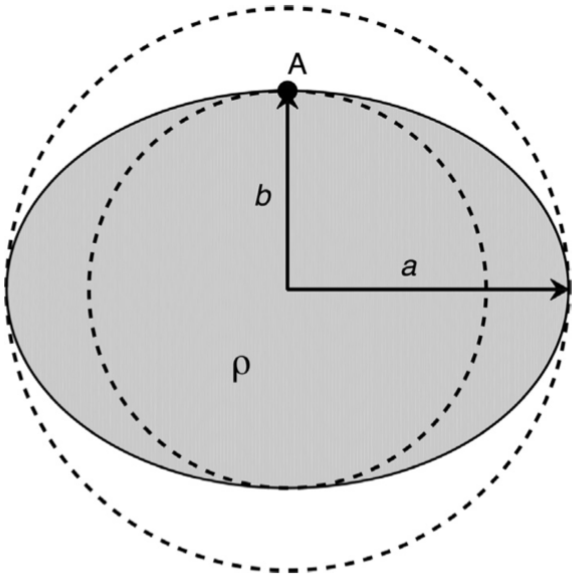
Ensuring Convergence in Calculations of Potential

Thomas Rimlinger

Introduction

- Want to calculate potential for arbitrary shape
- WAY TOO HARD – need simplifications!
  - Oblate spheroid
  - Constant density
- $r^2 = a^2/(1 + l^2\mu^2)$



One Approach

- Expansion in powers of a small parameter
- $l^2 = a^2/b^2 - 1$
- Closed form of surface potential is known
  - Features both a square root and an arctangent
  - Use a Taylor expansion!

$$U_{\text{surface,geophys}} = \frac{GM}{a}\left[1 + \frac{3}{10}\ell^2 - \frac{39}{280}\ell^4 + \frac{139}{1680}\ell^6 - \frac{2749}{49280}\ell^8 + \cdots\right]$$

- Diverges for $l > 1$

Another Approach

- Calculate potential at an "audit point"
  - No need for an expansion in r'/r

$$V_{\text{surface,audit}} = -\frac{GM}{b}\left[J''_{0,0}(b/a)^3 + \sum_{k=0}^{\infty} J'_{0,2k}(b/a)^{2k+1}\right]$$

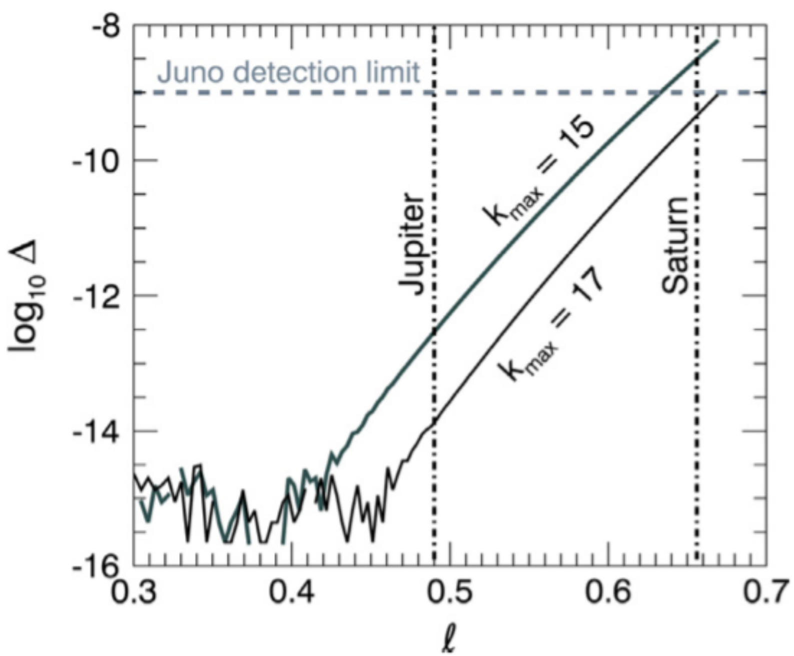
- Good news: $J'_{0,2k} = 0$ for all k > 1
  - Only three nonzero terms!

