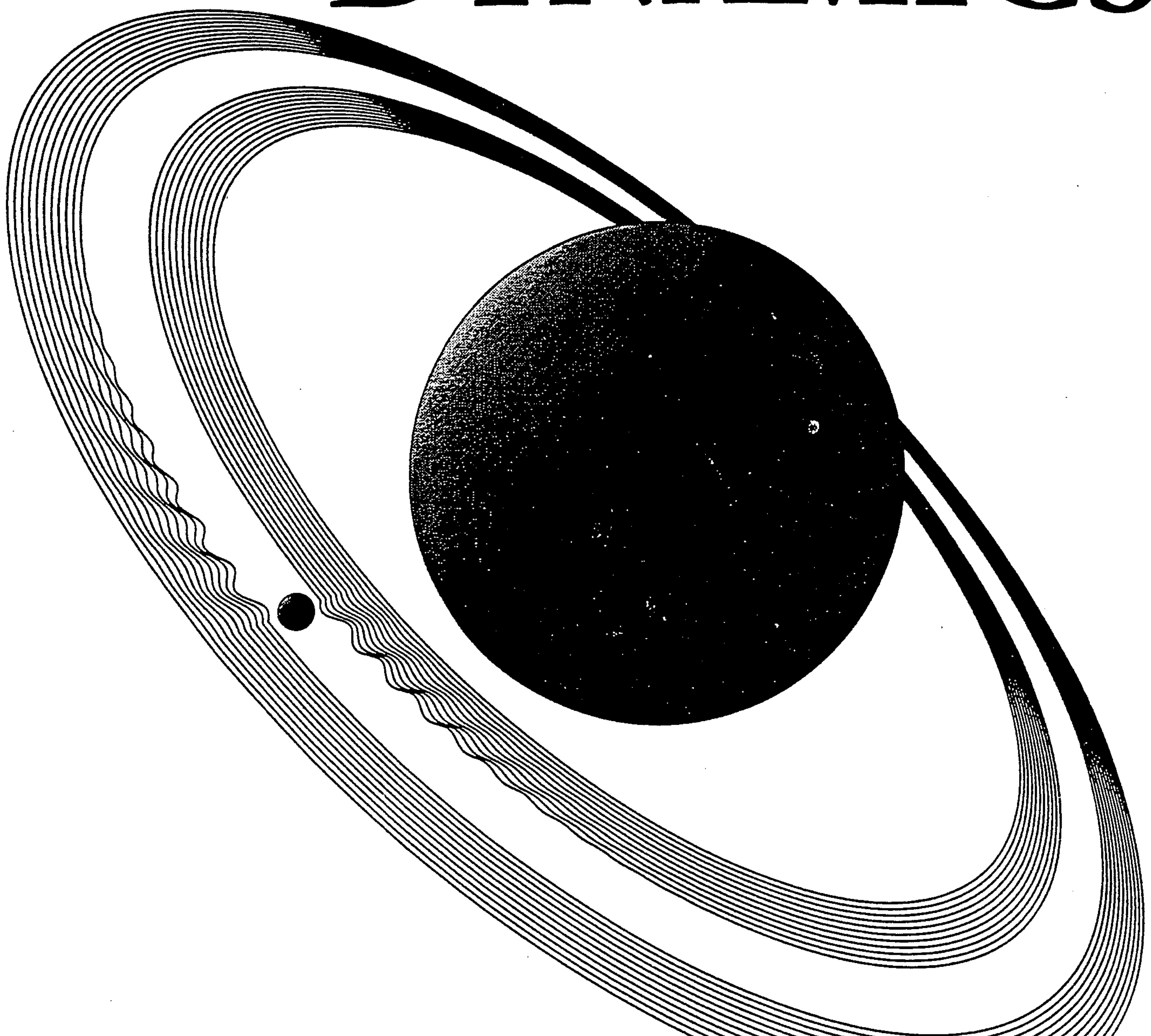
SOLAR SYSTEM DYNAMICS



C. D. Murray

and

S. F. Dermott

Perturbation Theory

- perturbing force is small compared to direct gravity.

⇒ orbital elements change slowly in time

- Arbitrary Perturbation = Secular Perturbations + Resonant Perturbations

Secular Perturbations ↑ do not depend on perturbee's longitude

Resonant Perturbations ↑ do depend on perturbee's longitude

Secular Perturbations:
- tidal evolution of the moon
- planetary perturbations
- oblate planet

Resonant Perturbations
- some moon-moon interactions

Secular Perturbations

Perturbations that do not depend on the position of the particle along its orbit or on the rotational state of the perturber.

Most figures in this packet are from a new book by Murray and Dermott. ([redacted]).

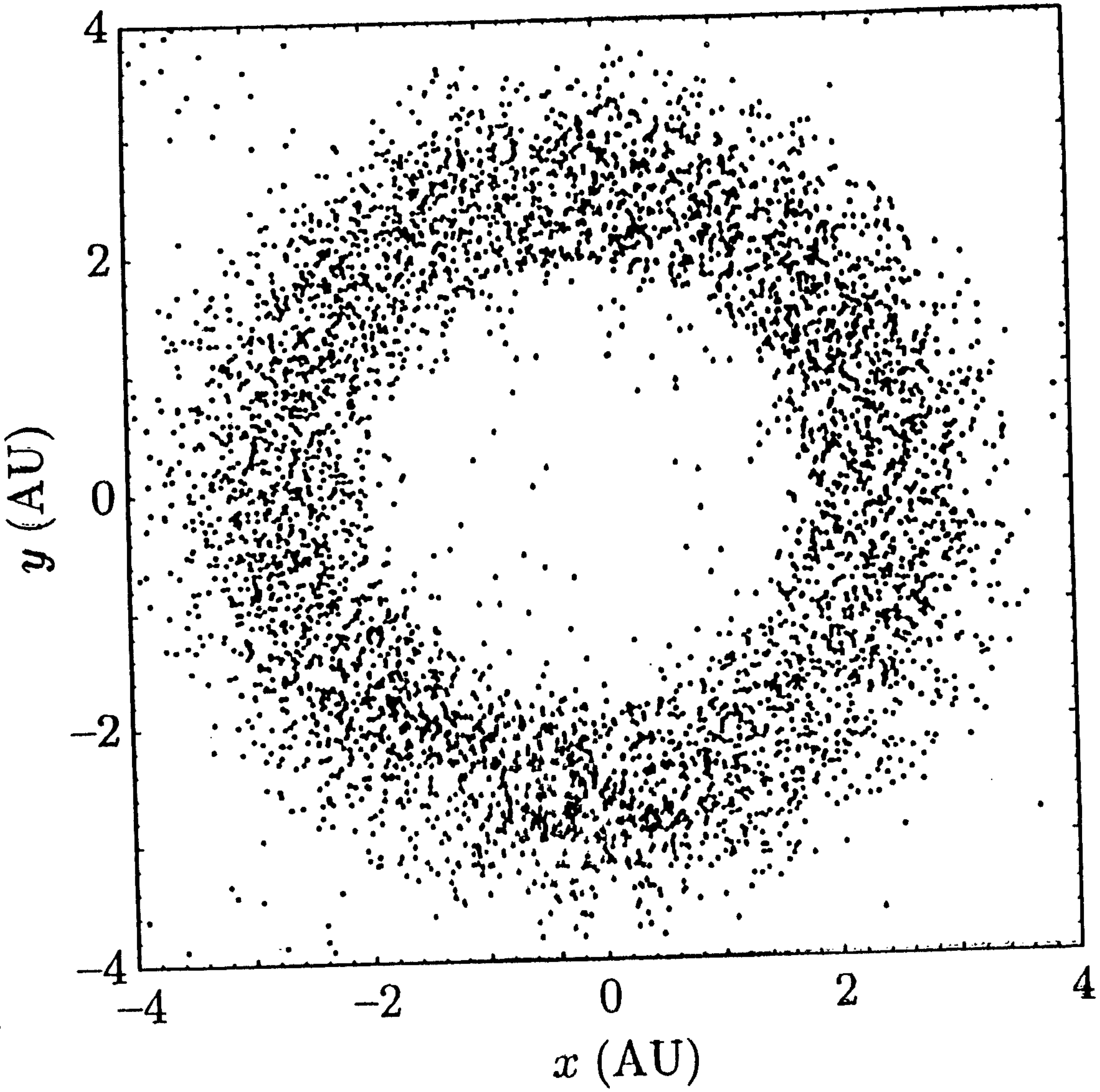


Figure 9.27 A plot of the x–y distribution of the known asteroids on 13 January 1993. Note the clear suggestion of a band of material corresponding to the main belt.

