

ASTR450 EQUATION SHEET

Basic Physics:

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v} \quad ; \quad \frac{d\mathbf{r}}{dt} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \quad ; \quad \frac{d^2\mathbf{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\frac{d^2\theta}{dt^2} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{\theta}}$$

$$\mathbf{F} = -\nabla V \quad ; \quad \frac{dE}{dt} = \int \mathbf{F} \cdot d\mathbf{v} \quad ; \quad \frac{d\mathbf{L}}{dt} = \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Central Force Motion:

$$\mathbf{F} = -f(\mathbf{r}, t)\hat{\mathbf{r}} \rightarrow \text{conserves } \mathbf{h} \quad ; \quad \mathbf{F} = -f(\mathbf{r})\hat{\mathbf{r}} \rightarrow \text{conserves } \mathbf{h} \text{ and } E$$

$$h = r^2\dot{\theta} \quad ; \quad \frac{v^2}{2} + V(\mathbf{r}) = C$$

$$\text{Stability of circular orbits: } 3f(a) + af'(a) > 0 \quad \text{or} \quad dh(a)/da > 0 \quad \text{or} \quad dC(a)/da > 0$$

Conics:

$$r = \frac{a(1 - e^2)}{1 + e \cos \nu} = a(1 - e \cos E) \quad ; \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$h = \sqrt{GMa(1 - e^2)} \quad ; \quad C = -\frac{GM}{2a} \quad ; \quad q = a(1 - e)$$

Anomalies:

$$M = E - e \sin E \quad ; \quad \tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

Jacobi's Integral:

$$C_J = \frac{G(M_1 + M_2)}{R^3} (x^2 + y^2) + \frac{2GM_1}{r_1} + \frac{2GM_2}{r_2} - v^2$$

Tisserand's Constant:

$$C_J \approx \frac{G(M_1 + M_2)}{a} + 2\frac{G(M_1 + M_2)}{R} \sqrt{\frac{a}{R}(1 - e^2)} \cos i$$

Meaning of the Orbital Elements

Angle	Measured From	Measured To
Longitude	Reference Direction	-
Argument	Ascending Node	-
Anomaly	Pericenter	-
Node	-	Ascending Node
Pericenter	-	Pericenter
True	-	Planet Location
Latitude	-	Planet Location
Mean	-	Planet Location*

* If the planet moved at a constant angular rate (its mean rate).

Symmetries:

(Orbits and Equations are Unaffected by the Following Transformations)

1. Change of Reference Direction:

Longitude Angles $\rightarrow$ Longitude Angles $+ \delta L$

2. Interchange of Pericenter and Apocenter:

$\mathbf{e} \rightarrow -\mathbf{e}$

Pericenter Angles $\rightarrow$ Pericenter Angles $+ \pi$

Anomalies $\rightarrow$ Anomalies $+ \pi$

3. Interchange of Ascending and Descending Node:

$i \rightarrow -i$

Node Angles $\rightarrow$ Node Angles $+ \pi$

Arguments $\rightarrow$ Arguments $+ \pi$

N-Body:

$$R_{ij} = G \left(\frac{1}{|\mathbf{r}_j - \mathbf{r}_i|} - \frac{\mathbf{r}_i \cdot \mathbf{r}_j}{r_i^3} \right) \quad ; \quad \nabla R_{ij} = G \left(\frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3} - \frac{\mathbf{r}_j}{r_j^3} \right)$$

$$\mathbf{P} = \sum_{i=1}^n m_i \mathbf{v}_i \quad ; \quad \mathbf{L} = \sum_{i=1}^n m_i \mathbf{r}_i \times \mathbf{v}_i \quad ; \quad E = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 - \sum_{i=2}^n \sum_{j=1}^{i-1} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Quote of the Semester:

“Doing ASTR450 homework assignments is like climbing Mt. Everest once a week.”