DRAG FORCES

Several drag forces operate in the magnetospheres of the giant planets. Most large satellites are driven slowly outward by tidal forces from the primary while small particles are affected by a host of processes /6/ including plasma, atmospheric, and Poynting-Robertson drags which, for dust grains, operate much more rapidly than tidal evolution. Because drag forces are typically much smaller than many other orbital perturbations, their effects on most orbital elements can often be ignored. Unlike most other perturbations, however, drag forces systematically affect an orbit's energy and therefore its size and mean motion. Furthermore, because of the limited radial extent of the resonance zone, we can approximate the functional form of the drag rate in this region by a simple constant $\dot{n}_{drag}$. The inclusion of drag forces requires that we replace equation (2a) with

$$\frac{dn}{dt} = -3en^2\beta\sin\phi + \dot{n}_{drag}. \tag{3}$$

RESONANCE TRAPPING

When $\dot{n}_{drag} < 0$, orbits evolve outward: near the 2:1 resonance, this evolution is toward the perturbing satellite (in the case of gravity) or toward synchronous orbit (in the Lorentz case). For this type of evolution, resonance trapping, in which the evolution in mean motion ceases, is possible (figure 1). Clearly trapping can occur only if the first term in equation (3) is equal and opposite to the second for some ϕ. Solving equation (3) for $\sin\phi$ in this case and substituting into equation (2b), we find

$$\left.\frac{de}{dt}\right|_{trapped} = \frac{-\dot{n}_{drag}A_1}{3ne}, \tag{4}$$

which is easily integrated yielding:

$$e = \left(e_0^2 - \frac{2\dot{n}_{drag}tA_1}{3n}\right)^{1/2}. \tag{5}$$

Linearizing equations (2a-c) around this solution, we find that it is stable against small perturbations. Note the remarkable fact that the rate of growth of the eccentricity given by equation (5) is independent of the resonance strength β. This result can also be obtained from equation (7) below, which expresses the conservation of energy in a rotating reference frame (see /2/). Thus the "square root growth" in time (equation 5) is a property shared by gravitational and Lorentz resonances of all types and orders. An example of resonant trapping and the associated eccentricity growth is shown in figure 2; for the parameters given in the figure caption, equation (5) reduces to $e \sim 0.00145N^{1/2}$ (N is the number of perturber orbits) in rough agreement with the figure. This behavior holds until $e \sim 0.5$ at which time higher-order effects become important.

JUMPS AT RESONANCE

When $\dot{n}_{drag} > 0$, inner orbits evolve away from the perturbing satellite (or from synchronous orbit). In this case trapping for low eccentricities is not possible as can be seen from equation (5) which implies that eccentricity becomes imaginary! Instead we shall find a different behavior at the resonant location.

Because drag forces are so small, the first term in equation (3) is usually far greater than the second; this fact allows us to obtain an adiabatic invariant. Ignoring the drag term for the moment, we divide equation (2a) by equation (2b) and find