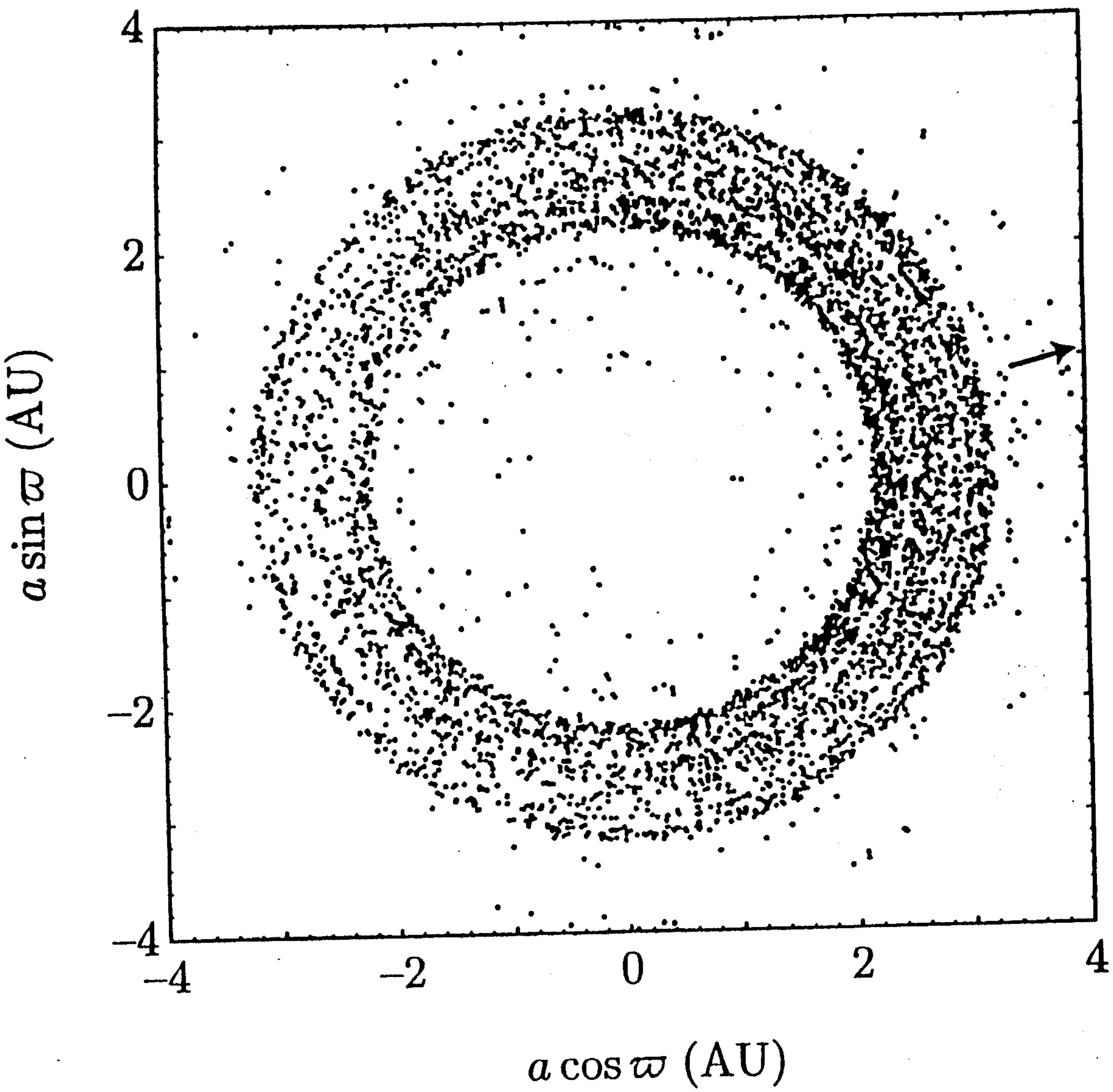


Figure 9.28 A plot of the $a\cos\varpi$–$a\sin\varpi$ distribution of the known asteroids using the same data used to derive Fig.9.27. Note the clear suggestion of a band of material corresponding to the main be'

