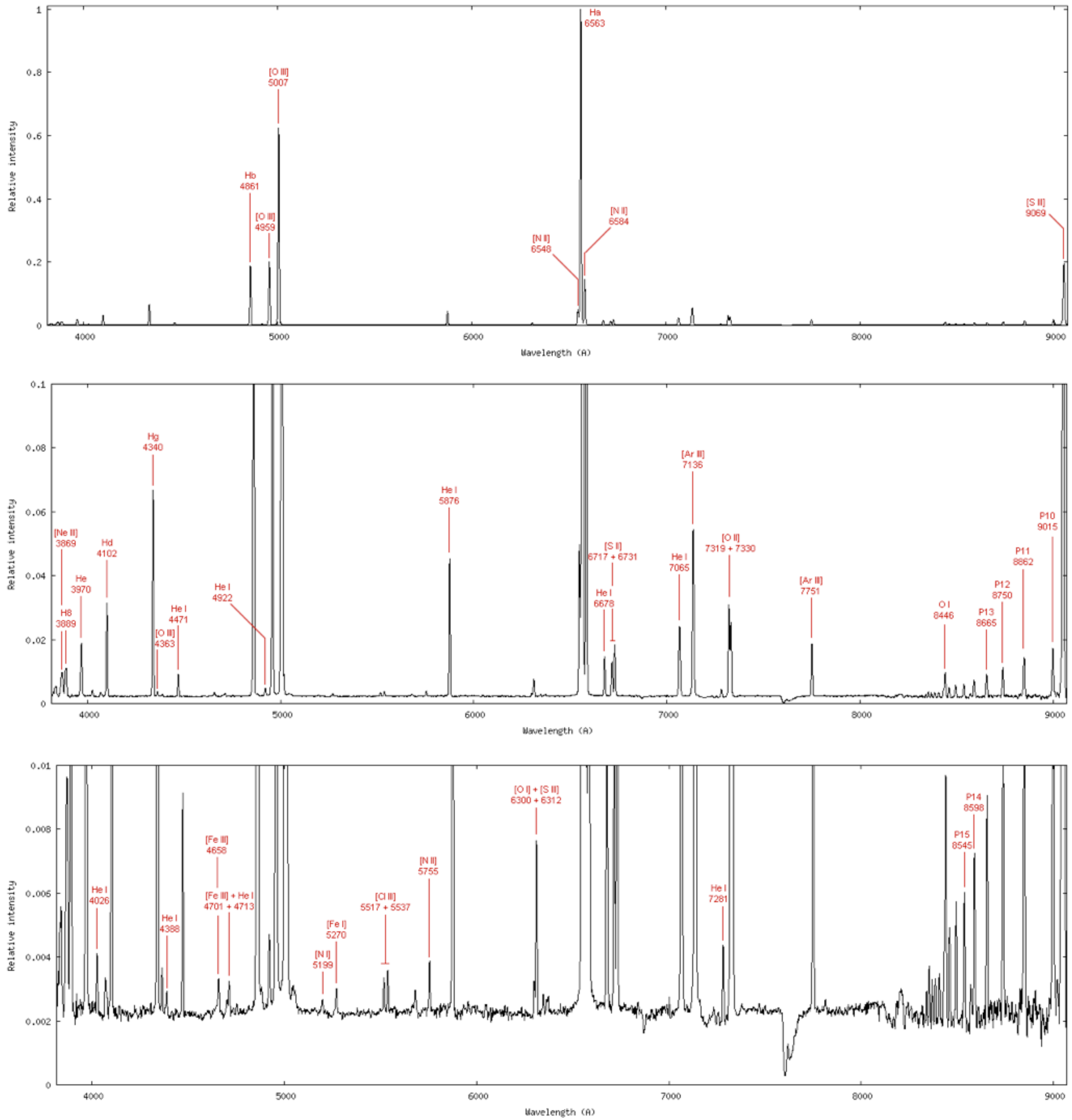
Collisionally Excited Emission Lines

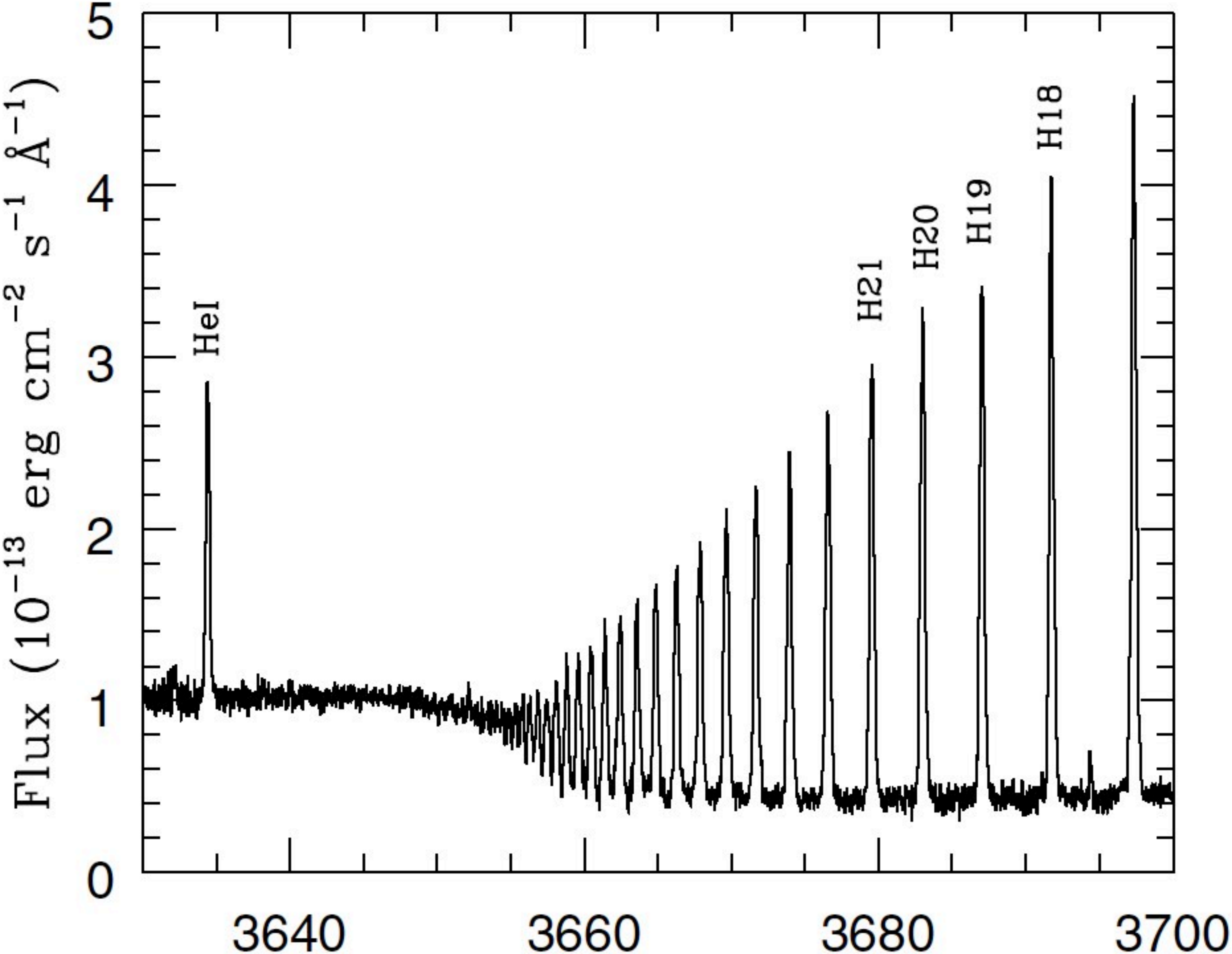
The spectra of nebular gas shows prominent lines of H and He which we have seen are produced by recombinations followed by cascades to the ground state. As a result, the strength of these lines depends upon the abundance of the recombining ion: N(H+) powers H I lines, N(He+) powers He I lines and N(He++) powers He II lines. But there are other equally strong lines of elements like oxygen, nitrogen and sulfur. If we look at a table of the relative abundances of some of these elements in the sun shown below (nebular abundances are similar) we see a puzzle. Oxygen is nearly 2000 times less abundant than hydrogen, yet some of the oxygen lines are stronger than the H lines. How can that be?

