ASTR 430 HW #4

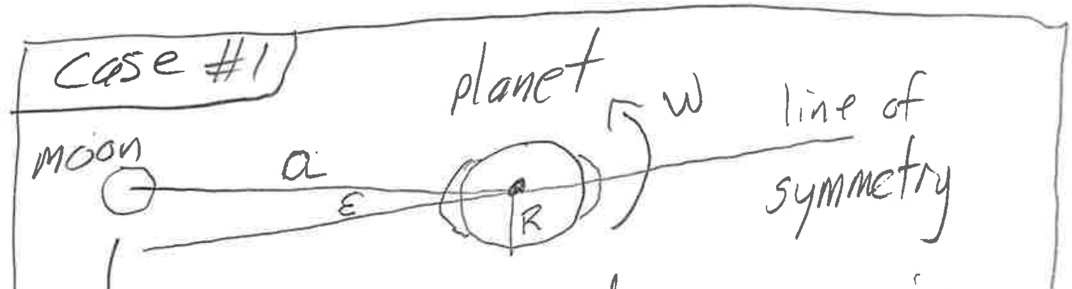
Hamilton

Problem 10-5

a) What do moons evolve inward?

It depends on the relative motions – planet's spin and moon's orbit.

Draw pics!



Here w and n are in the same direction w/ $w > n$

Because Earth's material takes some time to respond to the moon's gravity, the tidal bulges on Earth get rotated so that they lead the moon-planet line, so $\varepsilon > 0$.

Now the gravity of the near bulge and the far bulge almost cancels but not quite. The masses of the two bulges are nearly identical if $a >> R$; since gravity scales like $1/r^2$, the near bulge dominates. Its forces resolve like this

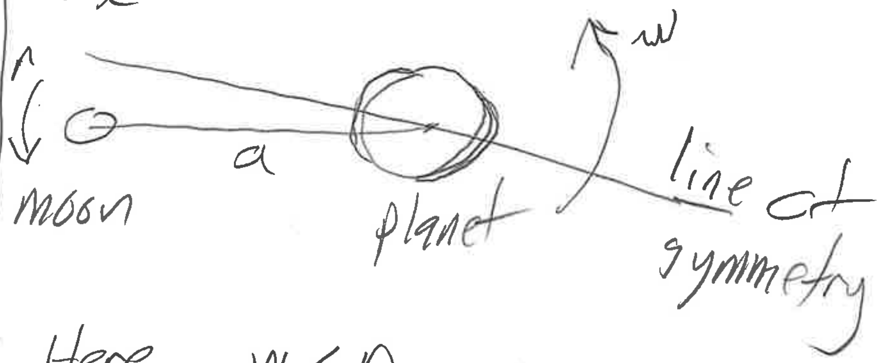
Now $\frac{dE}{dt} = \vec{F} \cdot \vec{v} > 0$

$\Rightarrow E \uparrow$ and from $E = \frac{G M_m M_E}{-2a}$

$a \uparrow$ too

The moon evolves outward

case #2



Here $w < n$

Now $\vec{F} \cdot \vec{v} < 0$, and the moon loses energy and evolves inward

Finally if the moon orbits retrograde

case #3

Here w and n are in opposite directions

$\vec{F} \cdot \vec{v} < 0$, and the moon evolves inward

So all retrograde moons evolve inward as do nearby moons, specifically those whose orbital periods are shorter than the planet's spin period.

b) <u>Two moons evolving inward?</u>

Phobos (orbits faster than Mars spins)

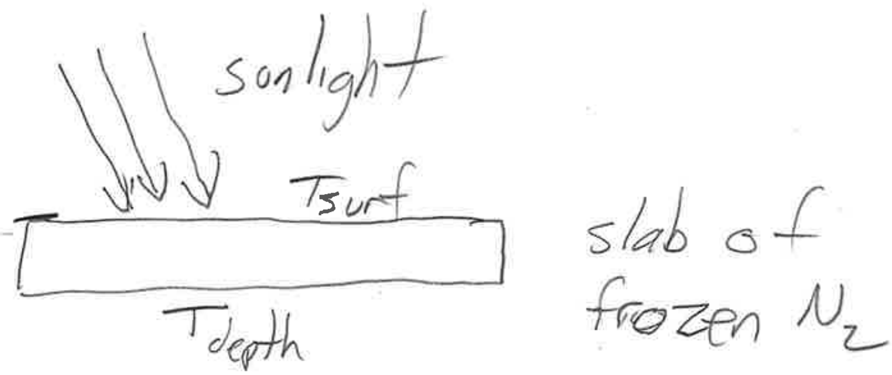
Triton (orbits Neptune retrograde)

Other small moons of the giant planet like Thebe and Adrastea at Jupiter. Distant retrograde moons evolve inward too, but over trillion year timescales. Tides are strongest for big close moons.

Problem 10-6

Geysers on Triton
solid state greenhouse

(Draw a picture!)



The idea is that if $T_{depth} > T_{surf}$ the N_2 can vaporize at depth. The gas then rises explosively since it is less dense than its surroundings. If the pressure is large enough, liquid N_2 is possible too. We'll return to this possibility. But first, how can we get $T_{depth} > T_{surf}$?

1) The N_2 slab must be transparent to sunlight so that the sunlight can deposit its energy at depth.

2) The N_2 slab must be opaque to thermal infrared generated at $T \sim 70$ K so that the energy cannot escape by radiation.

Just like CO_2 in our atmosphere! And like Earth's atmosphere, the slab need not block all outgoing IR, just most of it.