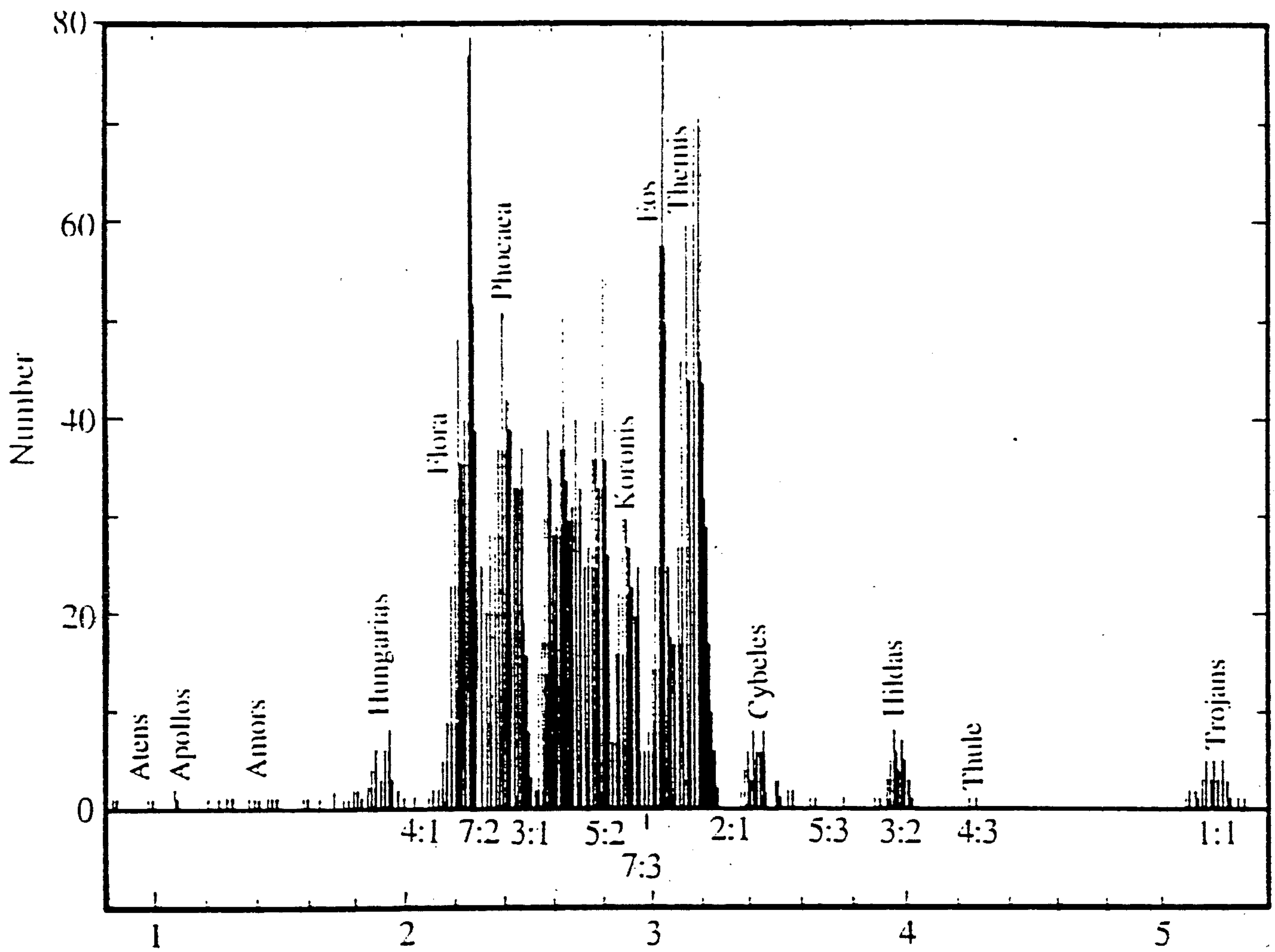


Figure VIII.19 Distribution of asteroid orbit semimajor axes. The main Belt, from 2.1 to 3.3 AU, is obvious. The resonance structure due to perturbations by Jupiter is indicated by the ratios of orbital frequency to that of Jupiter (7:2 means an asteroid that orbits the Sun seven times for every two Jupiter orbits).

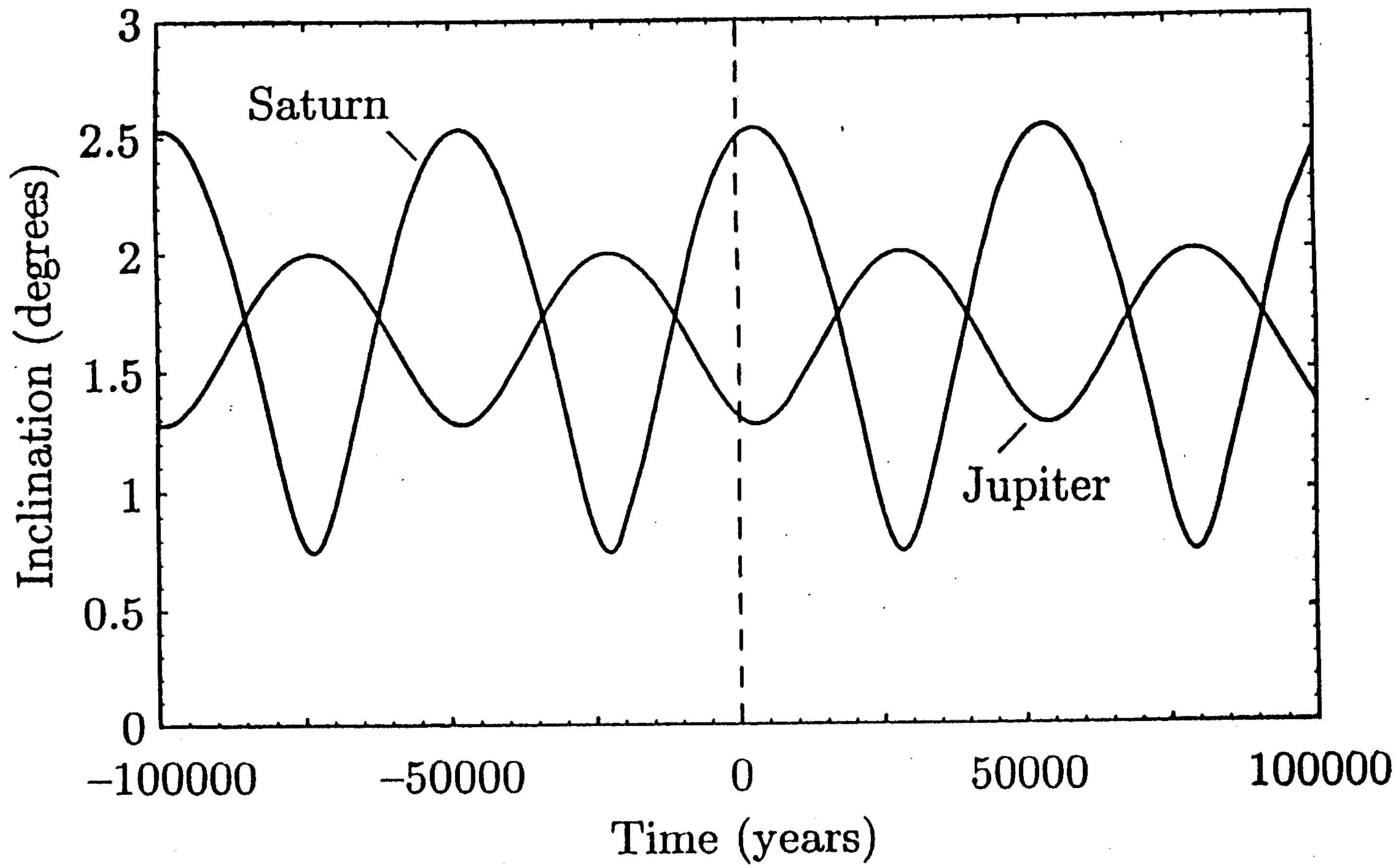


Figure 7.2 The inclinations of Jupiter and Saturn derived from a secular perturbation theory calculated over a timespan of 200,000 y centred on 1983.

