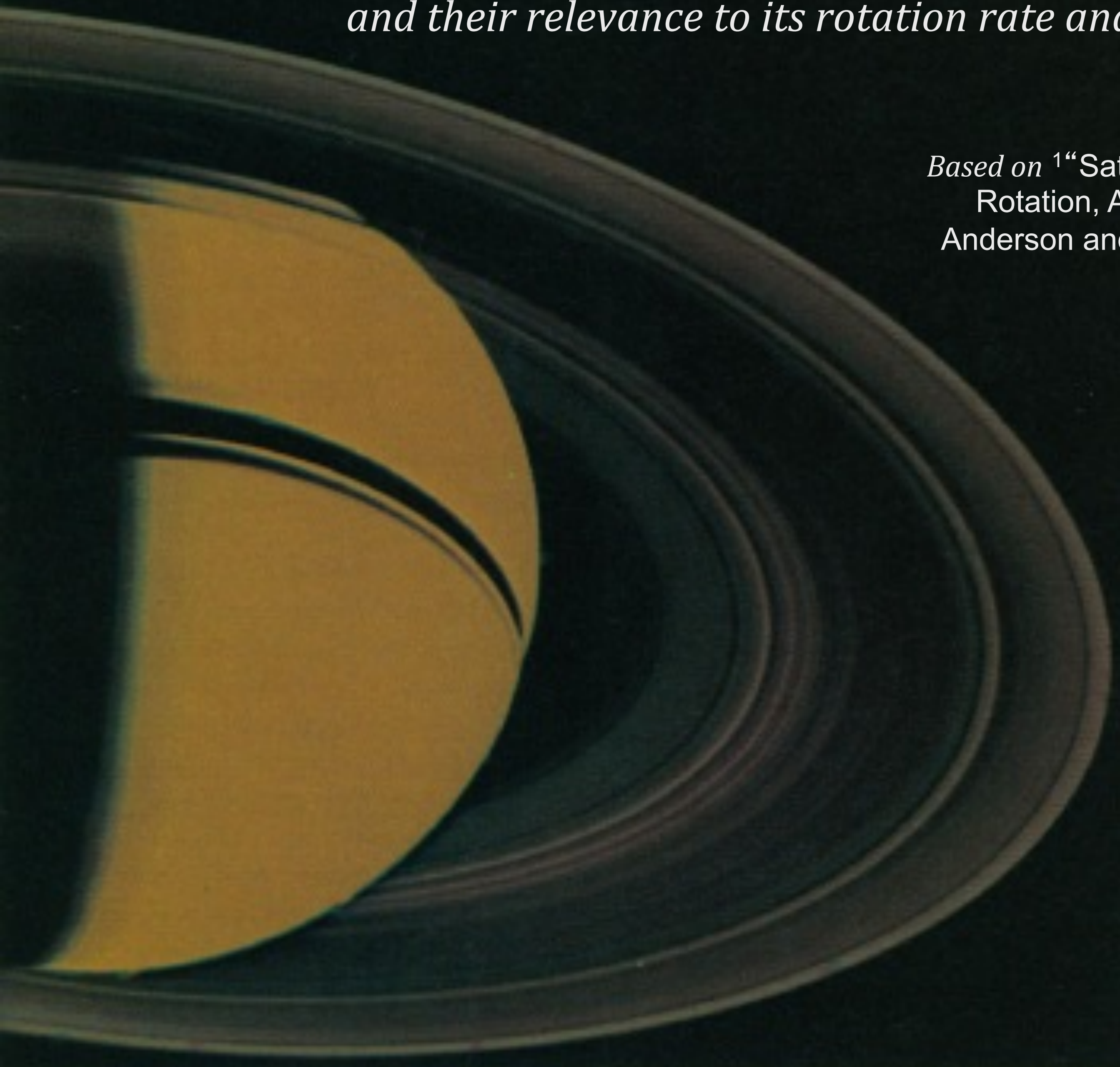
The Winds of Saturn

and their relevance to its rotation rate and figure



Based on [1]"Saturn's Gravitational Field, Internal Rotation, And Interior Structure" by J. D. Anderson and G. Schubert in *Science*, 317, 7th September, 2007

by

Daanish Maqbool

TERPS Conference

Dec. 5th, 2012

Background

- Saturn's rotation rate was originally derived from the periodicity of its radio emissions (e.g. Saturn Kilometric Radiation, SKR)

- But these radio emissions changed periods over time[2]:
  - Voyager: 10 hrs: 39 min: 24 sec.
  - Cassini: 10 hrs: 45 min: 45 sec.

[2]Gurnett, D. A. et al., "The variable rotation period of the inner region of Saturn's plasma disk," *Science*, 316, 20th April, 2007

Background

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- But these radio emissions changed periods over time[2]:
  - Voyager: 10 hrs: 39 min: 24 sec.
  - Cassini: 10 hrs: 45 min: 45 sec.
- These authors[1] set out to obtain Saturn's rotation rate without any radio emissions information (they get 10 hrs: 32 min: 35 sec.)

