ASTR 300 Stars and Stellar Systems Spring 2011

Homework Set No. 3 **Solutions**

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1. Why is it difficult to find out how common the most luminous stars are ? The least luminous stars ? (Chapt. 9, Review Question 13)

   See p 186 of your text. It is difficult to count the most luminous stars in a known volume of space because these stars are very rare. On the other hand, the least luminous stars are quite common, but are so faint they are hard to see even if they are close.

2. The parallax of the bright star Vega is 0.129 seconds of arc. What is the distance of Vega in parsecs ? In light-years ?

   We have the relation $d(pc) = 1/p(arcsec)$ so the distance to Vega is $d = 1/0.129 = 7.752$ parsecs. One parsec = 3.26 light-years, so $d = 3.26 \times 7.753 = 25.3$ ly.

   By how much, during the year, does the position of Vega shift relative to distant background stars ? If an astronomer were observing from the surface of Jupiter's moon Europa, by how much would Vega shift during the Jovian year ? (Hint: Jupiter is 5.2 AU from the sun.)

   A parallax of 0.129 means that when the observer moves 1 AU, the star shifts 0.129 arcsec. Since the *diameter* **of the Earth's orbit is 2 AU, the shift seen during the year will be $2 \times 0.129 = 0.258$ arcsec.**

   The radius of Jupiter's orbit is 5.2 AU so the diameter of its orbit is 10.4 AU. Thus the astronomer on Europa sees a shift of $10.4 \times 0.129 = 1.34$ arcsec.

3. A binary star system contains one star of mass 0.8 $M_\odot$ and another of mass 2.2 $M_\odot$. They are in circular orbits and the distance between the centers of the stars is 1.5 AU.

   (a) What is the period P of the binary?
   Kepler's third law is $(M_1+M_2)P^2 = a^3$. In this case $M_1+M_2 = 2.2+0.8 = 3$ and $a = 1.5$, so $P^2 = (1.5^3)/3 = 1.125$. Thus the period is $P = \sqrt{1.125} = 1.06066$ years.

   (b) Find the location of the center of mass (i.e., how far is it from the center of the more massive star?).
   We can use the equation given on Lect 10 & 11, slide 18 (or earlier at Lect 2 (cont), slide 12):

   $$a_1 = \frac{m_2}{m_1+m_2}\,a = \frac{0.8}{2.2+0.8}\,1.5 = 0.4 \ \text{AU}$$

   (c) Compare the gravitational force of the two stars on a small mass m located at the center of mass. Are the forces equal?
   The distance of the center of mass from the second star is just $a_2 = a - a_1 = 1.5 - 0.4 = 1.1$. Then we see that the gravitational forces on a mass m at the center of mass will be

   $$F_1 = G\frac{m_1\,m}{a_1^2} = G\frac{2.2\,m}{0.4^2} \quad \text{and} \quad F_2 = G\frac{m_2\,m}{a_2^2} = G\frac{0.8\,m}{1.1^2}$$

The forces are clearly not equal. Their ratio is

$$\frac{F_1}{F_2} = \frac{m_1}{m_2}\frac{a_2^2}{a_1^2} = \frac{2.2}{0.8}\frac{1.1^2}{0.4^2} = 20.8$$

4. A double-lined spectroscopic binary has a period of 30 days and the velocities of the two stars are 70 km/s and 200 km/s.

   (a) What is the ratio of the masses of the stars?

   The ratio of the masses is the same as the ratio of the velocities: $200/70 = 2.857$. The less massive star is the one moving at 200 km/s.

   (b) Assume we are in the plane of the orbit. What is the total separation of the stars? What is the total mass and the individual masses of the stars?

