ASTR 300 Stars and Stellar Systems Spring 2011

Homework Set No. 6 Solutions

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1. Astronomers believe that a 13 $M_\odot$ star will end as a neutron star. What is the maximum mass of a neutron star? What happens to the rest of the mass of the original star?

   The maximum mass of a neutron star is thought to be between 2 and 3 solar masses – surely below 3 $M_\odot$. A 13 $M_\odot$ star explodes as a core-collapse supernova in forming the neutron star, so 13 - 3 = 10 $M_\odot$ of material are expelled in that explosion.

2. According to the modern model of a pulsar, if a neutron star formed with no magnetic field at all, could it be a pulsar? Why or why not?

   We do not think a neutron star could be a pulsar without a magnetic field. The radiation which defines the pulsar is thought to be produced by electron- positron pairs, accelerated by the electric field which is in turn produced by the spinning magnetic field. Without the magnetic field, this could not happen. (See p 284, bottom)

3. Why would astronomers naturally assume that the first discovered millisecond pulsar was relatively young? (Chapter 14, Review Question 10)

   Astronomers had found that pulsars slow down with age as the pulsar wind carries angular momentum and energy away. Thus a very rapidly spinning pulsar was assumed to be very young. Later they realized that an old, slowly spinning pulsar could be "spun up" by mass transfer from a companion star.

4. What is "The Black Widow" pulsar and what puzzling objects does it help explain?

   "The Black Widow" pulsar is a millisecond pulsar with a very low mass companion which is being evaporated by the radiation of the Black Widow – soon it will have completely destroyed the companion and will be a solitary pulsar. (see p 289)
   The Black Widow is the key to understanding those millisecond pulsars that have no companion star. Astronomers were puzzled by them because they believe that all millisecond pulsars have been spun up by mass transfer from a companion star. This shows that some can be spun up but then destroy the star that spun them up.

5. Suppose that a neutron star has a radius of 10 km and a temperature of 1,000,000 K. How luminous is it? Give your answer in solar luminosities. At what wavelength would the radiation peak? (Use Wien's law.)

   We use the equation we have used many times before (p 173):
   $(L/L_\odot) = (R/R_\odot)^2\ (T/T_\odot)^4$.
   Now $T_\odot = 5800$ K, and we will need to get the radius of the neutron star in solar units:
   $(R/R_\odot) = (10/6.96 \times 10^5) = 1.437 \times 10^{-5}$. **Thus we get:**
   $(L/L_\odot) = (1.437 \times 10^{-5})^2\ (1 \times 10^6/5800)^4\ =\ 0.182$.
   So the neutron star is only about 1/5 the sun's luminosity.

To find the wavelength at which the radiation peaks, we use Wien's law (p 129):

$\lambda_{max} = 2.90 \times 10^6/T = 2.90 \times 10^6/10^6 = 2.90$ **nanometers.**

Recall that visible light has a wavelength of the order of 500 nm. We see this λ_{max} is much shorter; 2.9 nm is in the region of X-rays (p 101).

6. Your text gives the formula for the Schwarzschild radius of a black hole (p 292). Earlier in this course, you encountered the formula for the velocity of a body in a circular orbit of radius r about a body of mass M (p 82).
   Now, the last stable circular orbit about a black hole has a radius equal to 3 times the Schwarzschild radius. Combine these two expressions to find the velocity of a body in this last circular orbit. Express your result as a fraction of the speed of light. (This result isn't quite right since you are using Newton's gravity for V_c rather than Einstein's theory, but it's pretty close.)

   The Schwarzschild radius is $R_S = 2GM/c^2$ so the radius of the last stable orbit is $r = 3R_S = 6GM/c^2$. The circular velocity is given by $V_c = \sqrt{GM/r}$. Putting r into this equation gives $V_c = \sqrt{GM/(6GM/c^2)} = \sqrt{c^2/6} = c/\sqrt{6}$. Now, $1/\sqrt{6} = 0.408$, so V_c is about 40% the speed of light. (The correct result using Einstein's equations is actually $\frac{1}{2}$c.)

7. There is a black hole with a mass of $4 \times 10^6\ M_\odot$ at the center of our Galaxy.
   (a) What is the Schwarzschild radius of this black hole?
   (b) What is the circumference of the last stable circular orbit about this black hole? (See the preceding problem.)
   (c) Using the velocity you obtained above, find the period of a body that is in the last stable orbit about this black hole. Express your result in minutes.