[2]Gurnett, D. A. et al., "The variable rotation period of the inner region of Saturn's plasma disk," *Science*, 316, 20th April, 2007

Background

- They do this in the following way:
  - Using Saturn's polar radius (known using radio occultation data), assumed spin rate, and the *J* gravitational coefficients measured by Cassini, generate a geoid of constant gravitational potential:

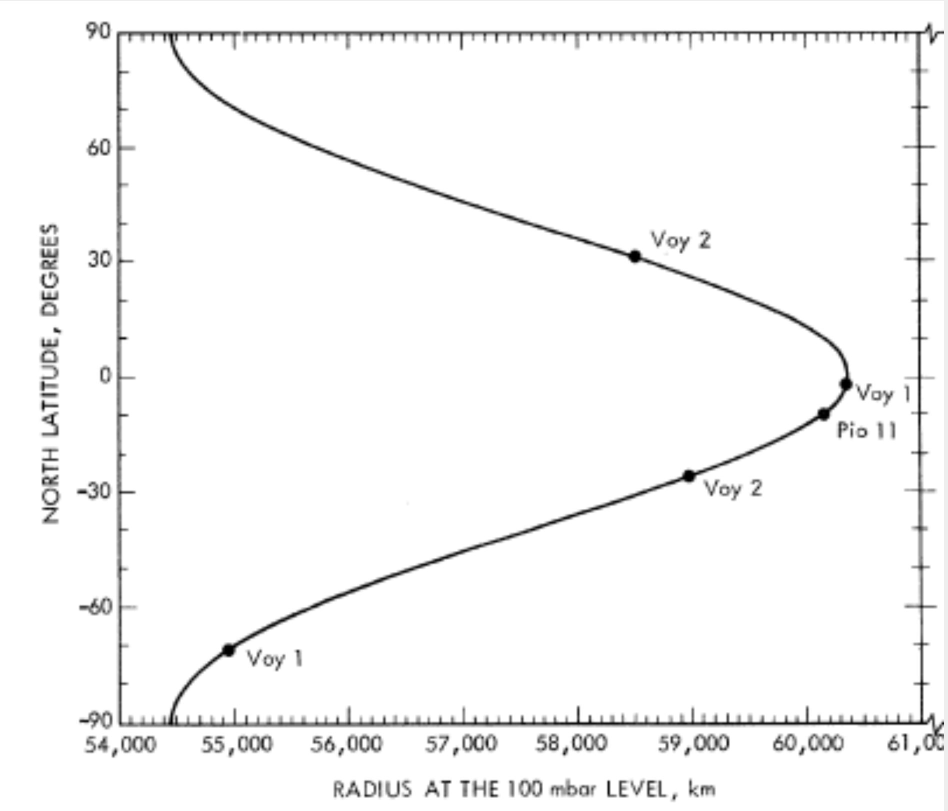
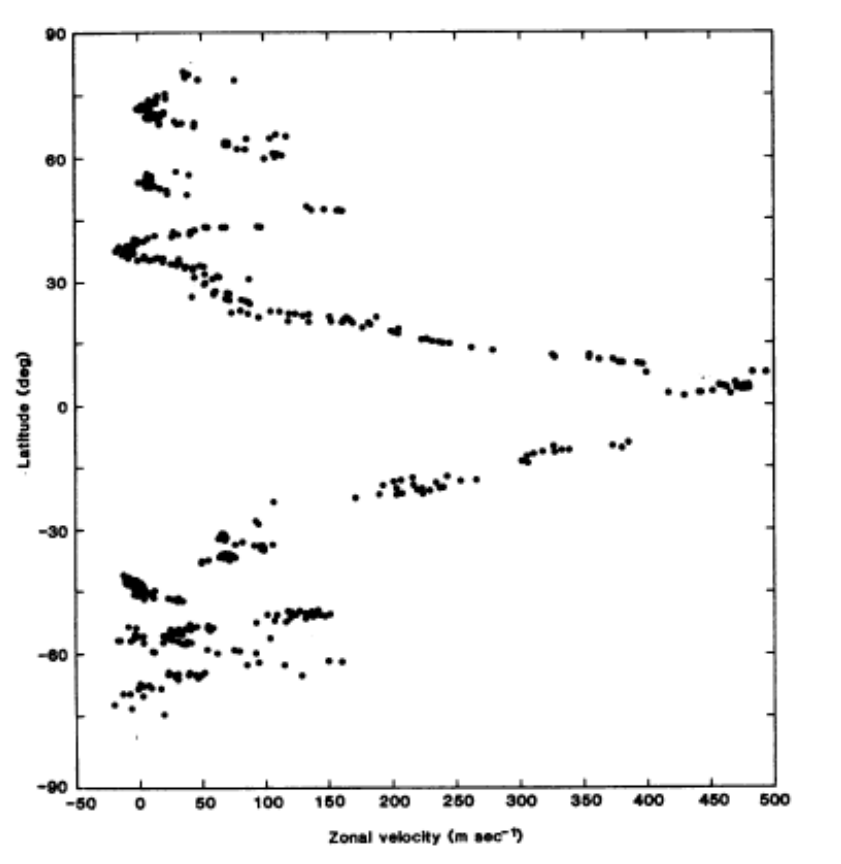


Image adapted from [3]Lindal, G. F., Sweetnam, D. N., and Eshleman, V. R., "The Atmosphere of Saturn: An Analysis of the Voyager Radio Occultation Measurements," *Astronomical Journal*, Vol. 90, no.6, June, 1985.

