ASTR 498N Stellar Evolution Spring 2014

Problem Set No. 3 Due Mar 25, 2014

1. Show that if the functional dependence of the opacity is given by

$$\kappa_\nu \;\propto\; \rho\, \nu^{-3}\, T^{-1/2} \;\;,$$

then the Rosseland mean opacity κ follows

$$\kappa \;\propto\; \rho\, T^{-3.5} \;\;,$$

which is just Kramer's law. (Hint: If you encounter a fearsome *definite* integral, remember that it's just a definite number – you don't have to actually evaluate it.)

2. Do problem 3.7 of the Phillips text.

3. We saw that the rate of thermonuclear reactions is determined by the integral over the product of two exponentials,

$$e^{-E/kT}\; e^{-(E_G/E)^{1/2}}$$

where E is the kinetic energy of the colliding nuclei and E_G is a constant called the "Gamow energy". Derive an expression for the energy E_0 where this product reaches a maximum (the "Gamow peak").

The result can be written as (you need not derive this particular form):

$$E_0 \;=\; 1.22 \left(Z_i^2 Z_j^2\, \mu\, T_6^2\right)^{1/3} \quad \text{keV}$$

where μ is the reduced mass of the nuclei in atomic mass units (AMU), and T_6 is the temperature in millions of degrees K.

Evaluate E_0 for the sun's core, where $T = 1.5 \times 10^7\ K$, for (a) the first step of the proton-proton chain, and (b) the first step of the CNO bi-cycle.

4. Do problem 4.3 of the Phillips text. Note that the upper and lower limits are obtained by assuming either all branch II or all branch I. Evaluate both the total number of neutrinos produced by the sun and the flux cm^{-2} reaching the earth for these two cases. Also, evaluate the flux for the "Standard Solar Model" of Bahcall as given in the text.