ASTR450 Homework # 9 – Perturbations
Due Thursday, November 9

1. **Impulse Approximation for Atmospheric Drag.** Consider an artificial satellite on an inclined elliptical orbit acted on by a drag force of the form $\mathbf{F} = -k\mathbf{v}_{rel}$, where $\mathbf{v}_{rel}$ is the velocity of the satellite relative to the atmosphere, and k is a positive constant. Recall that Earth's atmosphere decays exponentially with height (scale height $\approx$ 10km).
a) Approximate the range of eccentricities for which the drag force can be approximated by an impulse at pericenter.
b) Equatorial Orbits ($i = \Omega = 0$). Consider a non-rotating Earth first. Describe how the orbital elements a, e, i, Ω, ϖ vary in time. Show that the pericenter distance $a(1-e)$ is unchanged by the perturbation. Why must this be so? Now describe, quantitatively, how rotation of an assumed spherical Earth affects your answer.
c) Finally imagine orbits with $i \neq 0, \Omega \neq 0$ around a rotating Earth. Using the perturbation equations and other physical arguments, describe qualitatively how these orbits will evolve under atmospheric drag (i.e. how a, e, i, Ω, ϖ vary in time).

2. **Radial Perturbation Forces.** Consider a radial perturbation force of the form $\mathbf{F} = R\hat{r}$, where R is a function of the distance r.
a) Apply the perturbation equations to this force and obtain simplified expressions for da/dt, de/dt, di/dt, $d\Omega/dt$, and $d\varpi/dt$.
b) A radial perturbation to gravity, which is itself a radial force, is an example of a central force. So angular momentum must be conserved. Show that your equations conserve the angular momentum vector and describe the constraints that this imposes on these orbits.
c) Now let $R = Ar^n$ where A is a constant. Take the time average of your expressions over a single unperturbed Keplerian orbit (this step assumes that the perturbation is small). Show that $< r^n \sin\nu >= 0$ and argue, on physical ground, that $< \cos\nu >$ is negative (or zero) and that $< r^{-2} \cos\nu >= 0$. It can be shown that $< r^n \cos\nu >$ is negative for $n > -2$ and positive for $n < -2$. Use this fact to determine how the sign of your time-averaged $d\varpi/dt$ depends on A and n. Use the Central Force Integrator to check your results numerically.
d) Finally, consider the General Relativistic (GR) Perturbation $R = Ar^{-4}$ where A is a small negative constant. The integral $< r^{-4} \cos\nu >= a^{-4}e(1-e^2)^{-5/2}$. Solve, analytically, for the value of A that will give 30 degrees of precession per orbit for $e = 0.5$ (The true effects of GR on Mercury's orbit are almost exactly a million times weaker). Convert your prediction into the proper initial conditions for the Central Force Integrator, and test it! Start your orbit at pericenter and turn in a copy of your plot.