but also offers ways to control and handle the quasiparticles characteristic of the various states of matter that can be realized in solids.

That said, only a few experimental facilities will have the combination of technologies required to study quasiparticles that have fractional charges, or vortex–antivortex annihilation in two dimensions. But for those that do, Langer and colleagues' approach can be readily applied to investigate the properties of polarons in strontium titanate or other transition-metal oxides, or the 'heavy electrons' that occur in several materials owing to the coupling of mobile electrons to fluctuations of magnetic polarization[8–10]. According to some schools of thought, the quasiparticle concept does not apply in certain materials or under special conditions[11]. Collision experiments might therefore help to identify the boundaries of the quasiparticle concept. ■

Dirk van der Marel *is in the Department of Quantum Matter Physics, University of Geneva, CH-1211 Geneva 4, Switzerland.*
e-mail: dirk.vandermarel@unige.ch

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CELESTIAL MECHANICS

Fresh solutions to the four-body problem

Describing the motion of three or more bodies under the influence of gravity is one of the toughest problems in astronomy. The report of solutions to a large subclass of the four-body problem is truly remarkable.

DOUGLAS P. HAMILTON

The study of the orbital motions of bodies that are subject to their mutual gravitational attractions is crucial for understanding the movements of moons, planets and stars, and for navigating spacecraft to distant planets. The central problem is to determine the motions of n point masses interacting through gravitational forces that vary with the inverse square of their separation distances. This n-body problem is famous among astronomers and mathematicians, and is known to have no general analytical solution (that is, no solution that can be written down in terms of simple mathematical functions). Nevertheless, specific solutions have been eagerly sought and occasionally discovered. Writing in *Celestial Mechanics and Dynamical Astronomy*, Érdi and Czirják[1] report analytical solutions for a broad class of four-body configurations.

Isaac Newton solved the two-body problem in his 1687 masterwork, the *Principia*, but the three-body problem proved surprisingly complex and occupied many distinguished mathematicians over the next two centuries. Leonhard Euler and Joseph-Louis Lagrange found all analytical solutions to an important subclass of the three-body problem known as central configurations, but work by Heinrich Bruns and by Henri Poincaré in the late 1880s showed that a general arrangement of three or more bodies admits no analytical solution. Although the set of all possible central configurations of four bodies remains unknown, Érdi and Czirják have taken a large stride forward by solving all of those in which two of the bodies lie along an axis of symmetry.

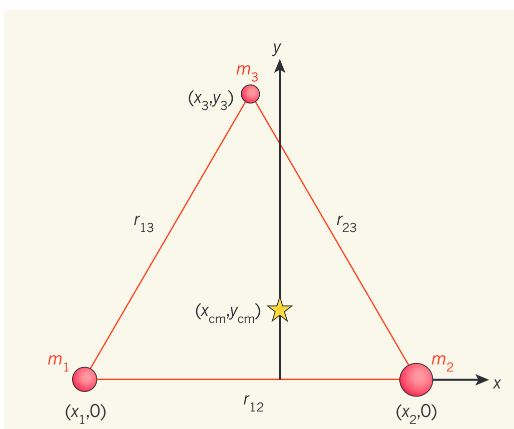

Figure 1 | A subclass of the three-body problem. The motions of three bodies with masses m_1, m_2 and m_3 under the influence of gravitational forces can be described analytically — that is, in terms of simple mathematical functions — for the special case in which the bodies are placed at the vertices of an equilateral triangle. Proof of this involves considering the accelerations in the y direction of two masses placed on the x axis. It emerges that the bodies orbit in such a way that the triangle rotates, expands or shrinks, and always remains in the xy plane. The yellow star represents the centre of mass of the three-body system. Distances between the masses are represented by the symbol r, with subscripts representing the masses; coordinates for masses and for the centres of mass are given as (x,y) pairs.

