Temperature and Density from Line Ratios

We saw the last time that collisionally excited lines have an exponential temperature dependance. Consider the simplest possible case of an ion with three levels, at a density low enough that every excitation is followed by a photon-emitting de-excitation. Then we see if we take the ratio of the two lines, the abundance of the ions and electrons cancels out and we are left with a quantity that depends on a few constants and a simple exponential function of the gas temperature:

$$j_{21} = n_e N_1 E_{12} \frac{\beta}{\sqrt{T}} \frac{\Upsilon_{21}}{g_1} \, \mathrm{e}^{-E_{12}/kT} \quad \text{and} \quad j_{31} = n_e N_1 E_{13} \frac{\beta}{\sqrt{T}} \frac{\Upsilon_{31}}{g_1} \, \mathrm{e}^{-E_{13}/kT}$$

so that the line intensity ratio becomes $R = \dfrac{j_{21}}{j_{31}} = \dfrac{E_{12}}{E_{13}} \dfrac{\Upsilon_{21}}{\Upsilon_{31}} \, \mathrm{e}^{E_{23}/kT}$

You can see that this is easily solved for T if we have the line ratio R. This example is too simple, of course. When the ion is excited to level three, there will be some probability that it will jump to level 2 instead of 1. So we must include a branching probability in the expression for j(3→1). In addition, if the ion jumps to level 2, that is an additional mechanism producing j(2→1) photons.

In practice, it is not difficult to set up the equations for an ion with n levels, where n=5 or more. Rather than an analytic solution, we can write a computer program to solve the equations for any temperature and electron density, including all possible collisional excitations and de-excitations and radiative cascade routes. The resulting line intensities are tabulated or graphed and allow us to read off the temperature from the observed line ratios.

Many ions have suitable ratios. Probably the most frequently used is the [O III] ratio found in high-excitation nebulae, j(5007Å+4959Å)/j(4363Å). The problem is that [O III] 4363 can be quite weak, especially if the temperature is low.