Background

- Now use wind velocity data from Voyager 2[3] to see how the geoid is perturbed (because it is for a body co-rotating with the surface).



Adapted from [4]Smith, B. A. et al., "A New Look at the Saturn System: The Voyager 2 Images," Science, Vol. 215, 29th January, 1982.

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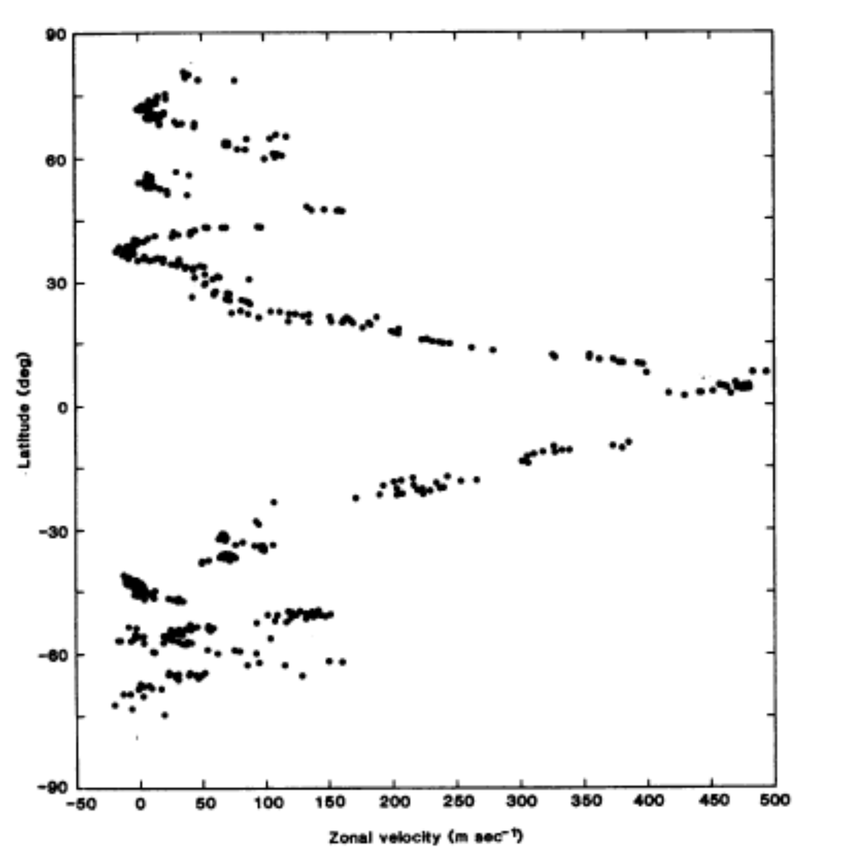
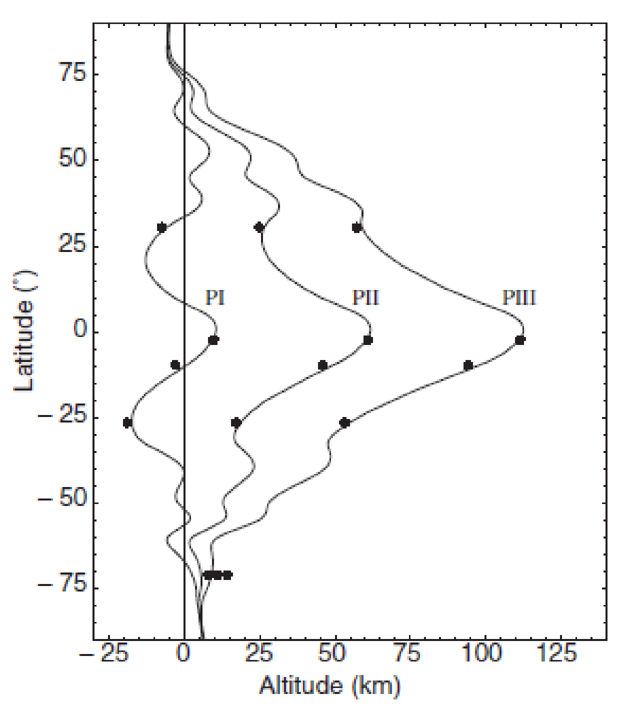
- Adjust spin rate until the discrepancy between the two geoids is minimized.





