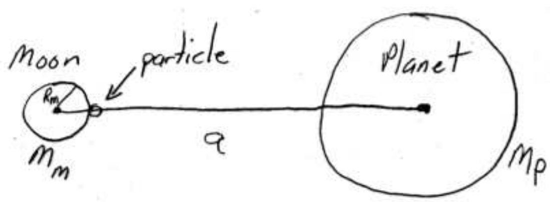
The Roche Limit

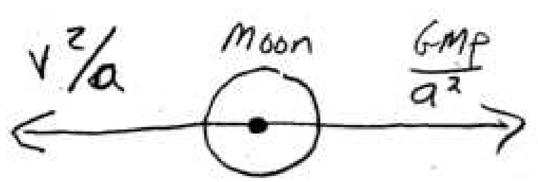
Most ring systems are found close to planets, often surrounded by small moonlets. The accretion of these objects into larger moons has been inhibited by tidal forces from the planet. The Roche limit is the distance from a planet within which accretion of material into large moons is prevented by planetary tides. In addition, a strengthless object wandering into the Roche zone will be torn apart as comet Shoemaker-Levy 9 was during its first close approach to Jupiter in 1992.

We can estimate the size of the Roche limit by considering the forces acting on a small test particle on the surface of a moon orbiting a planet. We assume that the moon is not rotating (its rotation is a small effect). Furthermore, real moons will deform under the tidal influence of the planet depending on their physical strength. We ignore this effect initially, but return to it at the end of this calculation.



We work in a reference frame centered on the planet that rotates with the average angular speed, n, of the moon on its uninclined, circular orbit of radius a around the planet. To use this reference frame, the planet's mass must be much greater than the moon's mass ($M_p >> M_m$) so that the center of mass of the system is nearly at the planet's center. First balance forces for the moon:

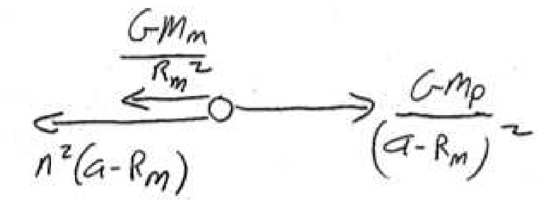
Outward Centrifugal Force = Inward Gravitational Force



$$\frac{v^2}{a} = n^2 a = \frac{GM_p}{a^2}$$
$$n^2 = \frac{GM_p}{a^3}$$

Here v is the circular velocity, G is the Gravitational constant, p and m subscripts refer to the planet and moon, respectively. The average angular rate, n, is just 2π divided by the Kepler period. Now balance forces for the particle.

Outward Centrifugal Force + Moon Gravity = Planet Gravity



$$n^2(a-R_m)+\frac{GM_m}{R_m^2}=\frac{GM_p}{(a-R_m)^2}$$
$$\frac{GM_p}{a^3}(a-R_m)+\frac{GM_m}{R_m^2}=\frac{GM_p}{a^2(1-\frac{R_m}{a})^2}$$

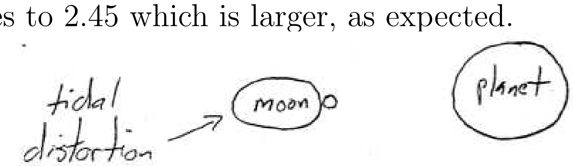
Now we Taylor expand the final term taking advantage of the fact that $a >> R_m$.

$$\frac{GM_p}{a^2}-\frac{GM_pR_m}{a^3}+\frac{GM_m}{R_m^2}=\frac{GM_p}{a^2}+\frac{2GM_pR_m}{a^3}$$
$$\frac{GM_m}{R_m^2}=\frac{3GM_pR_m}{a^3}$$
$$a^3=3\frac{M_p}{M_m}R_m^3$$

Now write M_p and M_m in terms of R_p and R_m.

$$a^3=3\Big(\frac{R_p^3\rho_p}{R_m^3\rho_m}\Big)R_m^3$$
$$a=R_p\Big(3\frac{\rho_p}{\rho_m}\Big)^{1/3}=1.45R_p\Big(\frac{\rho_p}{\rho_m}\Big)^{1/3}$$

So for a moon with the same density as its parent planet, tidal forces will pull material off the moon's surface if the moon orbits within about 1.5 planetary radii. Note that we have left the moon's spin rate and its tidal deformation out of the calculation. If we had included the effects of tidal distortion of the moon, the Roche distance should be larger. Why? Tides distort the moon along the planet-moon axis. An object at the surface of the moon along this axis is now further for the moon's center and hence feels a weaker binding force. When the calculation is done for a fluid, perfectly-deformable moon, the coefficient 1.45 changes to 2.45 which is larger, as expected.



Planetary Calculator

Results in SI units

Planet	2.456*(rho/2000)^(1/3)
MERCURY	3.42624415
VENUS	3.38795888
EARTH	3.44507004
MARS	3.0814188
JUPITER	2.1437242
SATURN	1.72253502
URANUS	2.1220141
NEPTUNE	2.29879183
PLUTO	2.4682191

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