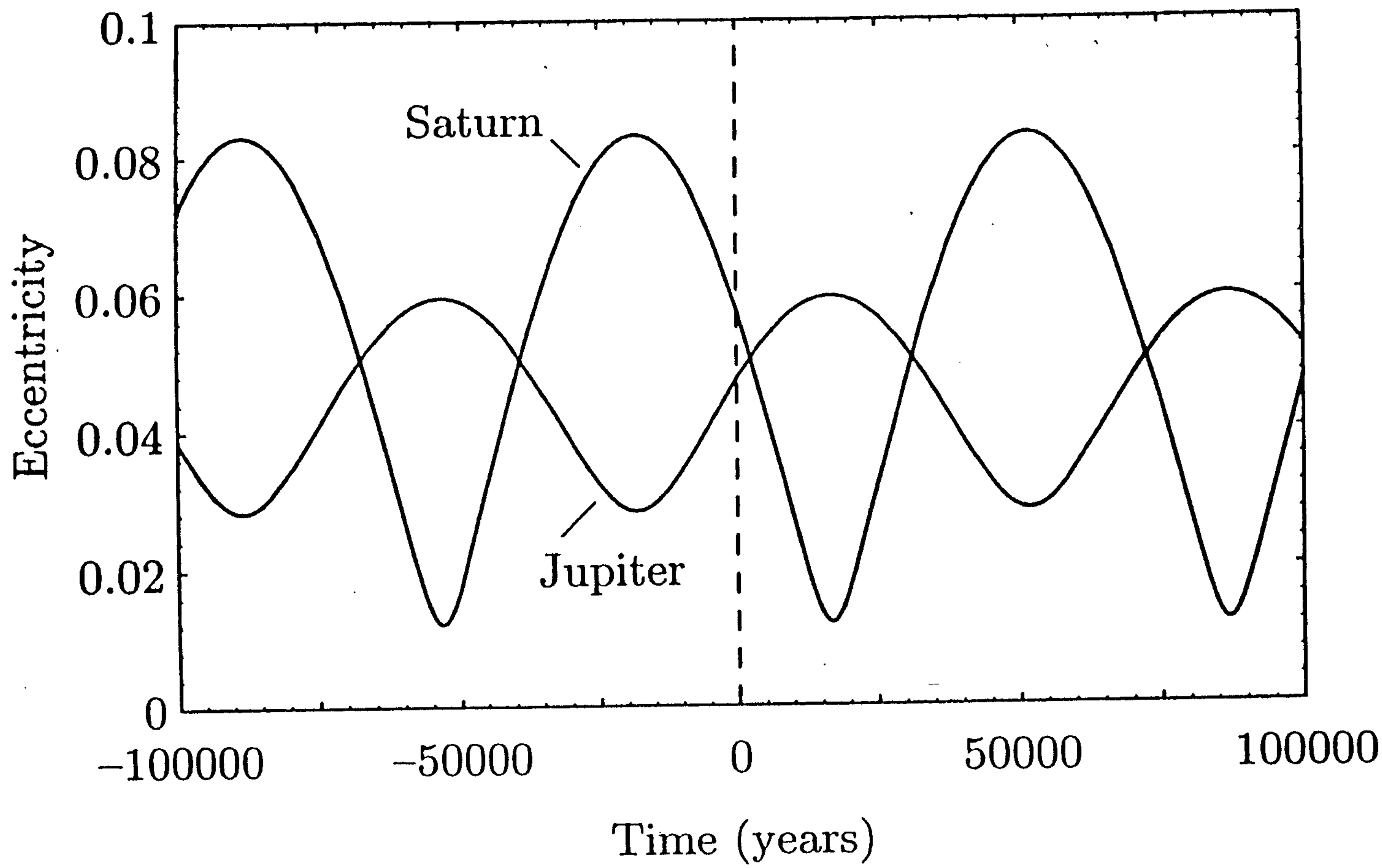


Figure 7.1 The eccentricities of Jupiter and Saturn derived from a secular perturbation theory calculated over a timespan of 200,000 y centred on 1983.

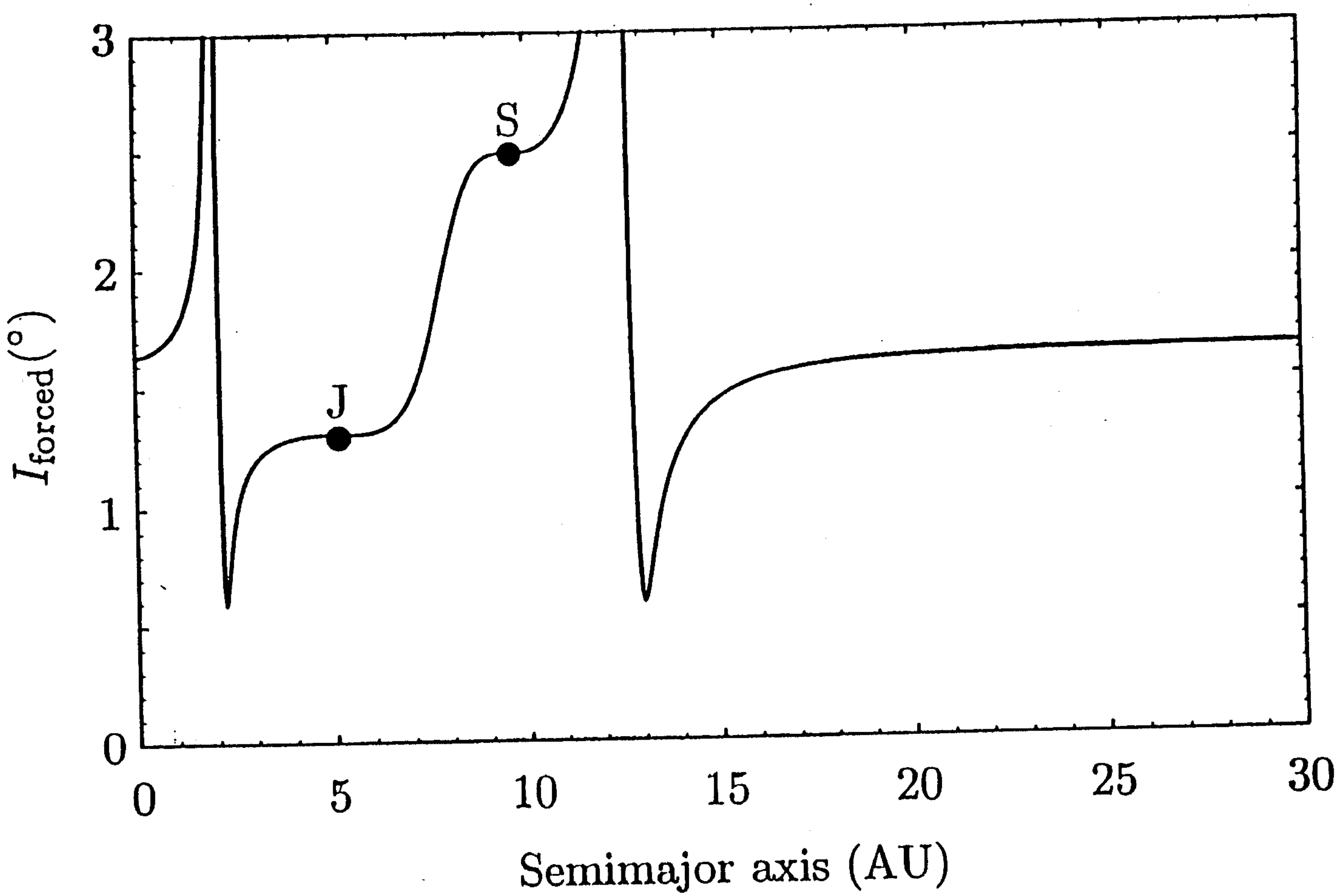


Figure 7.8 The variation in the forced inclination as a function of semimajor axis. The letters J and S denote the osculating values of the inclination of Jupiter and Saturn respectively at their semimajor axes. The singularities near 2 and 12 AU arise from the small divisor $B - f_2$ in (7.30).

