$$\frac{dn}{de} = \frac{3en}{A_1}, \tag{6}$$

which can be integrated to yield

$$\ln\left(\frac{n}{n_*}\right) = \frac{3e^2}{2A_2} \tag{7}$$

where n_* is an integration constant. Recalling that equation (2a-c) are accurate to only first order in eccentricity, we solve this equation to lowest order in e and find that

$$n_* = n\left[1 - \frac{3e^2}{2A_1}\right] \tag{8}$$

is a conserved constant of the motion (see /7/). Since the resonance zone is traversed quickly, equation (8) remains approximately constant during the passage. The half-width of the librating zone, $dn/2$, can be crudely estimated by setting the derivative of equation (1) equal to zero, taking $n = 2n' + dn/2$ and $\cos\phi = 1$, and solving for dn. We find $dn \approx 2n\beta A_2/e$. Inserting this into equation (6), and neglecting the difference between de and e, we find:

$$de = \left(\frac{2A_1A_2\beta}{3}\right)^{1/3}. \tag{9}$$

This case is displayed in figure 3; using the parameters from the figure caption, we calculate the jump amplitudes from equations (9) and (6) and obtain $de \approx 0.04$ and $dn \approx 0.012$ - values smaller than, but in rough agreement with, the figure.

DISCUSSION

Lorentz and gravitational resonances differ primarily in the magnitudes of the resonant strength β. For micron-sized dust grains around the jovian planets, β is orders of magnitude larger in the Lorentz case; thus Lorentz resonances are more effective at trapping dust particles and are able to induce larger orbital jumps than resonances due to a satellite's gravity. Slight additional differences arise when $A_i \neq 1$; most first-order Lorentz resonances have $A_i < 1$ which reduces the trapped growth rate (equation 5) and jump amplitude (equation 9). Despite this small difference between the two types of resonances, the equations that govern them are remarkably similar and, consequently, it is not surprising that orbital behavior at Lorentz and gravitational resonances is so alike.

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