   The Schwarzschild radius is $R_S = 3M$ for M in solar masses and R_S in km (p 298). Thus for the galactic center black hole we find $R_S = 1.2 \times 10^7$ km.
   Converting to AU we get
   $R_S = (1.2 \times 10^7\ km)/(1.5 \times 10^8\ km/AU) = 0.080$ AU. (This is only about 17 $R_\odot$, smaller than most giant stars!)
   Now the period P is just the distance around the orbit $d = 2\pi(3R_S)$ divided by the velocity $v = 0.408c$:
   $P = d/v = 2\pi\ 3R_S/(0.408c) = 46.2\ R_S/c = 46.2\ (1.2 \times 10^7)/(3 \times 10^5) =$ 1848 sec, using $c = 3 \times 10^5$ in km/s. So the orbital period is only about 30 min!

8. A neutron star and a white dwarf have been found orbiting each other with a period of 11 minutes. If their masses are typical, what is their average separation? Compare their separation with the radius of the sun, 7×10^5 km. (Hint: See Chapter 9)
   (Chapter 14, Problem 3)

   We will use the generalized form of Kepler's 3rd law: $(M_1 + M_2)P^2 = a^3$. Let's take the mass of the white dwarf to be $M_1 = 1M_\odot$ and the mass of the neutron star to be $M_2 = 1.4M_\odot$. Then $(M_1 + M_2) = 2.4M_\odot$. The number of minutes in a year is just $365.24*24*60 = 524946$, so the period in years is $P = 11/525,000 = 2.09\times10^{-5}$. Thus we have $a^3 = 1.05 \times 10^{-9}$. Taking the cube root we get $a = 0.00102$ AU. Since

one AU $= 1.5 \times 10^8$ km, the semimajor axis of the orbit is $a = 1.52 \times 10^5$ km. In terms of the sun's radius this is $1.52 \times 10^5/7 \times 10^5 \; = \; 0.22$. Thus the orbit of this system would easily fit inside the sun.

9. The speed of Saturn in it's orbit is 9.64 km/sec. Suppose a very distant observer saw the Sun's light decrease as Saturn passed across the Sun's disk.

    (a) How long would the decrease in light last?

    (b) By what fraction would the Sun's light decrease?

    (c) How long would the observer have to wait to see the next eclipse?

    (d) How close to the plane of Saturn's orbit would the observer need to be in order to see this eclipse? (I.e., how many degrees above or below Saturn's orbital plane?)

    (See Table A-10, p 424, for properties of planets.)

    (a) We saw that the sun's radius is about 7×10^5 km, so its diameter is 1.4×10^6 km. The time to cross the sun's disk is thus (time)=(distance)/(velocity)=$1.4 \times 10^6/9.64 = 145230$ sec. That is, about 40 hours.

    (b) The light will decrease by the ratio of the area of Saturn's disk to the sun's disk. The area of a circle of radius R is πR^2, so the ratio is

    $R^2_{Saturn}/R^2_{\odot} \; = \; (6.03 \times 10^4)^2/(6.96 \times 10^5)^2 \; = \; 0.0075$ **, a decrease of only 0.75%.**

    (c) Since the orbital period of Saturn is 29.46 years, we would have to wait nearly 30 years for the next eclipse.

    (d) Here we need to consider the triangle formed by the lines from Saturn to to the sun's center and to it's edge. This is a "skinny triangle", so the angle at Saturn (in radians) is given by $R_{\odot}/D_S$, where D_S is the distance from the sun to Saturn. Thus

    $angle \; = \; R_{\odot}/D_S \; = \; 7 \times 10^5 km/1.427 \times 10^9 km \; = \; 0.00049$ **radians.**

    This is only about 1.7 arcmin, so the chance of being close enough to the plane of the orbit is quite small.

10. Suppose a star of spectral type F3 has been found by the Kepler satellite to have a planetary companion which passes in front of the star every 12.3 days. From this star's spectral type, we know that its radius is 1.3 $R_{\odot}$. During the planet's transit, the star's light is seen to decrease by 0.31%. What is the radius of the planet it terms of the star's radius? What is its radius in units of Jupiter radii?

    This is like 9(b) above, except that we know the decrease (0.0031) but don't know the planet's radius, R_p. So the equation we need to solve is

    $R_p^2/R^2_{star} \; = \; 0.0031$ **or** $R_p^2 \; = \; 0.0031 \, R^2_{star}$

    Taking the square root gives the result: $R_p \; = \; 0.0557 \, R_{star} \; = \; 0.0724 \, R_{\odot}$.

    Since $R_{Jupiter}/R_{\odot} \; = \; 7.15 \times 10^4/6.96 \times 10^5 \; = \; 0.1027$,

    $R_p \; = \; (0.0724/0.1027) \, R_{Jupiter} \; = \; 0.705 \, R_{Jupiter}$.

    (Yes, the period of 12.3 days was a red herring.)