Atomic Number	Element Symbol	Relative Number N_X/N_H	Mass Ratio M_X/M_H
1	H	1	1
2	He	0.0955	0.382
6	C	0.000295	0.00354
7	N	0.0000741	0.00104
8	O	0.000537	0.00859
10	Ne	0.0000933	0.00188
16	S	0.0000145	0.000436
26	Fe	0.0000347	0.00194

The answer lies in the fact that the strong lines of the heavier elements are not produced by the recombination of ions, but rather by the excitation of the low-lying levels of these atoms or ions followed by the emission of radiation as they return to the ground level. The excitations are due to collisions of the ions with thermal electrons. The strength of the lines will depend upon the abundance of the ion (i.e., N(O+) powers O II lines) but also very strongly on the temperature of the nebular gas. The first excited levels of H and He are far above the ground state (10.2 eV for H) so that the electrons at T=10,000 K don't have the energy to excite them. However, as we will see, the atoms and ions of elements like O, N, C, Ne and S have low-lying levels that can be excited.

Optical and Near IR Spectrum of the Orion Nebula (M 42)





Flux (10^{-13} erg cm^{-2} s^{-1} Å^{-1})
0
1
2
3
4
5
3640
3660
3680
3700
HeI
H21
H20
H19
H18

Consider an atom or ion with levels 1 and 2. The rate of collisions upward, from 1 to 2, we call R_{12} and the reverse rate downwards, R_{21}. They can be written as

$$R_{12} = n_e N_1 \, q_{12} \qquad \text{and} \qquad R_{21} = n_e N_2 \, q_{21}$$

Then in terms of a *Maxwellian averaged collision strength* $\Upsilon_{21}(T)$ and statistical weights g_1 and g_2, the *rate coefficients* q_{21} and q_{12} can be shown to be

$$q_{21} = \frac{\beta}{\sqrt{T}} \frac{\Upsilon_{21}}{g_2} \qquad \text{and} \qquad q_{12} = \frac{\beta}{\sqrt{T}} \frac{\Upsilon_{21}}{g_1} \, \mathrm{e}^{-E_{12}/kT}$$

$$\text{where} \qquad \beta = \left(\frac{2\pi\hbar^4}{k \, m_e^3} \right)^{1/2} = 8.629 \times 10^{-6}$$

Note that if there were **only** collisions between the levels, then we would have equilibrium when $R_{12} = R_{21}$ which implies

$$N_1 \, q_{12} = N_2 \, q_{21} \quad \Rightarrow \quad \frac{N_2}{N_1} = \frac{q_{12}}{q_{21}} = \frac{g_2}{g_1} \, \mathrm{e}^{-E_{12}/kT}$$

This is just the Boltzmann equation from thermodynamic equilibrium. But if the collisions are not frequent enough – as they are usualy not in the ISM – then downward radiative transitions will depopulate N_2.

Because the life times of excited levels of H are so brief, we expect the vast majority of the H atoms to be in the ground state and thus downward collisions could be neglected. However, the transitions between the lowest levels of the heavier ions which give lines of [O I], [O II], [O III], [N II], [Ne III], etc. are forbidden -- their transition probabilities are much smaller (the use of square brackets designates forbidden lines). As a result, collisional de-excitations may compete with radiative decays at higher densities and such effects provide, as we shall see, a means to measure the density of the gas.

As we see with the energy levels of N II and O III, there are not only levels a few eV above the ground state which produce lines in the optical window (5000 Å ↔ 2.48 eV), but there is fine structure splitting of the ground state which gives rise to infrared lines. Since the probability of decay decreases with the energy of the transition, these levels are even more highly forbidden than the optical forbidden levels, and hence are likely to be sensitive to the gas density.

The key point about the collisionally excited lines follows from the exp(-E/kT) factor in the excitation rate. This means that the rate of excitation can be highly temperature dependent. In fact, these lines are responsible for most of the cooling as is implied by the strength of the forbidden lines in nebular spectra. The gas is being heated by photoionization: the temperature just keeps rising until the forbidden lines are strong enough to carry away all that energy, and this is what sets the temperature of H II regions and of planetary nebulae to about 10,000 K.

