Simulations of Planet Migration due to Planetesimal Scattering

Jessica Donaldson

Planetary migration is known to be one step in planetary system formation, both in our own solar system, and around other stars. Evidence of this includes the position of extrasolar planets that could not have formed *in situ* (Lin et al. 1996) and resonances between Neptune and Kuiper Belt objects (Malhotra 1993; Ida et al. 2000). Scattering of kilometer sized planetesimals is one process that produces migration. In this process, the scattering leads to a significant exchange of energy and angular momentum between the planet to the smaller body. As a result, the planet's semi-major axis will shrink or expand, depending upon the direction of energy exchange. The authors of this paper attempt to determine the dependence of migration on several different parameters, through simulations.

Planetesimals will be strongly scattered if they are within 1–3.5 Hill radii of the planet's semi-major axis (Kirsh et al. 2009). The lower boundary is due to the fact that planetesimals within one Hill radii of the planet's orbit will be in stable horseshoe orbits for a long time and will not be scattered. The outer boundary comes from the stability of solar orbits in the circular restricted three body problem (actually $2\sqrt{3} \rightarrow$ equation (2.28) in de Pater & Lissauer (2010)). Strong scattering also depends on the Tisserand parameter, which determines the planetesimal's energy, written here with respect to the planet's semi-major axis.

$$C_T = \frac{a_P}{a} + 2\sqrt{\frac{a}{a_P}}\sqrt{1-e^2}\,\mathrm{cos}i \tag{1}$$

where a planetesimal will be scattering if it is beyond the Hill radius of the planet's orbit and $C_T < C_T^{enc}$ where C_T^{enc} is calculated at $a = a_P + 3.5R_H$ and $e = i = 0$. The timescale for scattering is determined by the synodic period, the time between conjunctions of planet and planetesimal $T_{syn} = 2\pi/|n_{pl} - n_P|$.

Since the encounter zone is symmetric about the planet in semi-major axes, the synodic period is not the same in the inner and outer encounter zones. This causes a scattering bias, where planetesimals in the inner encounter zone are more easily scattered than planetesimals in the outer encounter zone. This would cause a planet to preferentially move inwards because it is transferring more of its angular momentum to move planetesimals out. However, there is also a mass asymmetry in the encounter zone. The area of the outer encounter zone is larger than the area of the inner encounter zone, so assuming a constant disk density, there is more available mass in the outer zone to be scattered. This would cause a planet to preferentially migrate outwards. The simulations in this paper show the that the scattering bias is more powerful than the mass asymmetry, and that planets are more likely to migrate inwards than out (Kirsh et al. 2009).

Ida et al. (2000) calculated the migration rate of one planet due to the scattering of planetesimals only on one side of the planet's orbit to be

$$\left|\frac{da}{dt}\right|_{fid} \approx \frac{4\pi\Sigma a^2}{M_\odot}\frac{a}{T} \tag{2}$$

The simulations show that this rate is still valid if the mass in the encounter zones is larger than the planet mass, there is little to no mass in the horseshoe zone, and the disk is dynamically cold. If $1 < M_P/M_{enc} < 10$ then the migration rate is damped by a factor of $(1 + \frac{1}{5}(\frac{M_P}{M_{enc}})^3)$. A larger horseshoe mass (mass of planetesimals with stable horseshoe orbits around the planet) also damps the migration because the planetesimals in horseshoe orbits can stabilize the planet against migration. The disk eccentricity can also damp the migration because the timescale for interaction is longer for planetesimals that have larger eccentricities. This effect damps the migration by a factor of $(1+(\frac{e_{RMS}a_P}{3R_H})^3)$ where R_H is the Hill radius of the planet.

Disks with a steep radial density distribution have much more mass in the outer encounter zone. If the density distribution is steep enough, it can overpower the scattering bias and cause the planet to migrate outwards. However, the outwards migration is slower than the inwards migration by about 34% because the scattering bias is still present.

Migration through planetesimal scattering should occur to some extent in all planet forming disks. This paper attempts to simulate all possible combinations of planets and planetesimal disks in order to shape a more general picture of planetary migration. The simulations determine the effects of various factors on planetary migration rates, such as disk eccentricity, planet and disk mass, location of the planet, and the disk surface density profile. However, the simulations neglect self-gravity of the planetesimals, and therefore, this model may be lacking a key component to fully understanding migration through planetesimal scattering.

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