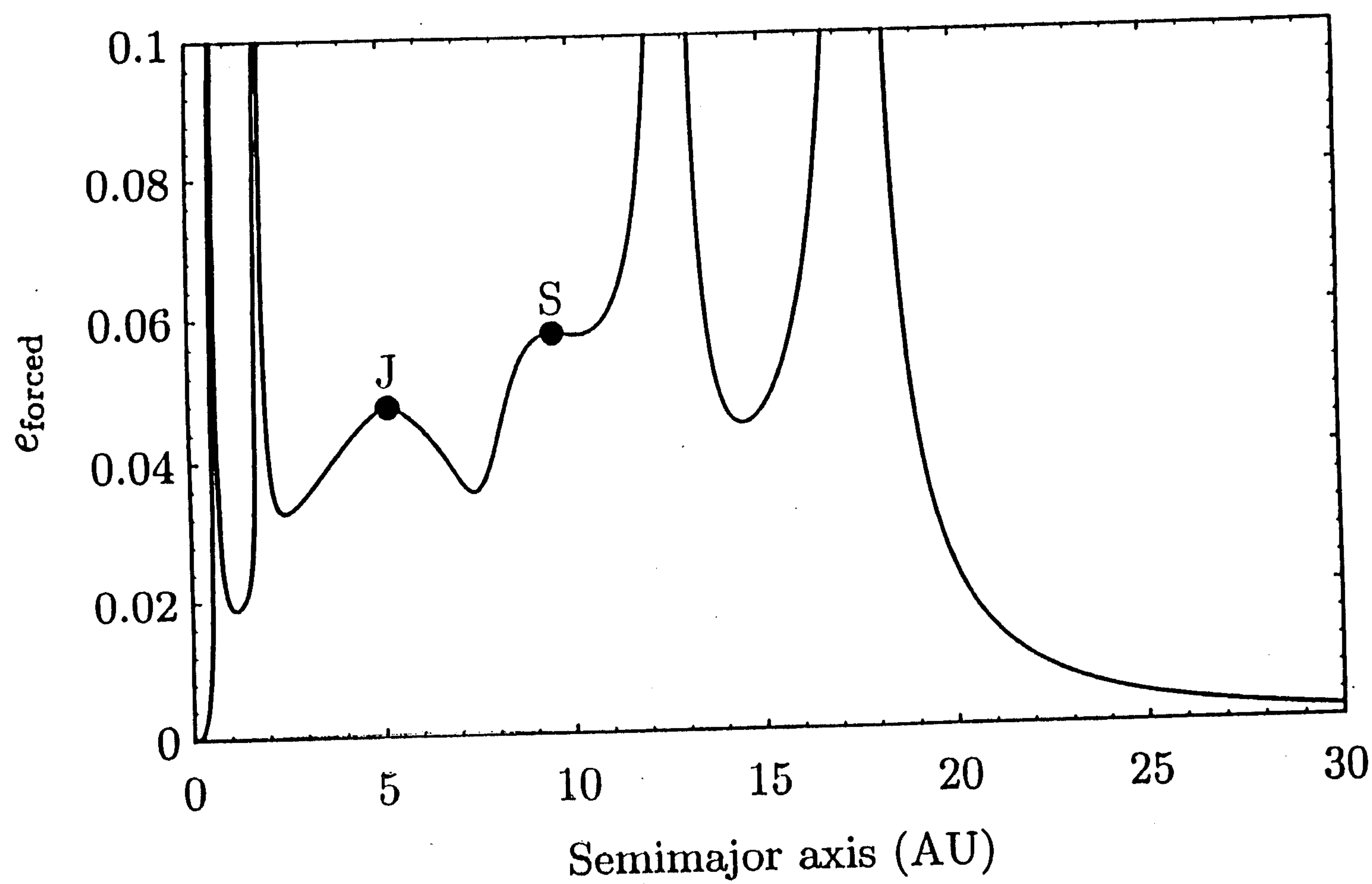


Figure 7.6 The variation in the forced eccentricity as a function of semimajor axis. The letters J and S denote the osculating values of the eccentricity of Jupiter and Saturn respectively at their semimajor axes. The singularities near 0.5, 2, 12.5 and 17.5 AU arise from the small divisors $A - g_i$ in (7.30).

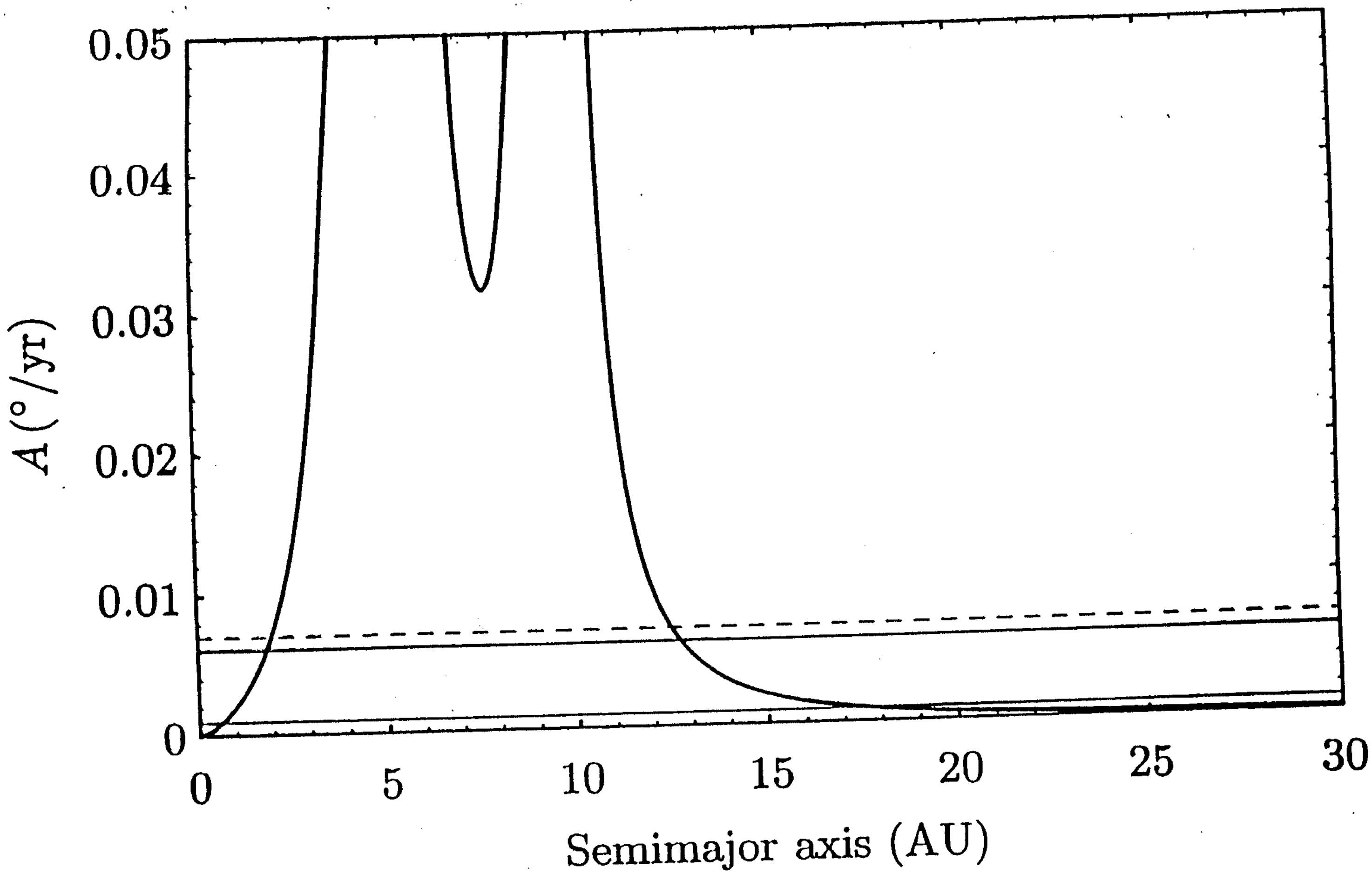


Figure 7.5 The variation of the frequency A, defined in (7.18), as a function of semimajor axis, derived from perturbations by Jupiter and Saturn. The horizontal solid lines denote the values of the two eccentricity–pericentre eigenfrequencies, $A = g_1 = 0.00096°/\mathrm{yr}$ and $A = g_2 = 0.0061°/\mathrm{yr}$; the dashed line denotes the value of the non-zero inclination–node eigenfrequency, $A = -B = -f_2 = 0.0071°/\mathrm{yr}$. Singularities in the plot correspond to the orbital semimajor axes of Jupiter and Saturn.