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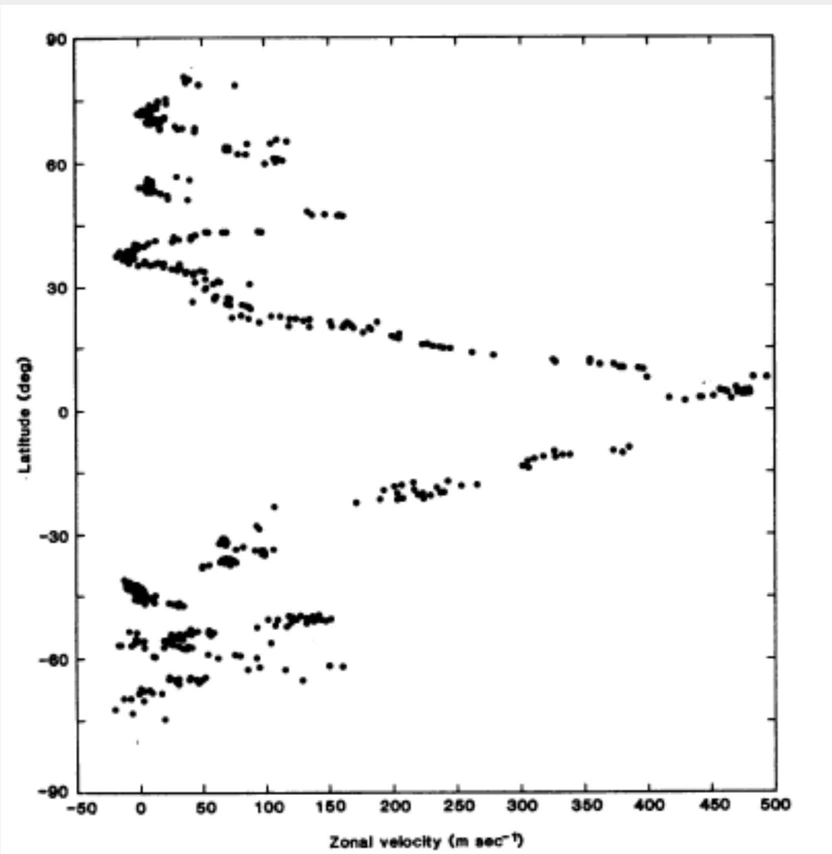

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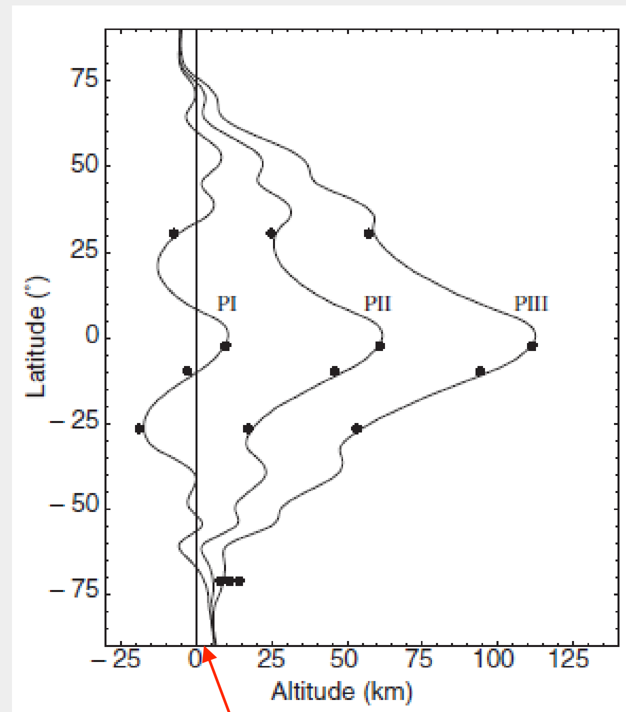

Adapted from ref. 1

Poles don't match up!

Centers of Mass and Figure are offset by ~10 km

Objective

- Is the discrepancy due to incorrect data fitting?
- Did they force the wind velocities to zero at the poles?
- Could this be due to uncertainty in the wind data?
- → Will address these questions

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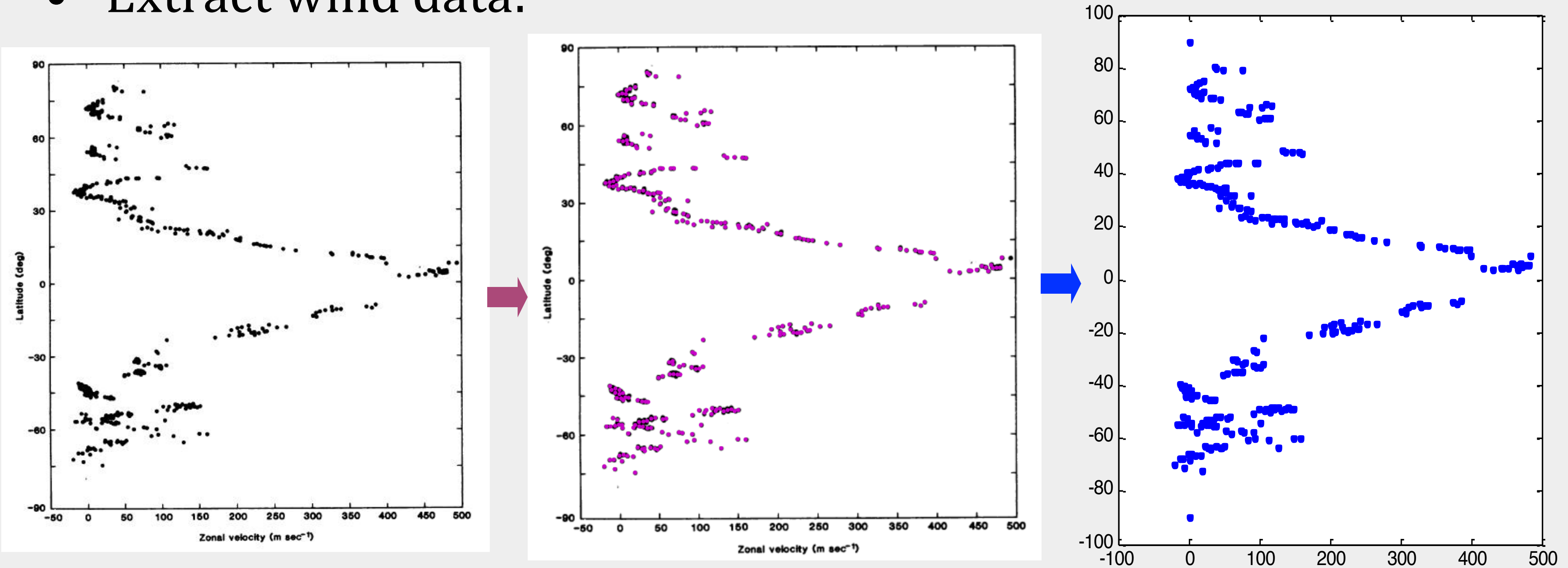
- Extract wind data:



Process

- Going from this data to the geoids is quite involved:

Calculate unperturbed geoid:

$$U(r,\phi) = -\frac{GM}{r} + \frac{GM}{r}\sum_{i=1}^{\infty} J_{2i}\left(\frac{R}{r}\right)^{2i} P_{2i}(\sin\phi) - \frac{1}{2}\omega_{\mathrm{III}}^2 r^2 \cos^2\phi,$$

Iterate:

$$U_{\mathrm{ref}} = U(r_p, \pi/2)$$

$$\Delta U = U(r_{\mathrm{ref}}, \phi) - U_{\mathrm{ref}}$$

$$\Delta r = \Delta U / g_r$$

where

$$g_r(r,\phi) = -\frac{GM}{r^2} + \frac{GM}{r^2}\sum_{i=1}^{\infty}(2i+1)J_{2i}\left(\frac{R}{r}\right)^{2i} P_{2i}(\sin\phi) + \frac{2}{3}\omega^2 r[1 - P_2(\sin\phi)], \quad (2)$$

$$g_\phi(r,\phi) = -\frac{GM}{r^2}\sum_{i=1}^{\infty} J_{2i}\left(\frac{R}{r}\right)^{2i}\frac{dP_{2i}(\sin\phi)}{d\phi} - \frac{1}{3}\omega^2 r\frac{dP_2(\sin\phi)}{d\phi}, \quad (3)$$

$$\psi = \arctan(g_\phi / g_r)$$

Calculate geoid perturbed by winds:

$$h(\phi) = \frac{1}{\langle g\rangle}\int_\phi^{\frac{\pi}{2}} V_W\left[2\omega_{\mathrm{III}} + \frac{V_W}{r_{\mathrm{ref}}(\phi)\cos\phi}\right] \cdot \frac{\sin(\phi + \psi_{\mathrm{ref}})}{\cos\psi_{\mathrm{ref}}} \mathrm{r}_{\mathrm{ref}}(\phi)\,\mathrm{d}\phi,$$

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Curve-fitting/ table lookup

numerical integration

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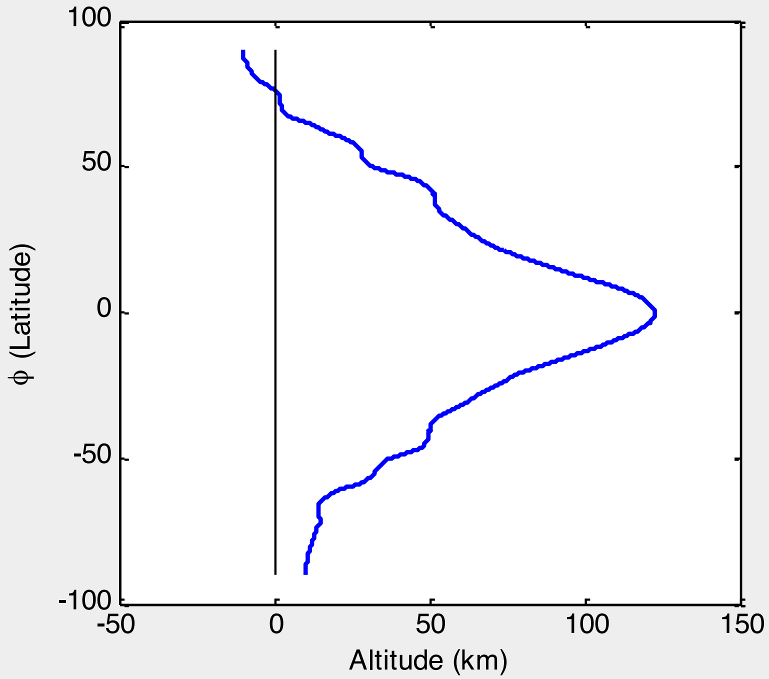
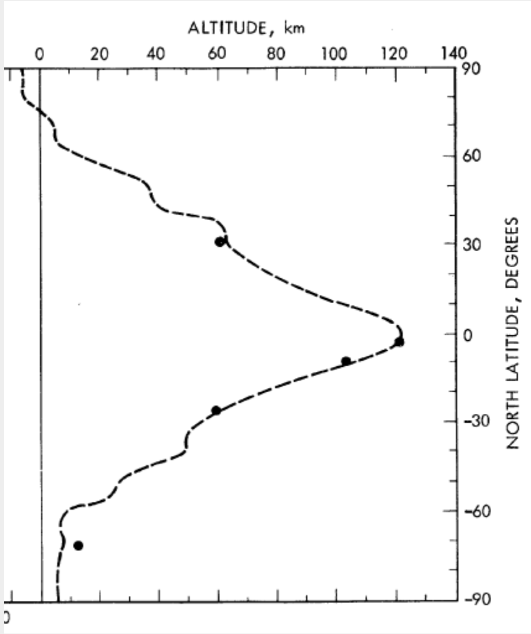
A lot of numerical processes means that getting the exact same answer as someone else is difficult

