of pericenter $\tilde{\omega}$ are sufficient. These three elements, respectively, describe the instantaneous size, shape, and orientation of an elliptical orbit; the Lagrangian equations that describe the time rate of change of such orbital elements are well suited to visualizing the results of orbital perturbations. The advantage of our approach is its simplicity: many non-intuitive effects of resonances, such as resonant trapping and jumps, will be elucidated.

RESONANCE EQUATIONS

The problem of determining the perturbing effects of one satellite on another is fundamental to celestial mechanics and has been studied for centuries. It is not solvable in closed form, but an approximate solution can be developed as a power series of small quantities. The typical procedure (*cf.* /5/, p. 339) is as follows. First, one evaluates the disturbing function, defined as the negative of the perturbing satellite's potential, at the position of the perturbed particle. Next, the disturbing function is written in terms of the orbital elements; this step requires complicated power series expansions in eccentricities, inclinations, and the semimajor axis ratio. Finally the changes to the orbital elements can be calculated with the potential form of Lagrange's planetary equations (/5/, p. 336) which relate the time rates of change of the orbital elements to derivatives of the disturbing function and to instantaneous values of the elements themselves.

We proceed in a similar manner for Lorentz resonances. Because the Lorentz force due to a magnetic field cannot be derived from a potential, we must calculate the electromagnetic force arising from an arbitrary magnetic field and express it in terms of orbital elements, an arduous task which requires power series expansions in the particle's eccentricity and inclination. These forces are then inserted into an alternate form of Lagrange's planetary equations (/5/, p. 327). The results of this calculation yield, as above, expressions for time derivatives of the orbital elements which are functions of the instantaneous values of these elements. We plan to submit the details of this calculation for publication in *Icarus*.

In both of the above derivations, secular terms (*i.e.*, those that do not depend on satellite longitudes) as well as periodic terms (with longitude dependence) appear. Secular terms are ubiquitous, whereas periodic terms, over long times, average to zero at all but a few resonant locations. In this paper we focus on one of these locations as an example: the 2:1 (first-order) eccentricity resonance. Near this location, the resonant argument ϕ is given by:

$$\phi = \lambda - 2\lambda' + \tilde{\omega}, \tag{1}$$

where λ and λ' are the longitudes of the perturbee and perturber, respectively. At the resonant location (defined by $\phi = 0$ - see figure 1), the perturbed body completes approximately two orbits for every one cycle of the perturbing force (the period of an exterior satellite in the gravitational case or the planetary spin period for Lorentz resonances). We ignore all periodic terms with different frequency dependencies (since they average to zero), and the secular perturbations (which are small compared to the strong 2:1 resonant terms).

The orbital elements most strongly affected by such a resonance are the abovementioned a, e, and $\tilde{\omega}$. Instead of the semimajor axis a, we use the unperturbed orbital mean motion $n \sim \dot{\lambda}$, which is related to the semimajor axis via $n^2a^3 = GM_p$, where G is the gravitational constant and M_p is the planetary mass (/5/, p. 131). Writing out the Lagrange perturbation equations to lowest order in eccentricity and inclination, we find that the effects of both the gravitational and Lorentz versions of the 2:1 first-order eccentricity resonance can be represented by a set of equations of the following form:

$$\frac{dn}{dt} = -3en^2\beta \sin\phi \tag{2a}$$