? (years)	D (meters)	h (meters)
Earth	64	16	350	836.0	163.0
Mars	1.6	0.000244	0.34	28.0	5.5
Venus	6500	16 million	23 million	83400	1200

d is maximum size impactor; D is smallest crater diameter; h is smallest crater depth.

Question 1 a,b,c) Please see the above Table. The trick here was to realize that the largest object that explodes in the air (airburst) is about the same size as the smallest one that can hit the ground. Accordingly, both objects release nearly the same amount of energy and we can use the results from the cratering collision to find this energy.

d) Titan's atmosphere is about ten times denser than Earth's and ten times thinner than Venus'. So it would likely be able to stop mid-sized impactors (600 meters in diameter) from reaching the surface intact. The impactors that do reach Titan's surface would create craters 10 times larger than their own size, or at least 6km in diameter. Titan should have few craters smaller than 6km in diameter.

Problem 8-3 Central Pressures

a) Eq. 3.2.1

$$P_c = \frac{3GM^2}{8\pi R^4}$$ ← underestimate: assumes constant density

see planetary calculator output

i) Jupiter : 11.0 Mbar
ii) Saturn : 1.95 Mbar

(Fig. 8.14 says 70 Mbar for Jupiter)

♃ is the Jupiter symbol

so $5 M_J$ and $1.1 M_J$ planet

$\Rightarrow$ 11.0 Mbar $\times\ 5^2 / 1.1^4$ (scale from Jupiter)

= 188 Mbar

iii) extra solar planet : 188 Mbar

Planetary Calculator

Results in SI units

convert to bars

Planet	3*G*M^2/(8*pi*R^4) /1E5 /1E6
MERCURY	0.245557911
VENUS	1.4091929
EARTH	1.71880898
MARS	0.247751696
JUPITER	10.9981921
SATURN	1.95225912
URANUS	1.40819794
NEPTUNE	2.22345025
PLUTO	0.00774128315

convert to Mbar

Back to Planetary Calculator!

Interesting that Neptune's central pressure exceeds Saturn's! And Earth's is also close.

b) Density of H_2 at those pressures

$$P = K\rho^2 \quad w/ \quad K = 2.7 \times 10^5 \ \frac{m^5}{kg\, s^2}$$

Jupiter: $1.1 \times 10^{12} = 2.7 \times 10^5 \rho^2$

$$\Rightarrow \rho^2 = 4.1 \times 10^6$$

$$\rho = 2020 \ kg/m^3$$

Mean density $\bar{\rho} = 1326 \ kg/m^3$ (Table E.3)

Saturn

$$1.95 \times 10^{11} = 2.7 \times 10^5 \rho^2$$

$$\rho = 849 \ kg/m^3$$

Mean density $\bar{\rho} = 569 \ kg/m^3$ (E.3)

Exoplanet

$$1.88 \times 10^{13} = 2.7 \times 10^5 \rho^2$$

$$\rho = 8340 \ kg/m^3$$

mean density? scale from Jupiter

$(1.1)^3$ * Jupiter's volume

5 * Jupiter's mass

$\Rightarrow$ density is $\frac{5}{(1.1)^3} = 3.8$ * Jupiter's $\bar{\rho}$

$= 7590 \ kg/m^3$

Estimated central densities are close to average densities in all cases, so lower bounds will not be bad estimates.

c) H → metallic @ 2Mbar

This matches Saturn's lower estimate pretty well. So we would guess Saturn-sized.

But we know that the real Saturn does have metallic H and that the equation gives a lower bound on pressure, so we guess that the transition is at a slightly lower mass.

Mass Saturn = $0.3\ M_J$

Transition Mass $\approx 0.2\ M_J$.

If made primarily of H, He, these planets would have very weak magnetic fields.

If they have substantial heavy elements, then they might have an ionic layer like Uranus and

Neptune and hence highly tilted and irregular fields like those planets. Field strengths would be weaker than Jupiter's.

Problem 8-4 Temp vs. altitude

a) How do we know Temp. vs. altitude?