Results

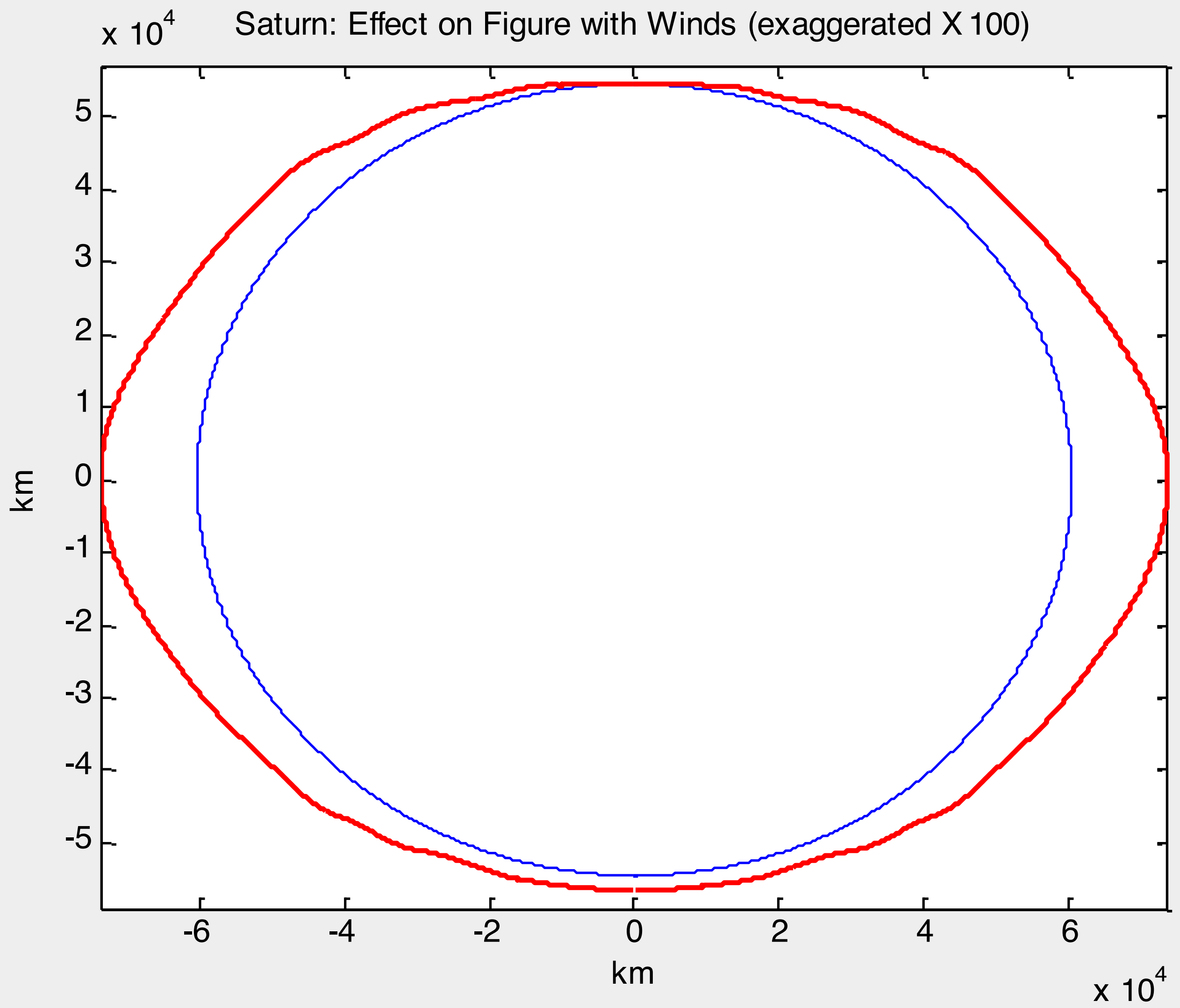




Adapted from ref. 2

- Results match well with published results (maximum altitude variation is within 5%).
- Setting the wind velocity to zero does not solve the centers of figure and mass discrepancy.