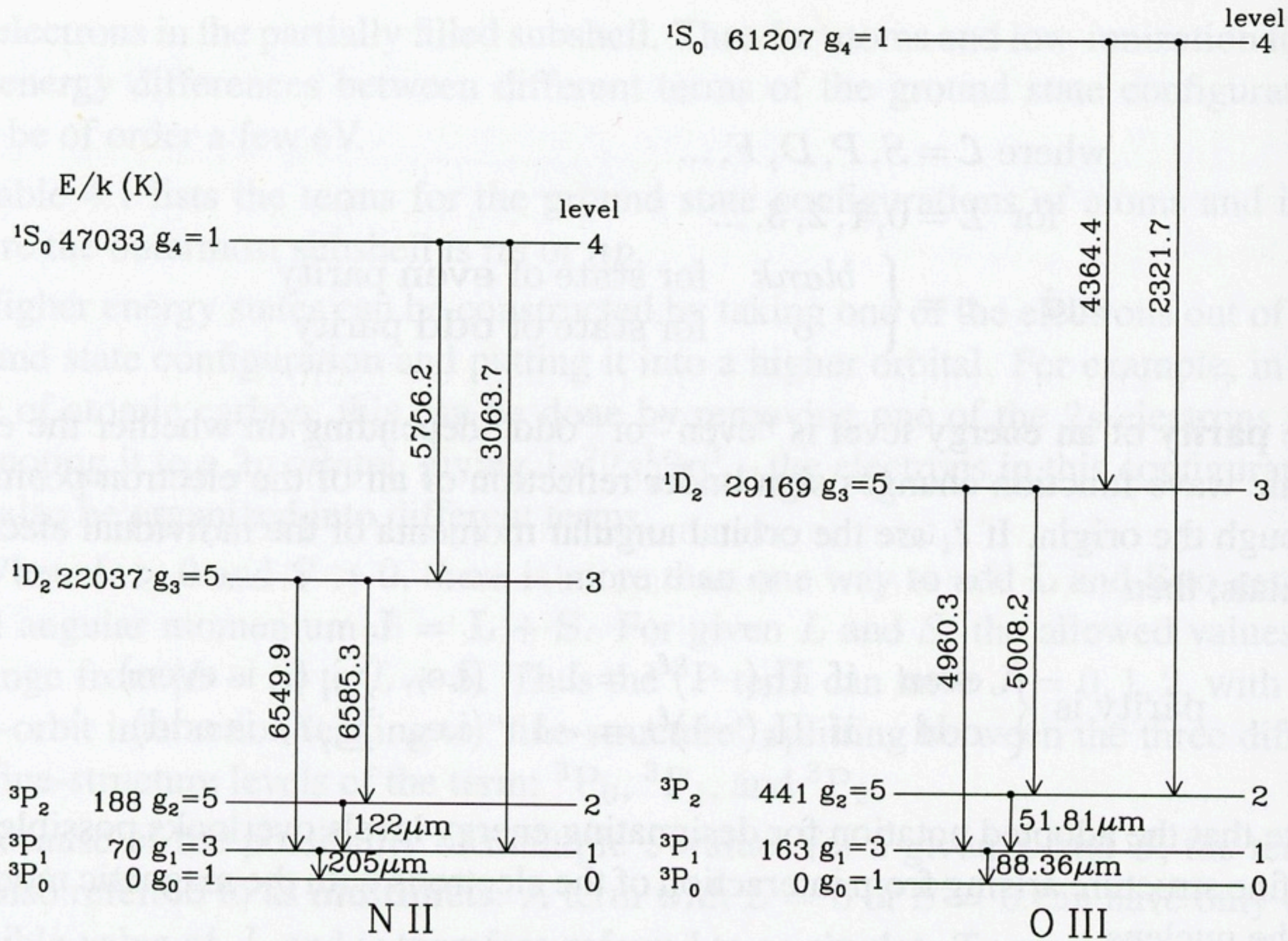


Figure 4.1 Energy-level diagram for the ground configuration of the $2p^2$ ions N II and O III. (Fine-structure splitting is exaggerated for clarity.) Forbidden transitions connecting these levels are shown, with wavelengths in vacuo.

If the density of the gas is low enough that there is little chance of collisional de-excitation before a radiative decay takes place (this critical density depends on the specific transition), then each collisional excitation will lead to the emission of a line photon, which carries that energy away. Thus the energy lost due to excitation $1 \rightarrow 2$ is

$$L_{12}(T) \;=\; n_e \; N_1 \; q_{12} \; h\nu_{12} \;=\; n_e \; N_{ion} \; E_{12} \; \frac{\beta \; \Upsilon_{21}}{\sqrt{T} \; g_1} \; e^{-E_{12}/kT}$$