ASTR450 Homework # 9 – The Last HW!
Due Thursday, May 9

1. Using the paper by Burns et al. (1979), fill in the missing steps in the derivation of Eq. 24 for da/dt. Follow the derivation of Eq. 28 for de/dt enough to understand it. Start from Energy and Angular Momentum and show your work!

The next two problems make use of the Planetary Satellite Integrator (PSI). Be sure to include some plots of PSI's output and describe what features they show in your writeup.

2. **Planetary Oblateness** a) Choose a moderately eccentric orbit ($e \approx 0.2$) at a semimajor axis of 6 Earth Radii and use the Planetary Satellite Integrator to test the orbit-averaged equations for planetary oblateness. Toggle Earth's equatorial bulge on and solar gravity off. Calculate the orbital period at 6 Earth Radii and then run an $i = 30°$ orbit for .01 years, saving values every .01 days. Describe how well the orbital elements are following the equations that we derived in class. Now change to a 1 year integration, saving points every 0.1 days; how well do the elements follow the equation on timescales long compared to the orbital period? Include these plots as Fig. 1.1 and Fig. 1.2 in your writeup and interpret.
b) Now follow a set of orbits with a given a and e, but different i's ranging from 0-180°, for 1 year saving values every 0.1 days. Plot the $\delta\Omega$ and $\delta\omega$ experienced by your orbit over 1 year vs. inclination and compare to the expressions that we derived in class (Use the Orbital Elements vs. Time plot). Be careful to verify that you are getting the inclination that you expect by checking the plot labels! What happens to the orbits as $i \to 0$ or $i \to 180°$ when Ω is undefined? Tweak these planar inclinations by a hundredth of a degree to determine what happens in the limit as 2D orbits are approached and include these on your summary plot.
c) A satellite has $a = 6.0R_p, e = 0.2$ and evolves under the influence of planetary oblateness only. In one Earth year, its node changes by $\Delta\Omega = -13°$ and its pericenter by $\Delta\omega = -8°$. What is the inclination of the satellite's orbit, and which planet does it circle?

3. **Solar Tides** a) Take a moderately eccentric orbit ($e = 0.2$) with $i = 30°$ and investigate five or six orbits of different sizes from 6 to 20 Earth Radii. Consider orbits where oblateness alone is important and produce a summary plot which tests the a dependence of the orbit averaged equations for $d\Omega/dt$ and $d\varpi/dt$.
b) Now toggle the equatorial bulge off and solar gravity on and repeat the experiments. Solar gravity has an a^n dependence. What value of the exponent n best fits your experiments? What features does solar gravity add to the orbital evolution?
c) Try to predict what you will see when both solar gravity and planetary oblateness are active and test your guess. At what distance are the two effects comparable in magnitude?