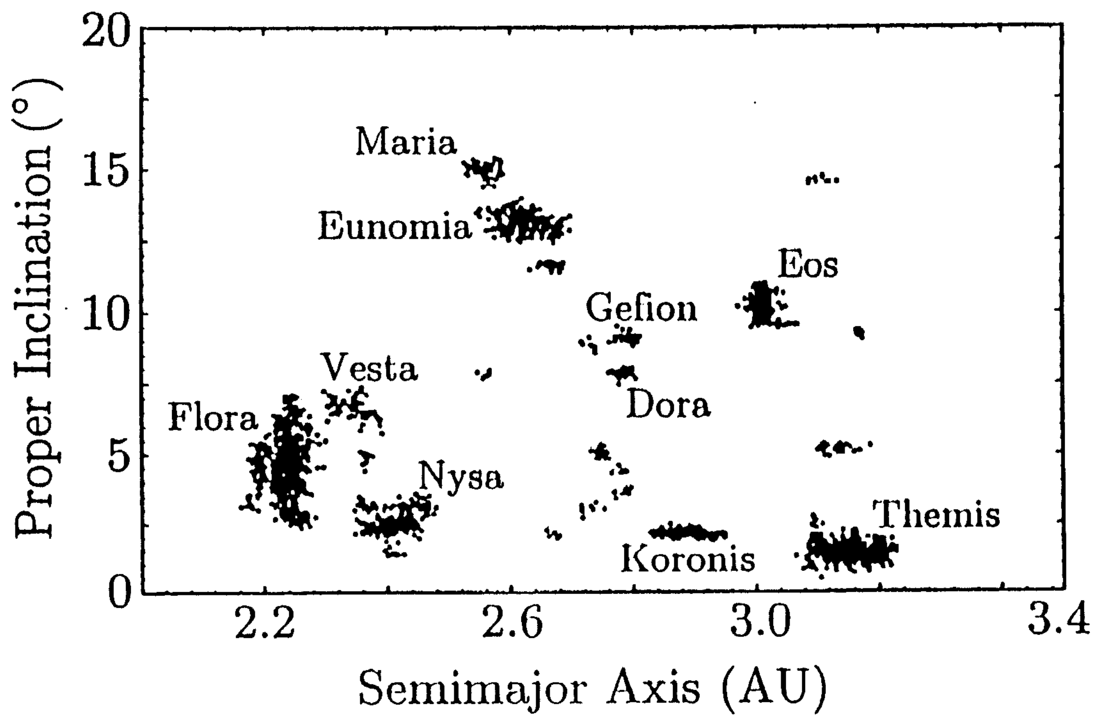


Figure 7.14 A plot of the proper (or free) inclination of asteroids in the range $2.0 \leq a \leq 3.5$ AU which have been identified as family members.

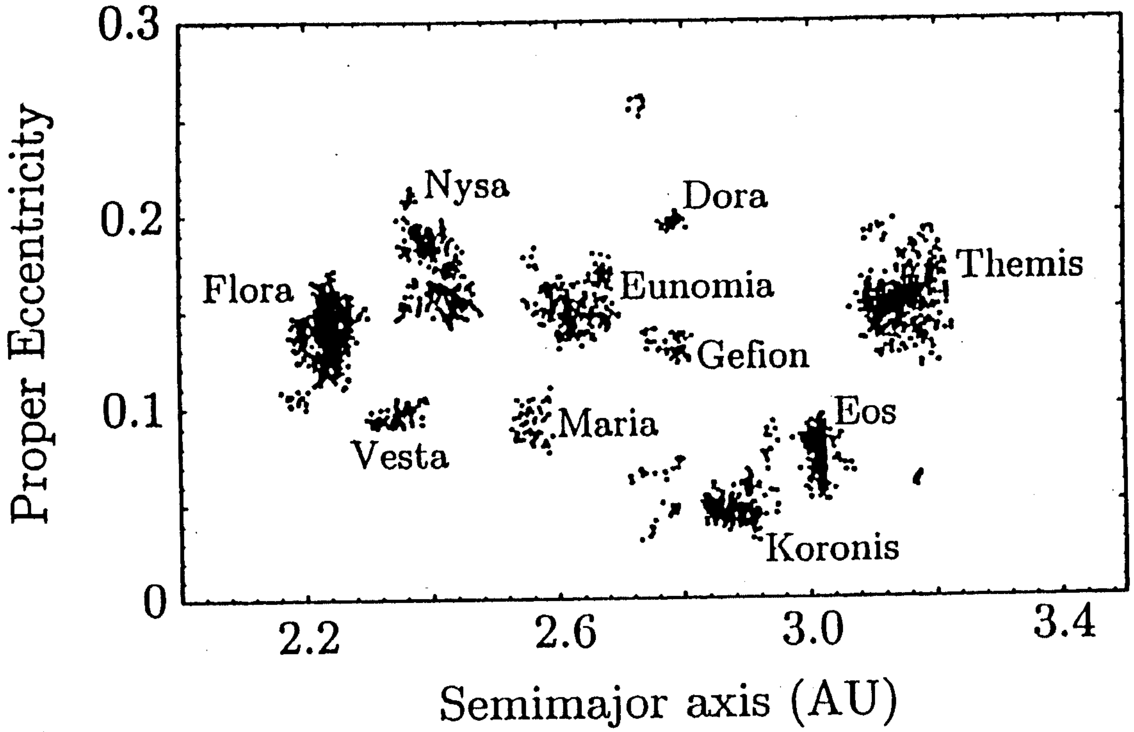


Figure 7.13 A plot of the proper (or free) eccentricity of asteroids in the range $2.0 \leq a \leq 3.5$ AU which have been identified as family members.

Resonances

Orbital Resonances

Jupiter - Asteroid Belt
Neptune - Kuiper Belt
Saturn moons - Saturn Ring
Jupiter - Trojan Asteroids
Mars - Trojan Asteroid
Io - Europa - Ganymede
Dione - Enceladus - Helene
Tethys - Mimas - Telesto - Calypso
Titan - Hyperion
Others in the past?
Neptune - Pluto

Solar B-field - Zodiacal Dust
Jupiter's Ring
Saturn's E Ring
Neptune Dust ?
Dust from Phobos

SPIN Resonances

Mercury
Venus
Moon + all tidally locked satellites
Hyperion & Nereid

Orbital Resonances

One dimensional analog

$$\ddot{x} + \omega_0^2 x = f \cos \omega t$$

if ω (forcing frequency) $\sim$ ω_0 (natural frequency) $\Rightarrow$ Resonant Forcing (large effects)

Ta

Orbits around planets

6 dimensions $(a, e, i, \Omega, \tilde{\omega}, \varepsilon)$

$\left\{ \text{forcing frequencies} \right\}$ $\left\{ \text{natural frequencies} \right\}$

