ASTR 300   Stars and Stellar Systems   Spring 2011

Homework Set No. 5   Solutions

1. The table in Figure 12-2 of your text (p 234) is from a computer model of the solar interior. A far more detailed table is given on the class web page; the table can also be found at this web address:

http://www.sns.ias.edu/~jnb/SNdata/Export/BS2005/bs05_agsop.dat

This model also includes the relative abundance of hydrogen, helium (^{4}He and ^{3}He) and some other isotopes. The star is divided into over 1000 zones, and the list starts at the center of the sun and goes outward (the table in Fig 12-2 goes from the surface inwards).

(a) Compare the values in Fig 12-2 at a fractional radius of $R/R_\odot = 0.1$ with the detailed model – how close are they? The columns called "X" and "Y(He4)" represent the fractional abundance, by mass, of H and ^{4}He. What are these values at $R/R_\odot = 0.1$? How do they compare to the values at the sun's surface (which reflects the sun's original composition)? How do you explain the difference?

Table 12-2 gives for R,T,ρ,M and L: 0.10 13.1 89.0 0.073 0.40
while the detailed model gives   0.10 13.0 87.4 0.076 0.44
So the agreement is pretty good.

At $R/R_\odot = 0.1$, X=0.5572 and Y(He4)=0.4278, while at the surface, X=0.7583 and Y=0.2291. Thus the He/H ratio at the surface is 0.30 while it is 0.77 in the interior. This is because the proton-proton chain operating over 4.5 billion years has converted a lot of the hydrogen in the interior into helium.

(b) The element with the highest density is osmium. It has a density of 22.6 gm/cm^3, twice as dense as lead. Yet the H and He inside the sun can be much denser. At what radius is the density of the solar material equal to that of osmium? What is the temperature at that point? Is that temperature high enough for hydrogen fusion?

That radius is $R/R_\odot = 0.245$. At that point the temperature is 7.95 million degrees K. At temperatures below 10 million degrees there can be little if any H fusion.

(c) In the inner part of the sun, energy flows outward by radiation. But at about $R/R_\odot = 0.7$, the material becomes too opaque and the convective zone begins. What is the temperature and density at this transition point? What fraction of the sun's mass is contained in the convective zone (the outer 30% of the sun's radius)?

At this radius, $T = 2.26 \times 10^6$ K and $\rho = 0.197$ g/cm^3. Since $M/M_\odot = 0.97495$ at this point, the mass remaining outside this point is $1 - 0.97495 = 0.02505$, which is the fraction of the sun's mass in its convective zone.

(d) What is the temperature and density in the zone where the abundance of the light helium isotope ^{3}He (He3) is greatest?

Y(He3) reaches a maximum value of 0.0033 at a $R/R_\odot = 0.275$, where $T = 7.2 \times 10^6$ K and $\rho = 16.1$ g/cm^3.

2. Why do some astronomers say that brown dwarfs are not really stars? Why are brown dwarfs hard to study?

Some astronomers would say that true stars must have nuclear fusion as an internal energy source. Brown dwarfs are too cool the burn hydrogen.
Even though there are nearby brown dwarfs, their luminosity is so low that their faintness makes them difficult to observe & study.

3. We saw there is a mass-radius relation for white dwarfs (see slide 12 from lecture 16). What is the mass of a white dwarf with the same radius as the Earth? The force of gravity is given by $g = GM/R^2$ for a body of mass M and radius R. By what factor is the gravity on the surface of a white dwarf greater than that on the Earth, if the two have the same radius?

The equation gives $R = 1R_\oplus$ for a mass of 0.89808 $M_\odot$ – 0.9 $M_\odot$ is close enough. Since the radius is $R_\oplus$ for both the Earth and the white dwarf, if we take the ratio of g_{wd} to $g_\oplus$ we have $g_{wd}/g_\oplus = M_{wd}/M_\oplus$. So all we need do is find the Earth's mass in solar mass units.
From Table A-5 of the text (p 422), $M_\oplus/M_\odot = 5.98 \times 10^{24}$ kg /1.99×10^{30} kg $= 3.005 \times 10^{-6}$. Thus we find that $g_{wd}/g_\oplus = 0.9/3 \times 10^{-6} = 300,000$.

