ASTR 300   Stars and Stellar Systems   Fall 2011

Homework Set No. 2   **Solutions**

1. Why do optical astronomers often put their telescopes at the tops of mountains, while radio astronomers sometimes put their telescopes in deep valleys? (Chapt. 6, Review Question 4)

    Optical astronomers hope to get above as much of the atmosphere as possible. For radio astronomers, interference from man-made radio signals is a major problem. Putting a radio telescope in a deep valley may shield it from such interference.

2. Optical and radio astronomers both try to build large telescopes but for different reasons. How do these goals differ? (Chapt. 6, Review Question 5)

    Optical astronomers build large telescopes mainly to gather as much light as possible. Large telescopes don't help increase angular resolution because that is set by the atmosphere. Radio astronomers need large telescopes to get good angular resolution, since the long wavelengths of radio radio waves would lead to poor resolution if the telescope diameter were not large.

3. An astronomer wants to put a telescope in space that will have a resolving power of 0.02 seconds of arc at visible wavelengths. What must the diameter of the mirror be to achieve this resolution?

    Your text (p 104) gives a formula for the resolution at visible wavelengths: $\alpha = 11.6/D$, where the resolution α is in seconds of arc, and D is the telescope diameter in centimeters.(Also, slide 17, Lecture 5.) Thus we have $0.02 = 11.6/D$, and we see that $D = 580$ cm $= 5.8$ meters. That's 19 feet – a big telescope!

4. We discussed the Stefan-Boltzmann law which gives the energy, E, radiated by a surface at temperature T:

    $$E = \sigma T^4 \text{ Joule/s/m}^2, \quad \text{where} \quad \sigma = 5.67 \times 10^{-8} \text{ Joule/s/m}^2/\text{degree}^4$$

    Suppose a space station has an exterior panel one square meter in area exposed to space. The panel is at a temperature of -20 C (-4 F).

    (a) What temperature units must be used in the Stefan-Boltzmann equation and what is the temperature of the panel in these units?

    This equation requires that the temperature be measured on the Kelvin scale. Thus we add 273 to the Centigrade value: (-20) + 273 = 253 K. (See Table A-4, p 421)

    (b) How much energy/s is radiated into space by the panel (in Joule/s)? (One Joule/s is a power of one Watt).

    Since the area is the standard one square meter, the energy radiated is just $E = \sigma(253)^4 = 232.3$ Joule/s, which is 232.3 Watts.

    (c) At what wavelength does the radiation from the panel peak?

    Using Wien's law (p 129, or slide 8 of Lecture 6) with $T = 253$ K, we find that $\lambda_{max} = 2,900,000/T = 2900000/253 = 11,463$ nm.
    This is in the far-infrared, as expected for an object a such a low temperature.

5. Suppose you are on Mercury, 0.39 AU from the Sun. How bright would the Sun appear compared to its brightness as seen from the Earth?

The brightness is given by the flux you see. From slide 8 of Lecture 7, the flux goes as the luminosity over the distance squared. For the earth $d = 1$AU, while for Mercury $d = 0.39$AU. The sun's luminosity is the same for both, so the flux at Mercury's distance is $1/0.39^2 = 1/0.1521 = 6.57$ times greater than the flux at the earth.

6. In class we derived a formula for the radius of a star, given its effective temperature and luminosity (Slide 17 of "Slides from Lecture 4". It is a bit more accurate to use 5777 K instead of 5800 K for the sun's effective temperature in that equation).

In the appendix of your text, you will find a table, Table A-7, which lists the properties of main sequence stars, including their effective temperature and their luminosity, mass and radius in solar units. I want you to check the self-consistency of this table: In particular, use the luminosity and effective temperature given in the table to **compute** the radius of the M5 star. Then compare your result with the tabulated value.

The equation we will use to find the radius is

$$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot} \left(\frac{5777}{T}\right)^2} \; .$$

Consider the M5 entry in Table A-7: $L/L_\odot = 0.01$ and $T = 2800$ K. Applying the formula we find

$$\frac{R}{R_\odot} = \sqrt{0.01} \left(\frac{5777}{2800}\right)^2 = 0.10 \times 2.06321^2 = 0.4257 \; .$$

On the other hand, Table A-7 gives $R/R_\odot = 0.3$, about 30% smaller. So the radius given is not really consistent with the given luminosity and temperature.

7. Table A-7 also lists the *average density* of these stars. Now the average density ρ is related to the mass and radius by the equation

$$\rho = \frac{Mass}{Volume} = \frac{M}{\frac{4}{3}\pi R^3}$$

From this equation we find that the average density of the sun is $\rho_\odot = 1.41$ gm/cm^3 , in agreement with the table for a G0 star. If we divide the above equation by the same equation evaluated for the sun, we obtain

$$\frac{\rho}{\rho_\odot} = \frac{M/M_\odot}{(R/R_\odot)^3} \quad \text{and solving for the radius:} \quad \left(\frac{R}{R_\odot}\right) = \left[\frac{1.41}{\rho} \frac{M}{M_\odot}\right]^{1/3}$$

Use this last equation along with the mass and density listed in A-7 to calculate the radius of the M5 star in this table. Compare your result with the radius listed in A-7.

Table A-7 gives 0.2 $M_\odot$ for the mass of the M5 star and 10 gm/cm^3 for its mean density. Using the formula above, we then get $R/R_\odot = [(1.41/10)*0.2]^{1/3} = [0.0282]^{1/3} = 0.304.$

Is this calculated value of the radius in better or worse agreement with the tabulated value than the one you calculated in problem 6 above?

It is in much better agreement. (It may be that the mean density was obtained from theory and used to calculate the radius given in the table.)

8. The highest-velocity stars an astronomer might observe have velocities of about 400 km/s. What change in wavelength would this produce in the Balmer gamma line? (Hint: Wavelengths are given on page 133.) (Chapter 7, Problem 10)

The equation for the Doppler shift is given on p 139 (or slide 16 of Lecture 7): $\Delta\lambda/\lambda = V_r/c$. Since $c = 3 \times 10^8$ m/s $= 3 \times 10^5$ km/s, we have $\Delta\lambda/\lambda = 400/300000 = 0.0013333$ Now from the figure at the top of page 133, the wavelength of $H\gamma$ is $\lambda = 434.0$ nm. Thus we find the wavelength shift is $\Delta\lambda = 0.0013333 \times 434.0 = 0.5787$ nm.