Jupiter and Saturn both emit substantially more energy in IR than they receive in visible sunlight

⇒ significant heat transport from depth

⇒ convection! ← (most efficient way to transport heat)

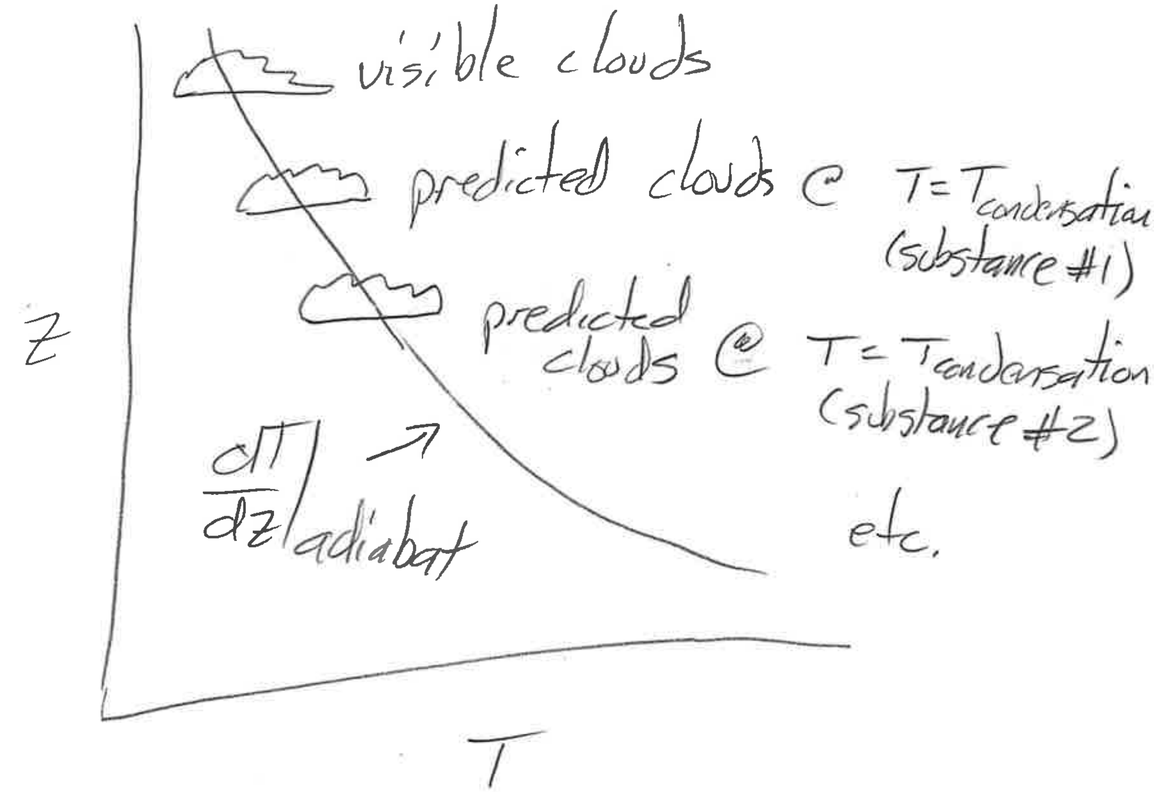
⇒ The T vs. z curve will follow an adiabat, why?

larger dT/dz diminished by convection

small dT/dz not convecting

The adiabat can be calculated.

b)



See section 4.4.1 and Eq. 4.27

$$\left.\frac{dT}{dz}\right|_{adiabat} = -\frac{\gamma - 1}{\gamma}\frac{g\mu}{k}$$

where g = gravity, μ = molecular mass

$\gamma = \frac{c_p}{c_v}$ = ratio of specific heats

k = Boltzmann's constant

plug in solar composion gas and calculate g, P profiles, then $\left.dT/dz\right|_{adiabat}$

c) Uranus emits in IR about what it receives in sunlight ⇒ no requirement for convection like Jupiter and Saturn.

A convecting planet is a requirement for assuming that dT/dz follows an adiabat.