3) Losses by thermal conduction and IR radiation must be slow so that heat builds up and T↑.

What would it take to get liquid N_2? Liquids can exist only if the pressure is above the triple point at which gases, liquids, and solids can coexist.

Nitrogen	Triton
$P_3 = 0.124$ Atm. } triple point	$g = 0.78\ m/s^2$
$T_3 = 63K$	$T_{surf} = 38K$
$\rho = 0.85\ g/cm^3$	

So we have to be deep enough before liquid N_2 can form. How deep?

$$P_3 = \rho g h$$

← pressure has units of energy density

$$(0.124)(10^5) = (850)(0.78)h$$

↑ convert to pa ↑ convert to kg/m³

$$\Rightarrow \quad h = 18.7 \text{ m}$$

So we would need ≥18.7 m of clear N_2 so that sunlight could be absorbed deep enough for liquid N_2 to form. <u>Unlikely</u>.

Frozen H_2O can be clear → ice
or white ⇒ snow

Frozen N_2 probably behaves similarly

Triton geysers likely powered by N_2 gas

Problem 10-4

Methane on Titan

a) Flux of Lyman-α UV at Earth

$\sim 1 * 10^{16}$ photons/$m^2 s$

Titan
9.5 AU
$R = 2575$ km

$\Rightarrow \frac{10^{16}}{9.5^2} =$ **$F = 1.1 * 10^{14}$ ph/$m^2 s$ at Titan**

photons intercepted by Titan:

$\pi R^2 \cdot F = \pi (2575000)^2 (1.1 * 10^{14})$

$= 22.9 * 10^{26}$

$= 2.3 * 10^{27}$ photons/s

(πR^2 ← Titan cross section)

photons per m^2 of Titan

$\frac{\text{previous answer}}{4 \pi R^2} = \frac{F}{4} =$ **$2.8 * 10^{13}$ ph/$m^2 s$**

($4\pi R^2$ ← Titan surface Area)

b) $CH_4 + UV \rightarrow CH_3 + H$

$CH_3 + CH_3 \rightarrow C_2H_6$; $H + H \rightarrow H_2$

<u>H_2 escape rate</u>

1 H_2 for every 2 Ly-α photons

$\Rightarrow 1.4 \times 10^{13}$ H_2 escapes/$m^2 s$

<u>CH_4 destruction rate</u>

1 CH_4 for every 1 Ly-α photons

$\Rightarrow 2.8 \times 10^{13}$ CH_4 created/$m^2 s$

<u>C_2H_6 creation rate</u>

1 C_2H_6 for every 2 Ly-α photons

$\Rightarrow 1.4 \times 10^{13}$ C_2H_6 created/$m^2 s$

c) <u>How long to deplete CH_4?</u>

weight of a $1 m^2$ column of atmosphere

$P = 1.5 \text{ bars} = N \mu_a g$

$g = 1.4 \, m/s^2$

$\mu_a = 28 \frac{g}{mole} = 0.028 \, kg/mole$

$$N = \frac{1.5 \times 10^5 \, pa}{0.028 \cdot 1.4 \, m/s^2} = 3.82 \times 10^6 \text{ moles}$$

Avagadro's Number is 6.023×10^{23}
(# molecules per mole)

so a m^2 column of Titan's atmosphere has $(3.8 \times 10^6)(6.023 \times 10^{23})$
$= 2.3 \times 10^{30}$ molecules

5% are methane ⇒ 1.1×10^{29} CH_4 molecules

$$\text{Lifetime} = \frac{N}{\dot{N}} = \frac{1.1 \times 10^{29}}{2.8 \times 10^{13}} = 3.9 \times 10^{15} \text{ s}$$

$= 1.2 \times 10^8$ years

short compared to age of Titan!

⇒ methane must be replenished

d) How many moles of ethane accumulate in this time?

3.8×10^6 moles, 5% methane

$\Rightarrow 1.9 \times 10^5$ moles of methane

2 to make an ethane

$\Rightarrow 9.6 \times 10^4$ moles of ethane

per m^2, of course

e) In 1.2×10^8 yrs,

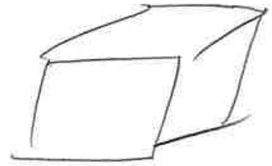
9.6×10^4 moles of ethane, $\rho = \frac{550 kg}{m^3}$

mass ethane $= 9.6 \times 10^4$ (C_2H_6 molecular weight)

$= 2.88 \times 10^6$ g

$= 2.88 \times 10^3$ kg

2C = 24
6H = 6
C_2H_6 = 30

$M = \rho V$ 

$= \rho A h$

$2880\ kg = \frac{550\ kg}{m^3} \cdot 1 m^2 \cdot h$

so $h = 5.23 m$ (in $1.2 * 10^8$ yrs)

Age of SS is $\frac{4.6 * 10^9}{1.2 * 10^8} = 38 \times$ longer

$\Rightarrow h = 200 m$ (in 4.6 Gyr.)

f) From google:

Boiling Point $CH_4 = 112 K$

$C_2H_6 = 185 K$

Freezing Point $CH_4 = 91 K$

$C_2H_6 = 90 K$

Temperature on Titan 90-94 K

9) Huygens Titan probe was designed to float. why?

The above calculation implies several hundred meters of accumulated liquid ethane ⇒ a global ocean.

But only lakes were found – why?

This is not fully understood! Relatively high freezing points imply that liquids can solidify at Titan temperatures. Further chemistry likely occurs with ethane, removing those molecules. Photochemical hazes may block UV light, protecting some methane in the atmosphere. This remains an active area of research!