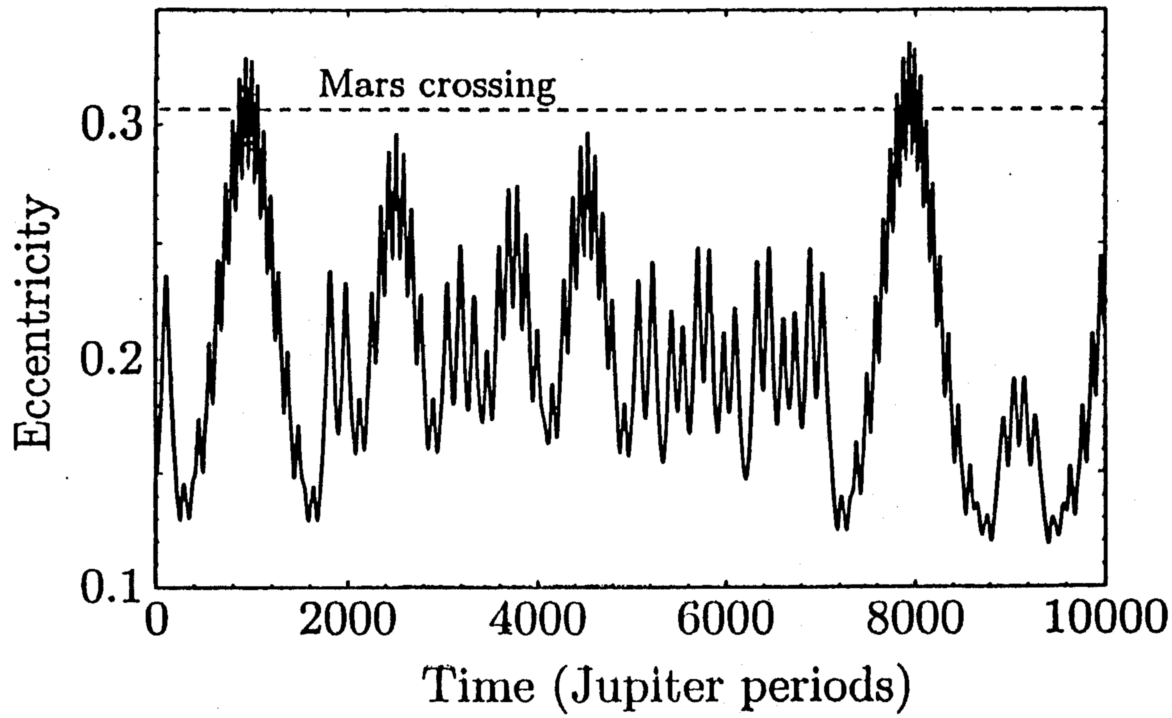


Figure 9.31 The evolution of a test particle's eccentricity at the 3:1 resonance showing the chaotic nature of the orbit and how it can cross the orbit of Mars. The initial values were $a_0/a' = 0.481$, $e_0 = 0.15$, $\varpi_0 - \varpi' = 0$ and $3\lambda' - \lambda_0 = \pi$, corresponding to the chaotic, 'representative' plane identified by Wisdom (1982).

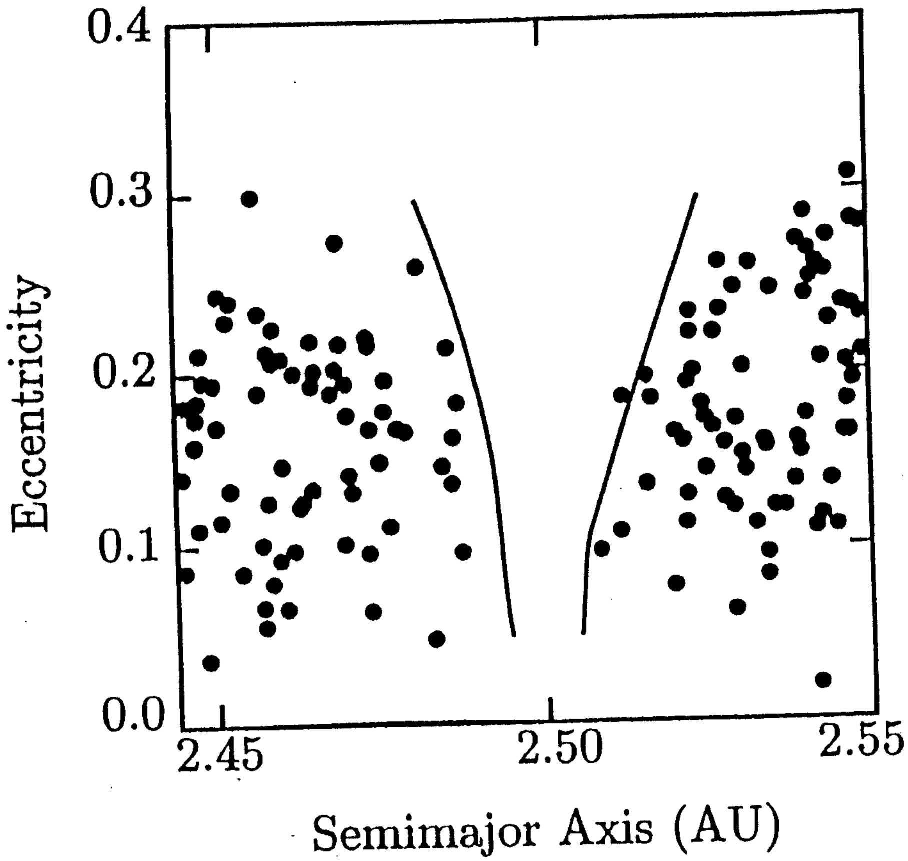


Figure 9.32 The positions of asteroids at the 3:1 resonance compared with the extent of the chaotic region. (Adapted from Wisdom 1983).

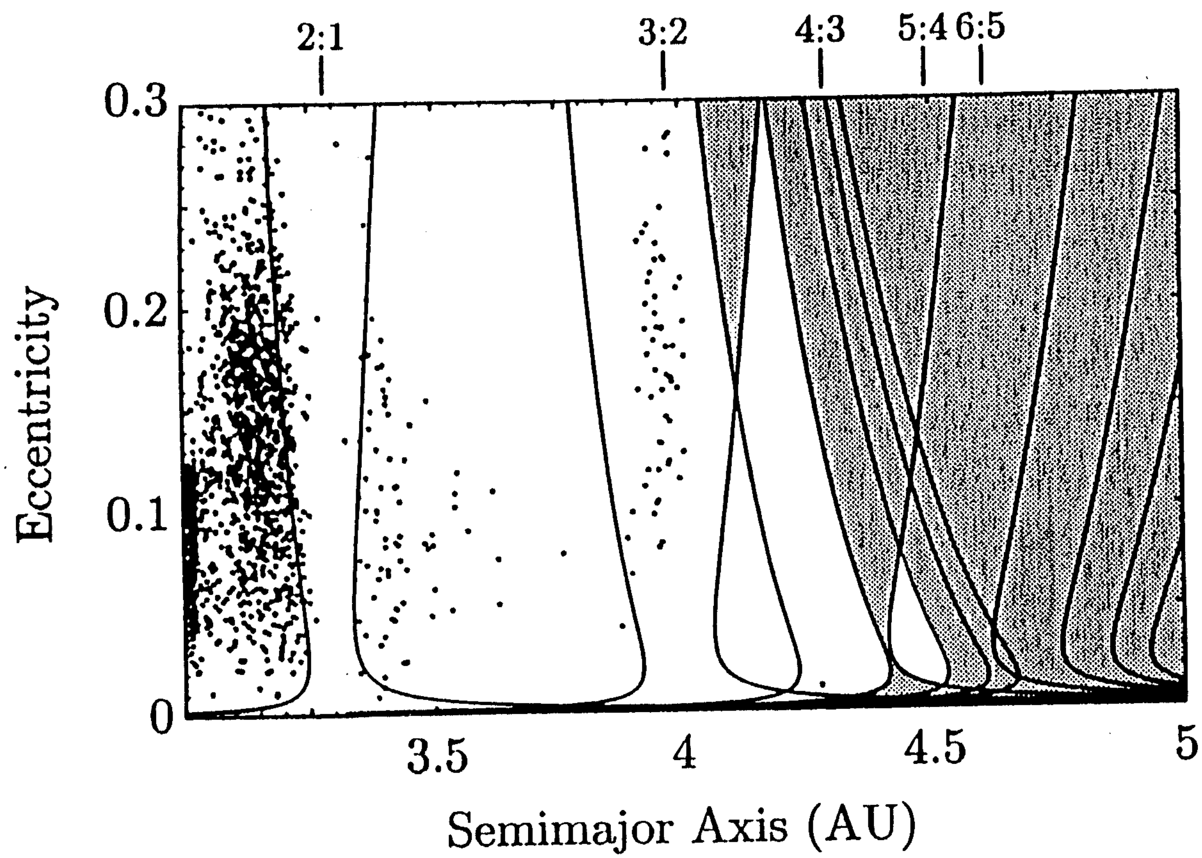


Figure 9.29 The overlap of adjacent first order resonances in the outer part of the asteroid belt. The curves denote the maximum libration widths in a–e space for all the first order resonances between 3 and 5 AU. The points denote the a–e values of asteroids in this region. The nominal locations of the resonance centres are indicated for the 2:1, 3:2, 4:3, 5:4 and 6:5 resonances. The shaded areas denote regions of resonance overlap. Note the prominent gap at the 2:1 resonance and the grouping at the 3:2 resonance.