In central configurations, each body must be subject to an acceleration directed towards the centre of mass of the system with a magnitude that is proportional to its distance from the centre of mass. All orbits of two bodies are central configurations, in which the objects each orbit their common centre of mass along ellipses that have identical shapes and orbital periods. Euler's solutions for linear arrangements of three bodies are also central configurations, as are Lagrange's solutions in which the three masses are placed at the vertices of an equilateral triangle. In the latter system, Lagrange showed that the vertices of the triangle can move in such a way as to preserve relative distances between the masses; the triangle can rotate around the centre of mass, expand or shrink, but must remain in its initial plane.

It is easy to show that equilateral triangles are the only possible planar central configuration of a three-body system by placing two of the masses (m_1 and m_2) on an x axis, and considering their accelerations in the perpendicular y direction (Fig. 1). The y acceleration on m_1 is due solely to the gravity of the third mass (m_3) and equals Gm_3y_3/r_{13}^3 — where G is the gravitational constant, y_3 is the coordinate of m_3 along the y axis and r_{13} is the distance between m_1 and m_3. The corresponding y acceleration on m_2 is Gm_3y_3/r_{23}^3. According to the definition of central configurations, these accelerations must separately equal λy_{cm} (where y_{cm} is the y coordinate of the centre of mass and λ is the common proportionality constant). By cancelling like terms in the two y accelerations, it immediately becomes apparent that r_{13} must be the same as r_{23}.

If the symmetry of the equilateral-triangle system is then exploited by choosing a new x axis to run along the line connecting m_1 and m_3, repeating the above argument shows that r_{12} must also be the same as r_{23}, and thus all three sides of the triangle must be equal in length. This proof extrapolates directly to four bodies: the only fully three-dimensional central configurations for

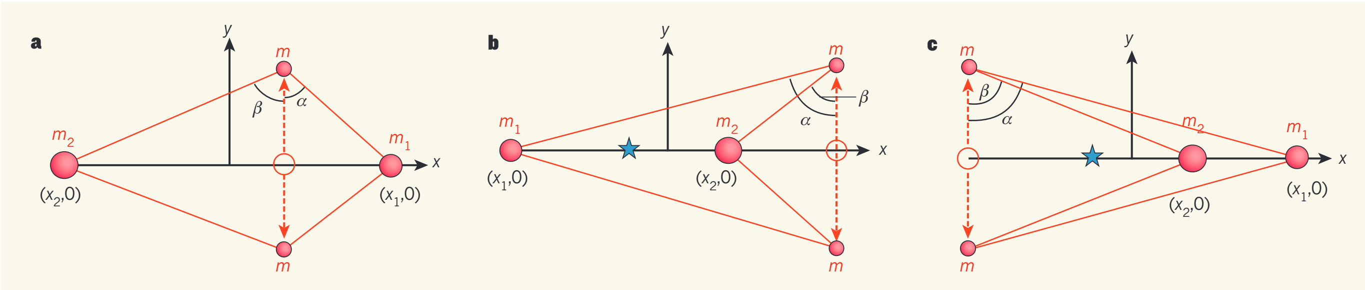

Figure 2 | A subclass of the four-body problem. Érdi and Czirják[1] solved a subclass of the four-body problem derived from systems of three masses initially located on the *x* axis. **a**–**c**, In each case, two masses (m_1 and m_2) are left on the *x* axis; the third mass (original position shown with open circle) is split into two equal halves, each of mass *m*, which are moved symmetrically in the *y* direction (dashed arrows). The centre of mass of each system is at (0,0) and the masses form convex (**a**) or concave (**b** and **c**) polygons when connected. The centre of mass excluding m_2 is shown with a blue star in **b** and **c**; its location inside or outside the polygon distinguishes the two cases. The positions of m_1 and m_2 were defined using rectangular coordinates, whereas those of the two off-axis masses were fixed by the angles α and β.

four bodies are those in which the masses are placed on the vertices of a tetrahedron such that all distances between the masses are equal.