   We need the period in seconds, since the velocity is in km/s. Now the period in seconds is just $P = 30 \times 24 \times 60 \times 60 = 2.592 \times 10^6$ s. In the time P, the star travels one circumference, $2\pi a$, of its orbit. Since $(time) \times (velocity) = (distance)$, we have $2\pi a = Pv$. Since we want the total separation a, we must use the total velocity $v = 70 + 200 = 270$ km/s. We then find that $a = vP/2\pi = 270 \times 2.592 \times 10^6/2\pi = 1.114 \times 10^8$ km. In AU, that is $a = 1.355 \times 10^8/1.5 \times 10^8 = 0.7426$ AU. Also, the period P in years is $P = 30/365.24 = 0.08214$ yr. Then Kepler's third law gives us the total mass:

   $$M_1 + M_2 = a^3/P^2 = (0.7426)^3/(0.08214)^2 = 60.7\ M_\odot$$

   We saw above that $M_1/M_2 = 2.857$. Now since we can write

   $$(M_1 + M_2) = M_2\left(\frac{M_1}{M_2} + 1\right) \quad \textbf{we have} \quad 60.7 = M_2(2.857 + 1).$$

   Solving for M_2 we find $M_2 = 60.7/3.875 = 15.7$ $M_\odot$, and then $M_1 = 60.7 - 15.7 = 45$ $M_\odot$.

   (c) If instead, the orbit is inclined at an angle of 45 degrees, what are the true velocities of the stars? In this case, what is their separation? What is the total mass and the individual masses?

   If the orbit is inclined, we will only see a fraction of the true orbital velocity. The fraction will in fact be $\cos(45^o) = 0.7071$. Thus $v_{observed} = v_{true}\cos(45^o)$, so $v_{true} = (1/\cos(45^o))v_{observed} = 1.414 * v_{observed}$. Since $v_{observed} = 270$ km/s, $v_{true} = 1.414 * 270 = 381.8$ km/s.

   We then see that the semi-major axis $a = vP/2\pi$ must increase by a factor of 1.414. The period remains the same, so the total mass, which goes as a^3/P^2, must increase by a factor of $(1.414)^3 = 2.82$. Thus the true total mass would be $2.82 * 60.7 = 171.7 M_\odot$. Likewise the individual masses would be $127.4 M_\odot$ and $44.3 M_\odot$.

   (By the way, $127 M_\odot$ is really high for any star – but maybe not impossible for a metal-poor star.)

5. If the orbital velocity of the eclipsing binary in Figure 9-20 of your text is 153 km/s and the smaller star becomes completely eclipsed in 2.50 hours, what is its diameter ? (Problem 14, chapter 9)

   The time for the smaller star to become totally eclipsed is $T = 2.5\,hr \times 60\,min/hr \times 60\,\sec/min = 9000\ s$. The distance the star travels in that time is just $d = Tv = 153\ km/s \times 9000\ s = 1.38 \times 10^6\ km$. This is the diameter of the smaller star. (Converted to solar radii, we find $d = 2\ R_\odot$.)

   P.S. You have noticed, haven't you, that answers to even numbered problems are given on page 449 of your text?

6. Use the table below to answer the following questions. You may also consult a standard H-R diagram. For each question, give a brief explanation (in one sentence).

Star	Spectral Type
Aldebaran	K5 III
Alpha Centauri A	G2 V
Antares	M1 I
Canopus	F0 II
Fomalhaut	A3 V
Regulus	B7 V
Sirius	A1 V
Spica	B1 V

   (a) Which star has the greatest luminosity ?

   Look at Figure 9-9 of your text. The only supergiant (class I) is Antares, M1 I. It is the most luminous.

   (b) Which star has the highest surface temperature ?

   Spica. B1 spectral type is hotter than Regulus which is B7.

   (c) Which star has the lowest surface temperature ?

   Antares. M type stars are the coolest. '

   (d) Which star is the most similar to the Sun ?

   Alpha Centauri A. It is type G2 V and so is our Sun.

   (e) Which star is a red supergiant ?

   Antares again. It's a supergiant (I) and type M are red.

   (f) Which star has the largest radius ?

   And again it's Antares – look at Figure 9-9.

   (g) Which star has the smallest radius ?

   That would have to be Alpha Centauri A. Of all the class V stars, it is the furthest down the main sequence.