Results


Saturn: Effect on Figure with Winds (exaggerated X 100)
x 10^4
5
4
3
2
1
0
-1
-2
-3
-4
-5
km
-6
-4
-2
0
2
4
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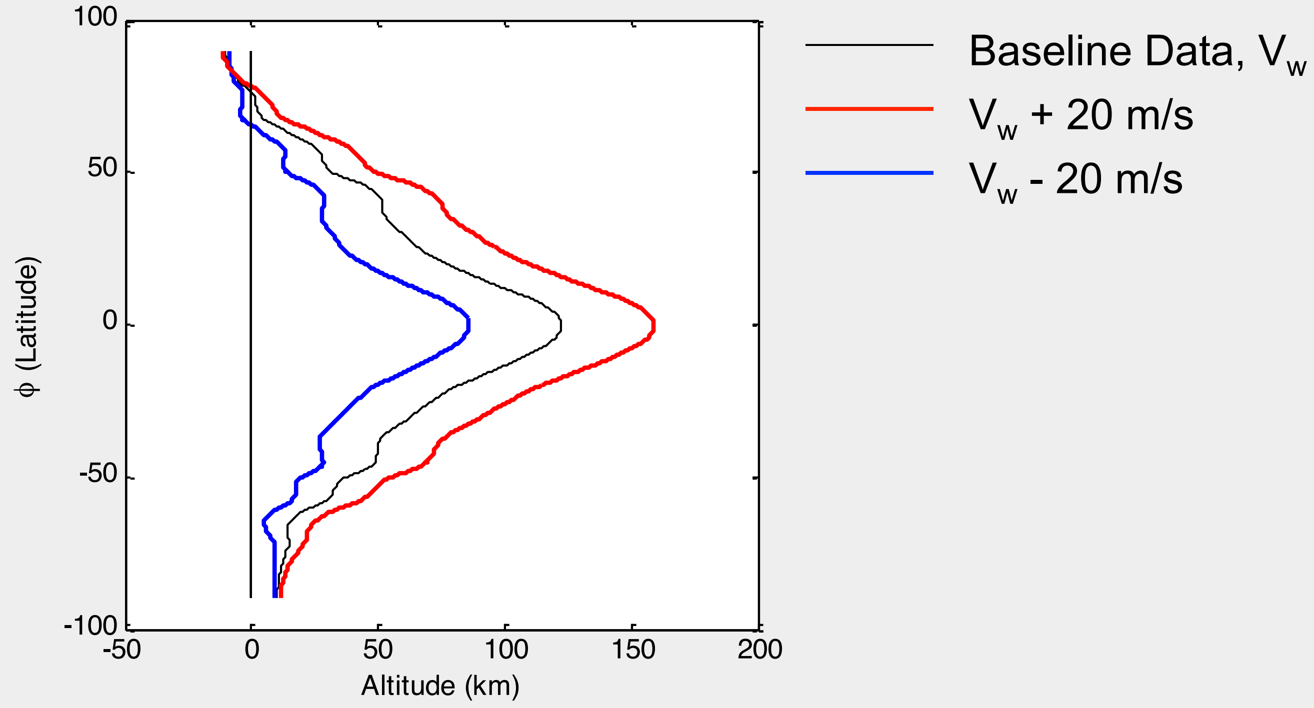
km
x 10^4

Results

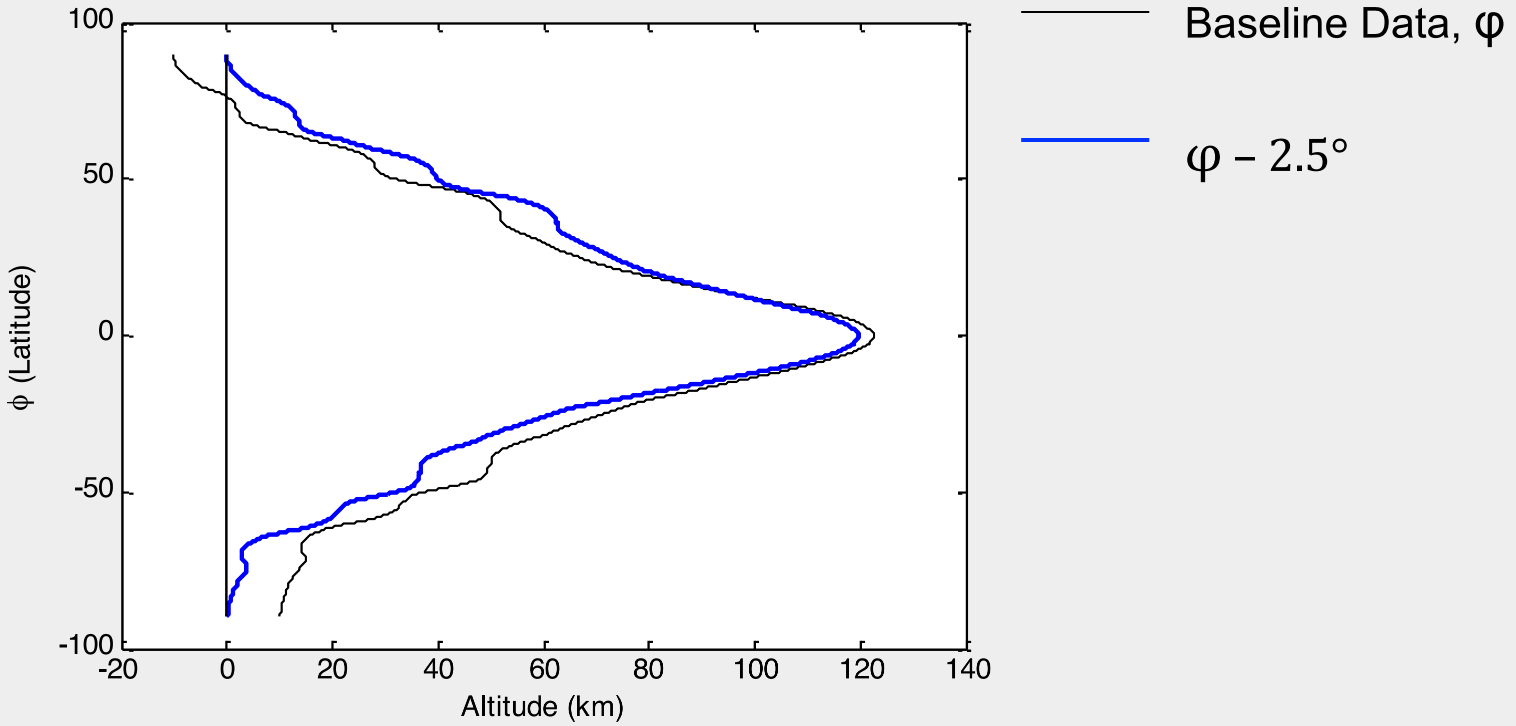
- Maybe the discrepancy is due to uncertainties in the wind measurements:
  - Perturb velocity and location (latitude)