But two-dimensional central solutions of four bodies are much more difficult to identify. In their seminal work, Érdi and Czirják find three examples of such solutions, which can be visualized by considering a system of three masses distributed on a line. Each solution is found by splitting one of the masses into equal halves and moving the fragments up and down so that the resulting distribution of four masses is symmetric about the *x* axis (Fig. 2). The four-sided polygon formed by connecting the on-axis masses to the off-axis masses is convex when the central mass is split (Fig. 2a), and concave when one of the other masses is split (Fig. 2b,c). Érdi and Czirják's two concave cases differ according to whether the centre of mass of the system excluding m_2 is enclosed by the polygon or not.

The next step would normally be to specify the masses and then to seek all arrangements of those masses that satisfy the conditions for a central configuration. Érdi and Czirják, however, chose to tackle the inverse problem: given the positions of the bodies, they computed the masses that make the configuration central. And, rather than working with rectangular coordinates for the two off-axis masses, the authors chose to recast the problem in terms of a pair of angles that fix the position of those masses relative to the ones on the *x* axis (Fig. 2). These are both inspired choices that make the problem analytically tractable. If one or more of the four masses is set to zero, the angles take on values that are consistent with straight lines and equilateral triangles; in this way the four-body edifice of Érdi and Czirják's work is rooted in the three-body bedrock of Euler and Lagrange.

Central configurations are dynamic equilibria that can be stable (such as a ball at the bottom of a smooth bowl) or unstable (as for a ball perched atop a round hill). Euler's straight-line configurations are all unstable so that, like the ball on the hill, the configuration cannot persist when tweaked. Lagrange's equilateral-triangle solutions are stable if one of the three masses contains more than about 96% of the total mass of the system, but unstable if the mass is more evenly distributed. Thus, Lagrangian configurations for a system that incorporates the Sun, Jupiter and a suitably placed asteroid are stable, as would be those for Earth, the Moon and a modestly sized future space station. By contrast, Pluto's massive moon Charon prevents any central configurations involving these bodies and a smaller moon from being stable. Whether the new four-body central configurations are stable is an interesting, unexplored question and is an inviting direction for future research.

Érdi and Czirják's solution to a large subclass of the central four-body problem is a major advance that encompasses and greatly extends many previous four-body results, including: arrangements of four[2] and three[3] identical masses; kite-shaped configurations of diagonally opposite pairs of equal masses[4]; and the limiting case of three bodies plus a massless test particle[5]. Just as three-body configurations serve as limiting cases for Érdi and Czirják's four-body configurations, the authors' solutions could, in turn, be used as limiting cases for ambitious future extensions of the *n*-body problem: perhaps three masses along a line plus two symmetrically placed equal masses; a test particle plus planar configurations of the type considered in the present work; or even planar arrangements of four different masses. ■

Douglas P. Hamilton *is in the Department of Astronomy, University of Maryland, College Park, Maryland 20742-2421, USA.*
e-mail: dphamil@astro.umd.edu

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NEUROBIOLOGY

Wired for sex

Analysis of a sensory neural circuit in the roundworm *Caenorhabditis elegans* reveals that its wiring is sex-specific, and arises through the elimination of connections that are originally formed in both sexes. **See Article p.206**

DOUGLAS S. PORTMAN

If presented with a human brain, even the most meticulous neuroanatomist would be hard-pressed to identify the sex of its former owner. There are clearly male–female differences in some brain regions, but these can be subtle and variable, and their causes and consequences remain largely unclear. Over the past five years, work in several organisms[1–3] has suggested that altered neural connectivity between brain regions might be a hallmark of male–female differences. On page 206 of this issue, Oren-Suissa *et al.*[4] provide clear evidence for sex differences in neural wiring in the roundworm *Caenorhabditis elegans*. Moreover, they report that these differences arise through sex-specific eradication of neural connections and are controlled by the genetic sex of the nervous system itself.

In *C. elegans*, males are males, but females are properly called hermaphrodites. They