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LORENTZ AND GRAVITATIONAL RESONANCES ON CIRCUMPLANETARY PARTICLES

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ABSTRACT

Micron-sized circumplanetary dust particles are subject to various non-gravitational perturbations, principally solar radiation pressure and electromagnetic forces, which are typically a few percent as strong as the planetary gravity. Individually, these perturbations can cause some orbital evolution, but when the perturbations act in concert the excursions can be much larger. We demonstrate this effect for a single example, the coupling between resonances and drag forces. Throughout this work, we emphasize the parallels between satellite-satellite gravitational resonances and their electromagnetic counterparts (Lorentz resonances).

INTRODUCTION

A dynamical system typically has a set of natural frequencies at which it can rotate or vibrate. When such a system is forced at one of these natural frequencies (or a multiple of it), the amplitude of oscillations grows as a result of the cumulative effect of in-phase perturbations; the system is said to be in resonance. A child on a swing provides a familiar example of a resonant system. If the swing (initially at rest) is pushed at an arbitrary frequency or at random times, the amplitude of oscillation is likely to remain small; if, however, the swing is pushed once per period, the oscillation amplitude will grow quite large. In an entirely similar manner, charged dust grains oscillate wildly near the locations of "Lorentz resonances" which occur at those positions where the electromagnetic force sensed by an orbiting particle (and arising from a planet's spinning magnetic field) has a component that matches a natural frequency of the orbit /1/. The abrupt vertical expansion of the jovian ring into its halo and the disappearance of the halo itself /1,2/ have been ascribed to the action of these Lorentz resonances on orbiting dust grains.

Gravitational resonances occur when the orbital periods of two objects are nearly a simple ratio of integers. Many features in the main saturnian ring system have been successfully attributed to gravitational resonances with exterior satellites. For example, the 2:1 resonance with Mimas defines the inner edge of the Cassini division, which divides the A and B rings, while the sharp outer edge of the A ring occurs at a 7:6 resonance with the moon Janus. Satellites themselves are often found in resonances with one another; examples include the saturnian pairs Enceladus/Dione, Titan/Hyperion and Mimas/Tethys, as well as the jovian triple Io/Europa/Ganymede (see /3/ for a qualitative physical description of these gravitational resonances).

In this paper we wish to illustrate how resonances couple with drag forces. This idea is not new; indeed it has been extensively studied in the context of satellite evolution where tidal effects from the central body create small drags on satellite orbits. This problem has been thoroughly treated using Hamiltonian mechanics (see *e.g.,* /4/). The purpose of the current paper is twofold. First, we wish to draw parallels between the extensively studied satellite (gravitational) resonances and their less well known relatives, Lorentz resonances. Secondly, we will reproduce some results of the Hamiltonian theory using the Lagrangian orbital perturbation equations /5/, which are written in terms of the orbital elements. The latter quantities provide a physically meaningful description of an orbit; for orbits confined to a particular plane, the semimajor axis a, the eccentricity e, and the longitude