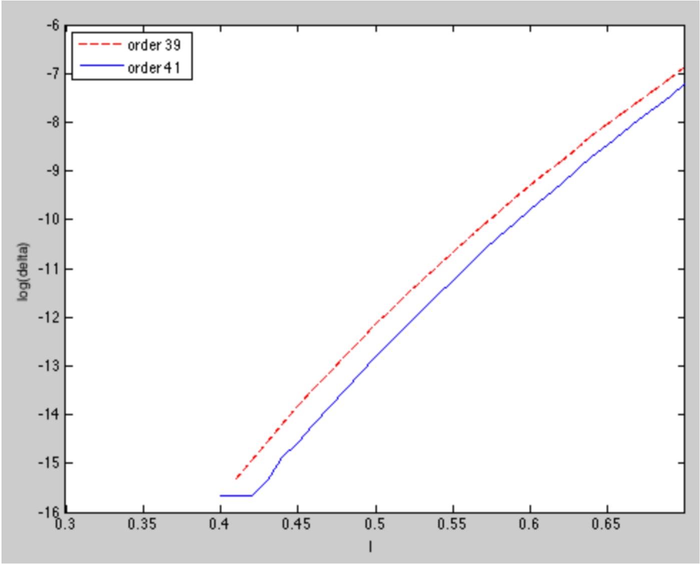
$$J''_{0,0} = \frac{2\pi\rho a^3}{3M} \qquad J'_{0,0} = -\frac{2\pi\rho a^3}{M}\int_0^1 d\mu\,\xi(\mu)^2$$

$$J'_{0,2} = -\frac{4\pi\rho a^3}{M}\int_0^1 d\mu P_2(\mu)\ln\xi(\mu)$$

Comparing Approaches

- Plot the residuals vs. l



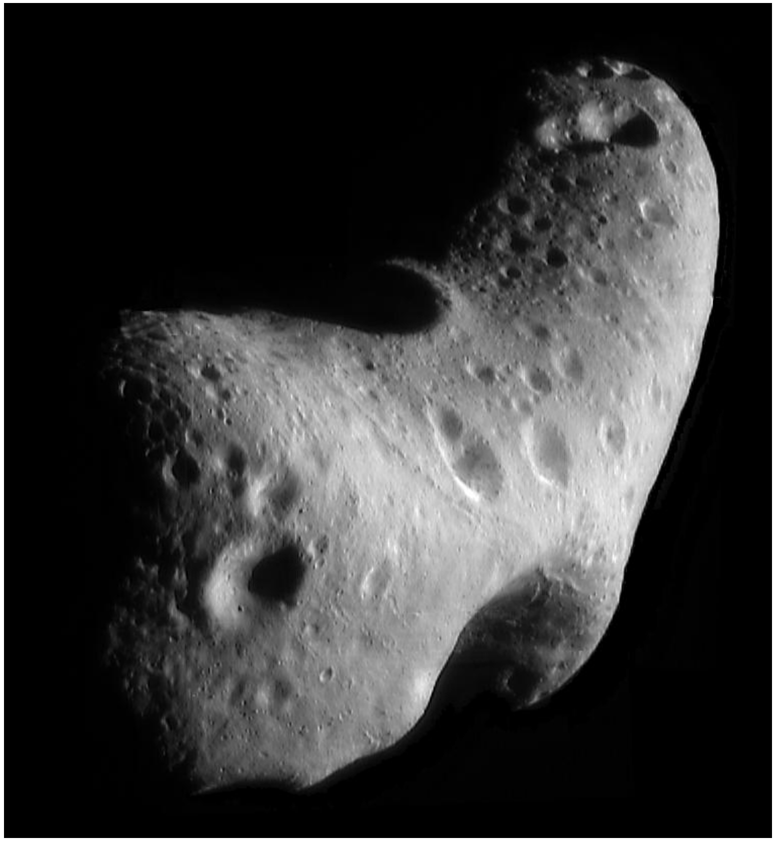


Applicability

- When a > √(2)*b, we have a problem!
  - Implies that l > 1, so the series diverges
- Not an issue for any of the planets
- But a big problem for some asteroids!
  - Mostly just for the smaller ones, such as…

433 Eros

- 33 x 13 x 13 km
- Must be careful when calculating the potential
- Many other small bodies like this
  - 216 Kleopatra
  - 243 Ida
  - 25143 Itokawa



Works Cited

- Hubbard, W., Schubert, G., Kong, D., Zhang, K.. 2014. On the convergence of the theory of figures. Icarus **242**, 138-141.
- Eros data taken from http://nssdc.gsfc.nasa.gov/planetary/factsheet/asteroidfact.html
- Eros picture taken from http://photojournal.jpl.nasa.gov/catalog/PIA02923