# ASTR430 Homework #2
## Due Thursday September 28, 2023

1. Problem 6-5 from the textbook.

2. Problem 6-7 from the textbook.

3. Build a Planet!

a) Evaluate the moment of inertia $I$ of a uniform density spherical planet of radius $R$ about its spin axis by integrating the expression $dI = r_{xy}^2 dm$ over the volume of the sphere ($dI$ is the differential moment of inertia due to mass element $dm$ at distance $r_{xy}$ from the spin axis). Write your answer two ways: i) in terms of $R$ and $\rho$, the mass density and ii) in terms of $R$ and $M$, the mass of the planet.

b) Will the moment of inertia increase or decrease for an oblate planet (one with an equatorial diameter greater than its polar diameter)? What about for a differentiated planet with a dense core and a less dense mantle? Explain your answers.

c) Use your answer from a) to get the moment of inertia of a two-layer planet with a core of radius $R_c$ and density $\rho_c$, and a mantle with outer radius $R$ and density $\rho_m$. Write your answer in terms of $R$, $R_c$, $\rho_c$, and $\rho_m$. Apply your result to get a constraint on the interior structure of Mars using the measured $I_{Mars} = 0.365MR^2$. Write the constraint in terms of $R$, $R_c$, $\rho_c$, $\rho_m$, and $\bar{\rho}$, where $\bar{\rho} = 3.93$ g/cm$^3$ is the average density of Mars.

d) Write down an expression for the total mass of the planet in terms of $R$, $R_c$, $\rho_c$, and $\rho_m$. Eliminate mass in favor of $\bar{\rho}$ to get a second constraint on the interior structure of Mars.

e) Parts c) and d) give two constraints on the three unknowns $R_c$, $\rho_c$, and $\rho_m$. If we assume a core density for Mars, the system reduces to two equations in two unknowns. Eliminate the core radius from your two equations to get a single equation that relates the two unknown densities. Assume an iron core with density $\rho_c = 7.5$ g/cm$^3$, and guess different $\rho_m$'s until you find a solution (This equation cannot be solved analytically). What core radius $R_c$ does your answer suggest?