4. Look again at slide 12 from lecture 16. What is the radius of a 0.5 $M_\odot$ white dwarf? What is the radius of a 1 $M_\odot$ white dwarf? (Use the formula to calculate your results and check them against the graph.) Why is there no reference to the surface temperature of the white dwarf in this diagram? If these two dwarfs have the **same** temperature (say 10,000 K), which is more luminous? What is the ratio of their luminosities?

For a 0.5 $M_\odot$ white dwarf the radius is 1.537 $R_\oplus$, while for a 1 $M_\odot$ white dwarf the radius is 0.8788 $R_\oplus$, which agree with the graph.
The radius of a white dwarf does not depend upon its temperature, only its mass.
If they have the same temperature, each unit of area on the surface of one radiates the same as a unit of area on the other. Thus the ratio of their luminosities goes as the ratio of their surface areas, which in turn goes as the ratio of their radii squared. The 0.5 $M_\odot$ white dwarf, being larger, will be more luninous, and the ratio of luminosities will be $(1.537/0.8788)^2 = 1.749^2 = 3.06$.

5. What happens to the radius of a white dwarf as it cools off? A white dwarf with a surface temperature of 18,000 K has a luminosity of 1/100 $L_\odot$. What is the radius of this object (in solar radii)? (Hint: Use the equation on slide 17 from lecture 4, not the mass-radius relation.) When it has cooled to 9000 K, what will its luminosity be then?

The radius doesn't change. White dwarfs are supported by the pressure of degenerate electrons, which does not depend upon temperature.
We use the equation we've used many times (e.g. Lect 4, slide 17):

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{5800}\right)^4 \quad \rightarrow \quad \frac{1}{100} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{18000}{5800}\right)^4$$

$0.01 = (R/R_\odot)^2 \times 92.76$ so that $(R/R_\odot)^2 = 1.078 \times 10^{-4}$ and $R = 0.0104 R_\odot$

Since the radius does not change, the change in luminosity as the white dwarf cools is entirely due to the change in temperature. The temperature changes by a factor of $18000/9000 = 2$, so the luminosity decreases by a factor of $2^4 = 16$. Thus the luminosity drops from $0.01 L_\odot$ to $0.01/16 = 6.25 \times 10^{-4} L_\odot$.

6. If the stars at the turnoff point in a star cluster are of spectral type F0, about how old is the cluster?

From Table 12-2 we see that an F0 star has a mass of 1.7 $M_\odot$. The same table says it will spend 3×10^9 years on the main sequence, so that is the age of the star cluster.

7. Use the formula in chapter 12 of your text to compute the life expectancy of a 0.4 solar-mass star. Why might this be an underestimate if the star is fully mixed?
(Chapter 13, Problem 1)

The time on the main sequence compared to the sun varies as $1/M^{2.5}$. Since $1/0.4^{2.5} = 1/(0.4)(0.4)\sqrt{0.4} = 9.9$, such a star should have a main sequence lifetime of 9.9 times that of the sun, that is, $9.9 \times (10 \times 10^9)$ yrs, nearly 100 billion years. But a star like the sun only burns a fraction (about 10%) of its hydrogen before leaving the main sequence. While a 0.4 $M_\odot$ star may burn a larger fraction of its hydrogen, we do not think it would burn it all before leaving the main sequence. If, however, it were fully mixed, it would burn *all* its hydrogen and could thus live much longer than our 100 billion yr estimate.

8. Presumably, all the white dwarfs in our galaxy were produced by sun-like stars of medium mass. Why couldn't any of these white dwarfs have been produced by the deaths of the lowest mass stars?
(Chapter 13, Review Question 2)

A one solar mass star will spend 10 billion years on the main sequence. The universe is only 13-14 billion years old. From the formula $T = 1/M^{2.5}$ and the sun's lifetime, we see that a star of 0.9 solar masses should spend 13 billion years on the main sequence. So no stars of lower mass would have had time to become white dwarfs, even if they formed just after the big bang.

9. The coolest stars at the centers of planetary nebulae are about 25,000 K. Why don't astronomers see planetary nebulae with cooler central stars? (Hint: What kind of photons excite the gas in a planetary nebula?)
(Chapter 13, Review Question 5)

To be seen as a planetary nebula, the ejected envelope of the star must be ionized and heated by the ultraviolet photons of the central star. Stars with temperatures below 25,000 K produce very little such radiation – the nebula doesn't become visible until the star's surface temperature exceeds 25,000 K.