ASTR120 Challenge Problem #2 – (Hamilton)
Optional, due before Midterm #2

In this challenge problem you will work out how Mars' tilt and eccentricity affect its seasons.

a) For an untilted Earth on a circular orbit at 1 AU, work out the average energy hitting a square meter of surface perpendicular to the sunlight per second (your answer should be in $W/m^2 = J/m^2s$). Call your answer F_0 for the intercepted energy flux.

b) Now solve for F for different latitudes by accounting for the fact that the square meter of surface is tilted with respect to the incoming sunlight and hence intercepts a smaller fraction of this light. Give your answer in terms of F_0 (your answer to part a) and a trig function of the latitude λ. Since on an untilted Earth, all points of the surface would receive 12 hours of sunlight a day, the intensity of sunlight received is directly related to the temperature of the surface. Give values for noon for $\lambda = 0°, 30°$, $\lambda = 60°$, and $\lambda = 90°$ ignoring the absorbing effect of the atmosphere. Leave your answer in terms of F_0, Does your answer make sense?

c) For a planet on an orbit of eccentricity e, work out the maximum and minimum heating rates at apocenter and pericenter. Evaluate for $e = 0.1$ and $e = 0.2$ in terms of F_0.

d) For a planet tilted by an angle ϵ, the λ in your equation is replaced by $\lambda - \lambda_{sun}$, where $\lambda_{sun} = \sin(\epsilon)\sin(2\pi t/T)$, T =1 year is Earth's orbital period, and t is time. What is the date (month and day) corresponding to $t = 0$? Replace λ_{sun} with its maximum value, and recalculate your answer to b) for a tilt $\epsilon = 25°$. Again, leave all answers in terms of F_0. This gives the heating in mid summer. Repeat for mid winter. Make a large table of your results for an untilted planet, tilted planet in summer, tilted planet in winter, and the average of the summer and winter values.

e) Use your answer to part d) to determine which effect is most important for the different latitudes on Mars ($e \sim 0.1, \epsilon \sim 25°$). Compare the ratio of your fluxes: pericenter/apocenter to summer/winter. The effect with the bigger ratio controls the seasons!