

ASTR450 Homework # 6 – Two and Three Body Problems Due Thursday, April 4

Reading: Read Chapter 8. Also, test your knowledge of orbital elements with the 2D and 3D orbit viewers available from the class web page.

1. (Moderate) a) Orbital Elements. Find the six orbital elements $(a, e, i, \Omega, \omega, \nu)$ for an asteroid which, at time $t=0$, has $(X, Y, Z) = (2.5AU, 0, 0)$ and $(V_x, V_y, V_z) = (V_E/\sqrt{10}, -V_E/\sqrt{5}, 0)$. Here V_E is the speed of the Earth in its orbit, and an AU is the astronomical unit. Take the reference plane to be the XY plane and the reference direction to be \hat{X} . It is easiest to use dimensionless units; take $V_E = 30\text{km/s}$ to be the unit of velocity, and the AU to be the unit of distance, and $GM = 1$ to define the unit of mass (so in dimensionless units, $V_E = 1, GM = 1$, and the Earth-Sun distance is one).

b) Find the pericenter distance, the apocenter distance (if it exists), and the semilatus rectum of the orbit, and use these quantities, and your orbital elements, to draw a reasonably accurate sketch of the orbit relative to the X and Y axes.

2. (Moderate) Danby: Page 136, Problem 1. Given that the Moon's eccentricity is 0.05, what total fraction of the Moon's surface is visible to observers on Earth over a month due to this effect?

3. (Moderate) Chapter 8. Rewrite Tisserand's Criterion so that it applies to i) parabolic-parabolic transitions and ii) parabolic-elliptical and parabolic-hyperbolic transitions. Answer the following questions using the approximations that i) the Tisserand Constant is exactly conserved and ii) Jupiter affects the comet only when it is very close to Jupiter. Can Jupiter put parabolic $i = 0$ comets onto circular uninclined orbits? Can a prograde parabolic ($i < 90^\circ$) comet be perturbed onto a retrograde ($i > 90^\circ$) parabolic orbit? Onto a retrograde elliptical or hyperbolic orbit?