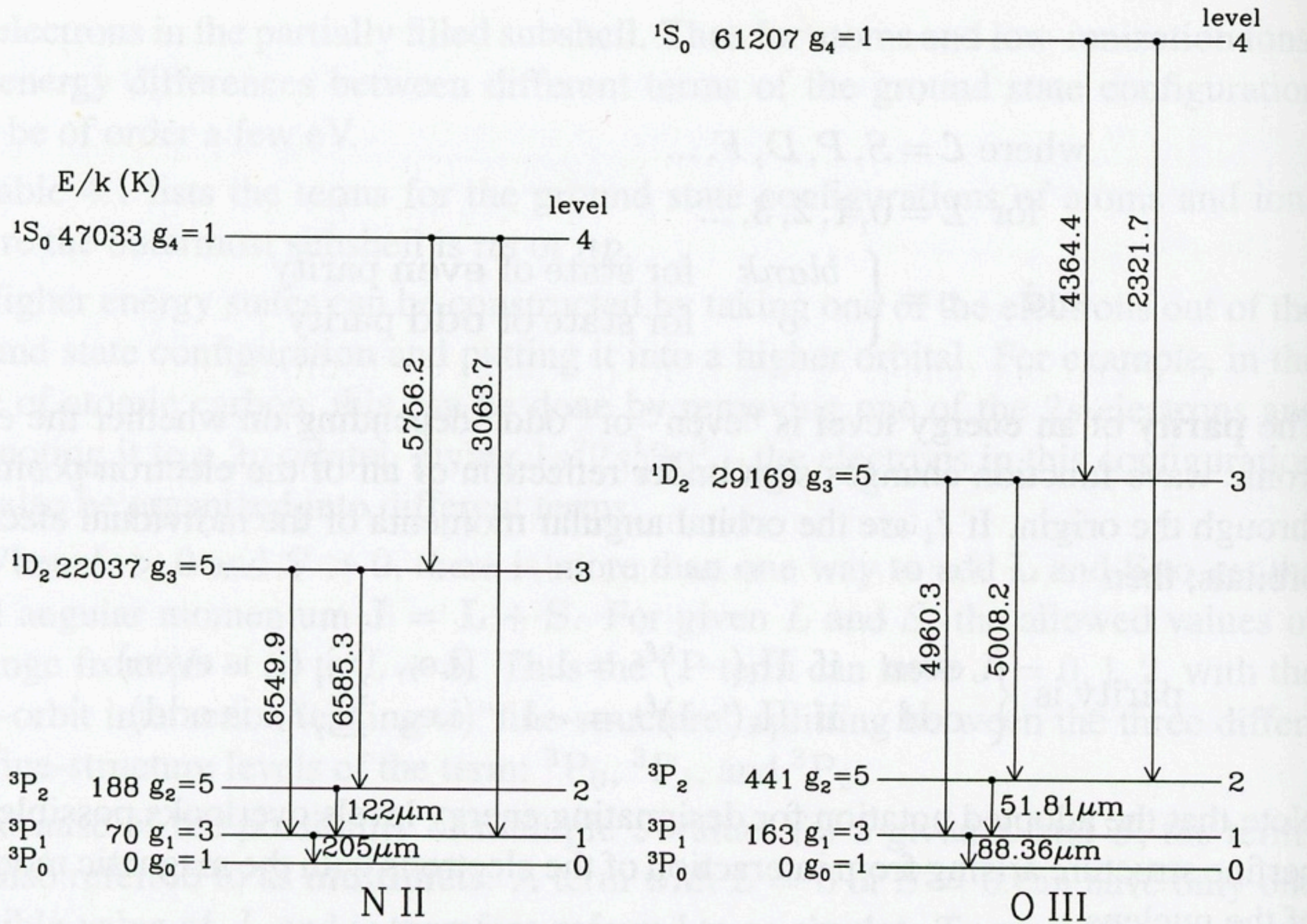

Figure 4.1 Energy-level diagram for the ground configuration of the $2p^2$ ions N II and O III. (Fine-structure splitting is exaggerated for clarity.) Forbidden transitions connecting these levels are shown, with wavelengths in vacuo.