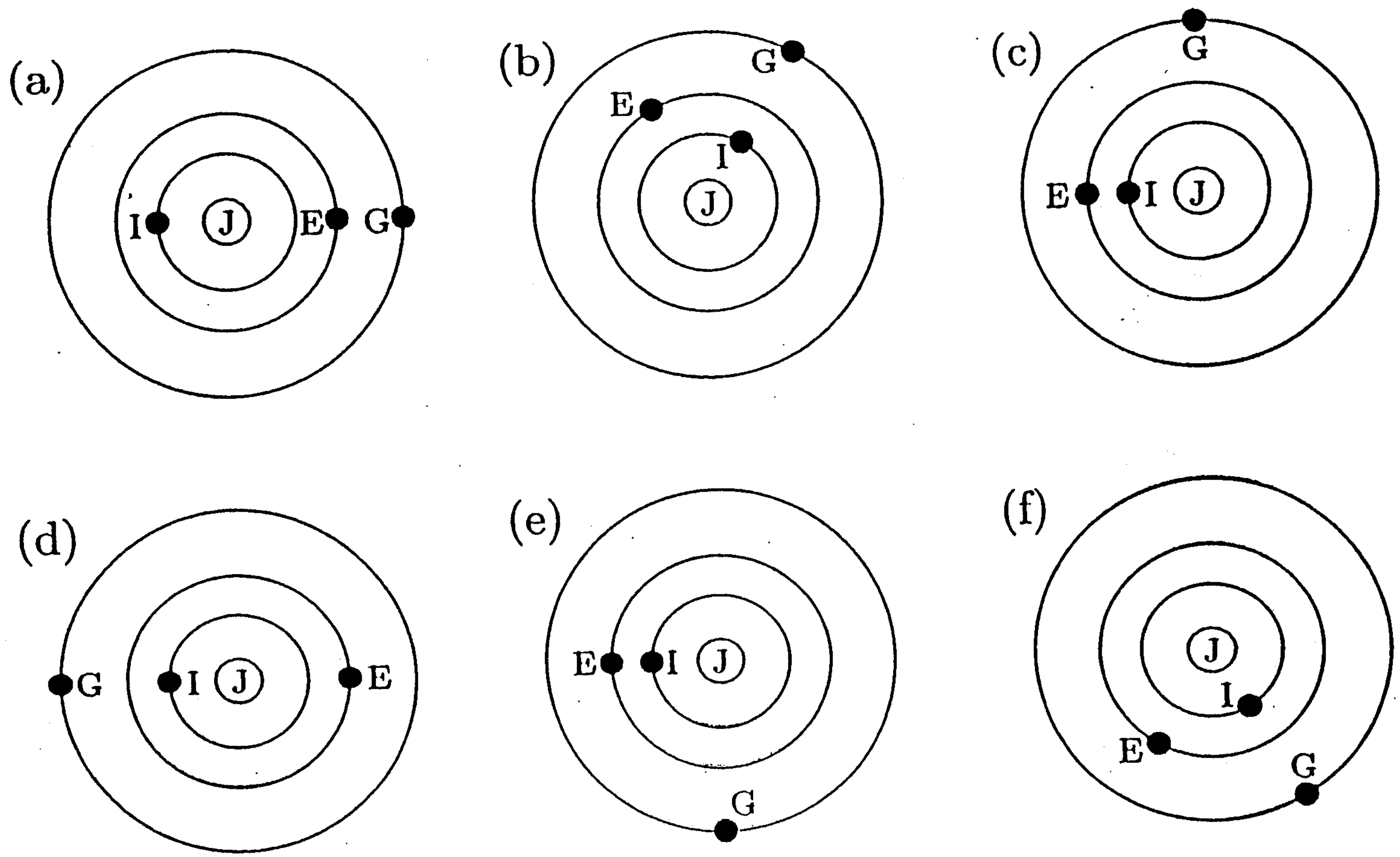


Figure 8.24 The sequence of conjunctions for the Galilean satellites. The configurations at times (a) $t = 0$, (b) $t = T_{\rm rep}/6$, (c) $t = T_{\rm rep}/4$, (d) $t = T_{\rm rep}/2$, (e) $t = 3T_{\rm rep}/4$, and (f) $t = 5T_{\rm rep}/6$. The letters J, I, E and G denote Jupiter, Io, Europa and Ganymede respectively.

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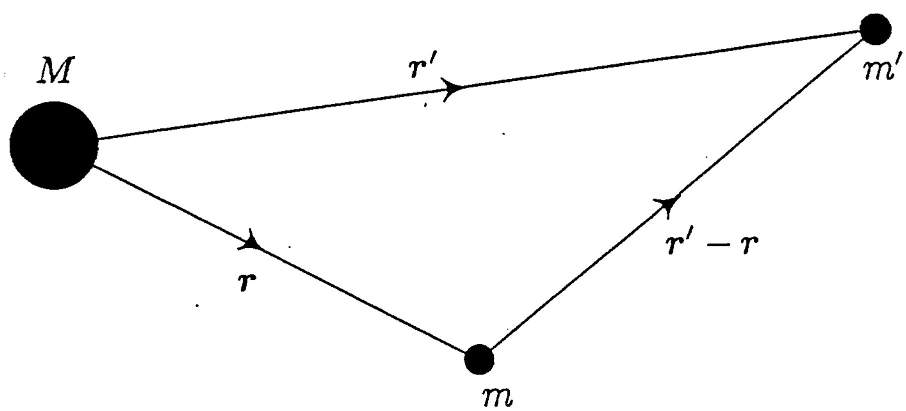
QMW Maths Notes 15



EXPANSION OF THE PLANETARY DISTURBING FUNCTION TO EIGHTH ORDER IN THE INDIVIDUAL ORBITAL ELEMENTS

Carl D. Murray and David Harper

Planetary Perturbations

1. Obtain the perturbing potential

2. Translate to orbital elements

3. Taylor expand in small quantities
   $(e, e', i, i', a'/a)$

4. Combine trig functions to obtain a sum over terms of the form
   $$f(a'/a, e, e', i, i') \cos(A\lambda + B\lambda' + C\tilde{\omega} + D\tilde{\omega}' + E\Omega + F\Omega')$$

5. Take derivatives to obtain the time-rates of change of the orbital elements

The Disturbing Function

$$R = \frac{\mu'}{|\vec{r}\,' - \vec{r}|} - \mu' \frac{\vec{r} \cdot \vec{r}\,'}{(r')^3}$$

$\Downarrow$

translate to orbital elements

$\Downarrow$

$$R = \sum_{e,e',i,i'} f(a,a',e,e',i,i') \cos(A\lambda + B\lambda' + C\tilde{\omega} + D\tilde{\omega}' + E\Omega + F\Omega'$$

<u>One Term</u> : β [redacted] $e^c e'^d i^e i'^f \cos\phi$

with $\phi = A\lambda + B\lambda' + C\tilde{\omega} + D\tilde{\omega}' + E\Omega + F\Omega'$

β = function of (a,a') = strength.

<u>d'Alembert Rules</u> :

$$A + B + C + D + E + F = 0$$

$c \geq C \qquad e \geq E$

$d \geq D \qquad f \geq F$

$E + F$ - even