- Perturbing velocity does not have an appreciable effect:



Results

- But perturbing the latitude does have a significant effect.
- Tends to twist the distribution.
- A downward latitude shift of 2.5° brings the poles to the right place:



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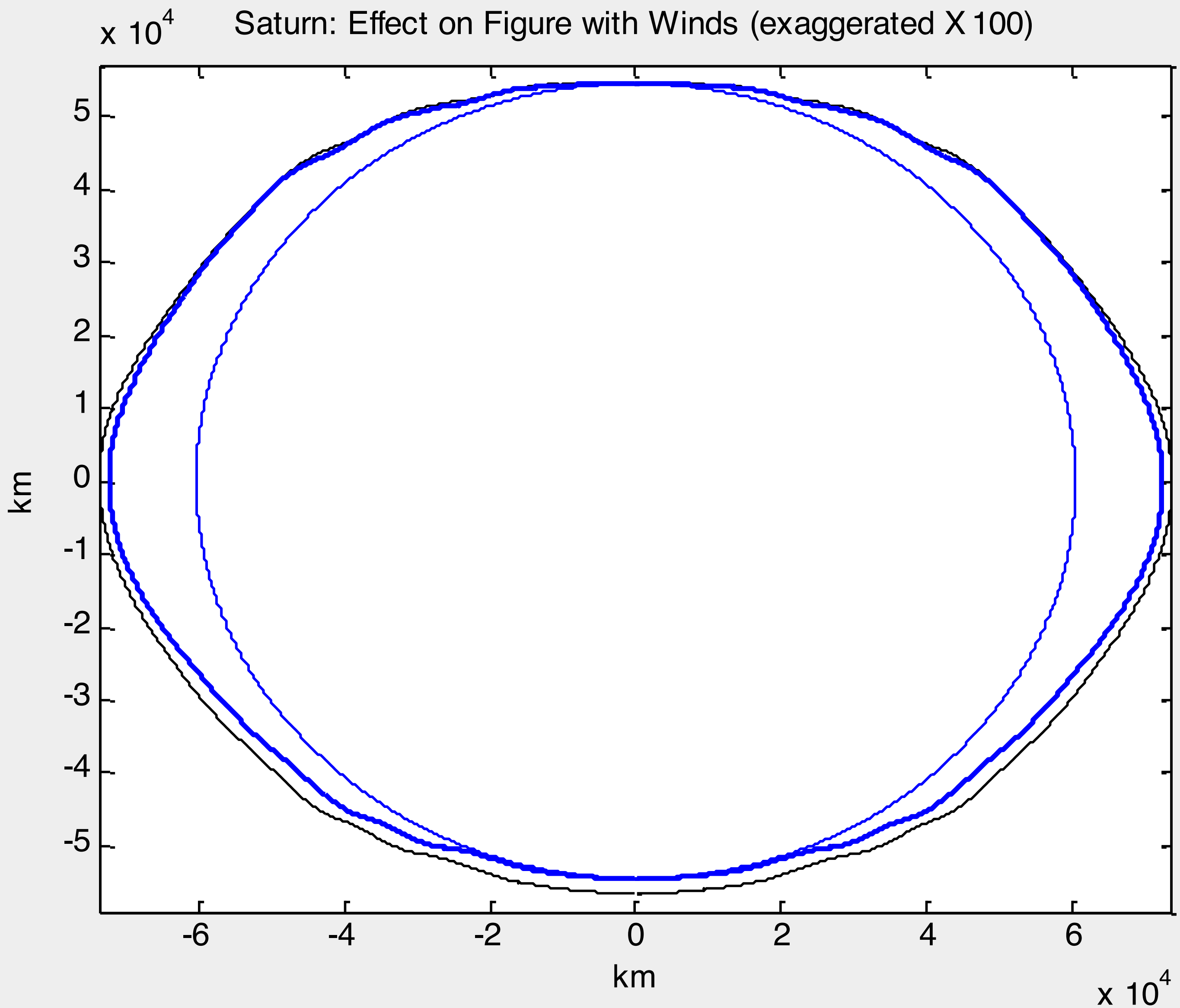
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Questions?

Image Adapted from ref. 4