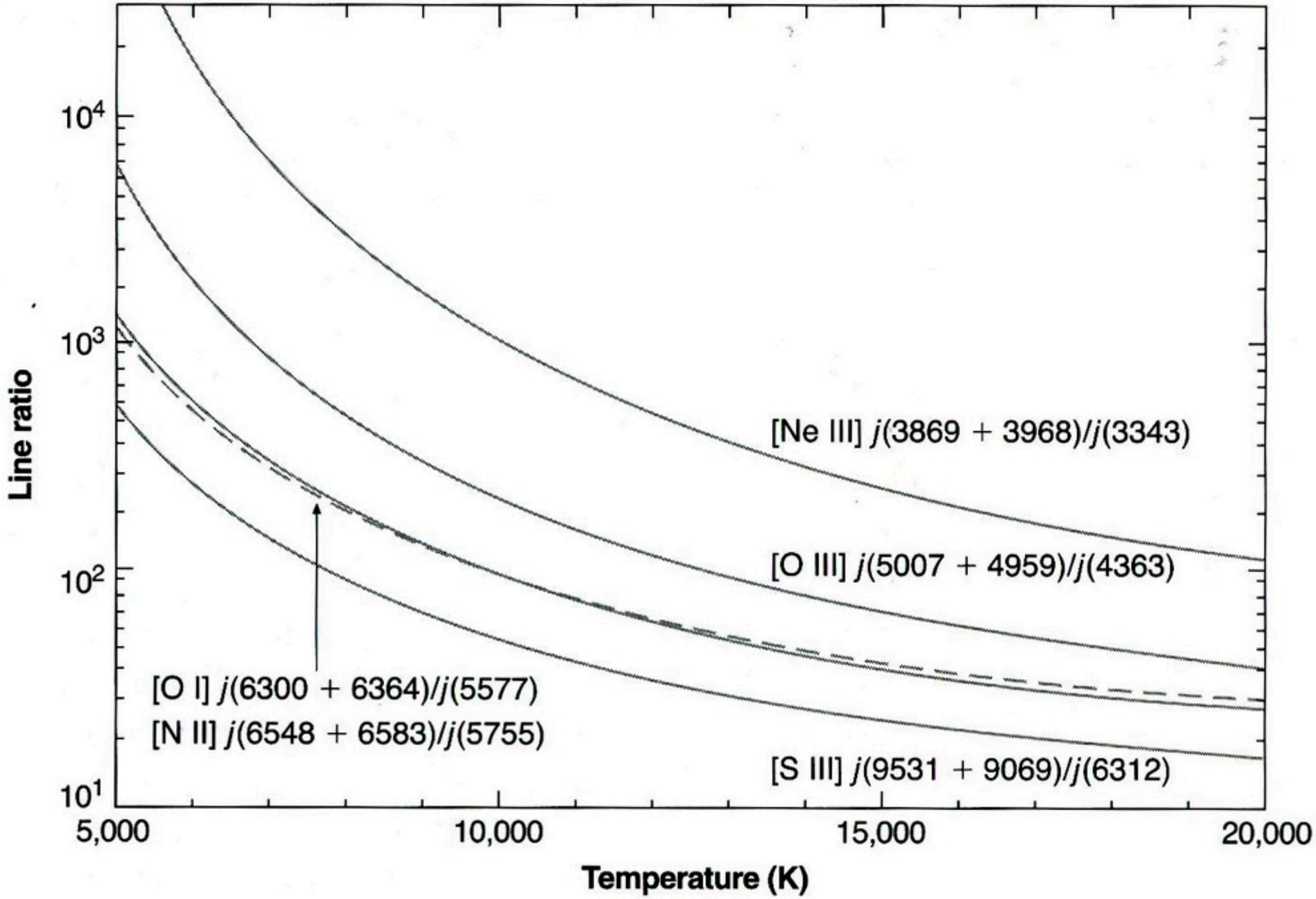

Figure 5.1

Four temperature sensitive forbidden line ratios are shown as a function of the electron temperature. The [O I] (solid line) and [N II] (dashed) ratios are nearly coincident, partially because of their similar excitation potentials. The ratios are shown in the low density limit ($n_e = 1\ \mathrm{cm}^{-3}$).

Other line ratios are sensitive to the gas density, but not very sensitive to temperature. Good examples of this are the [O II] doublet at 3729Å and 3726Å. Ionized sulfur has the same electron configuration (p^3) and has the doublet [S II] 6716Å, 6731Å which is also a useful density indicator.

To understand the behavior of this doublet, we first note that the $^2D^0$ levels have nearly the same energy, so the excitation rate will depend on the collision cross sections, but will be nearly independent of temperature. So at low densities, where each excitation is followed by a radiative decay, producing a 3729Å or 3726Å photon, we find a ratio of 1.5. (This is because the collision strengths are proportional to the statistical weights of the levels, 6/4=1.5.)

So what happens at higher densities? In that case the collisions will set up a Boltzmann like situation where the levels will be populated according to their statistical weights, 3:2. Then these populated levels decay according to their Einstein A rates, and the resulting intensity ratio is

$$R = \frac{n(^2D^o_{5/2})}{n(^2D^o_{3/2})} \frac{A_{\lambda 3729}}{A_{\lambda 3726}} = \frac{3}{2} \; \frac{3.6 \times 10^{-5}}{1.5 \times 10^{-4}} = 0.34$$

Solving the proper n-level problem shows how the ratio goes from 1.5 to 0.34 as a function of the electron density. The following graph gives the results for both the O II and S II doublet. There are many other density-sensitive ratios that can be used depending upon the density and temperature conditions of the nebula.

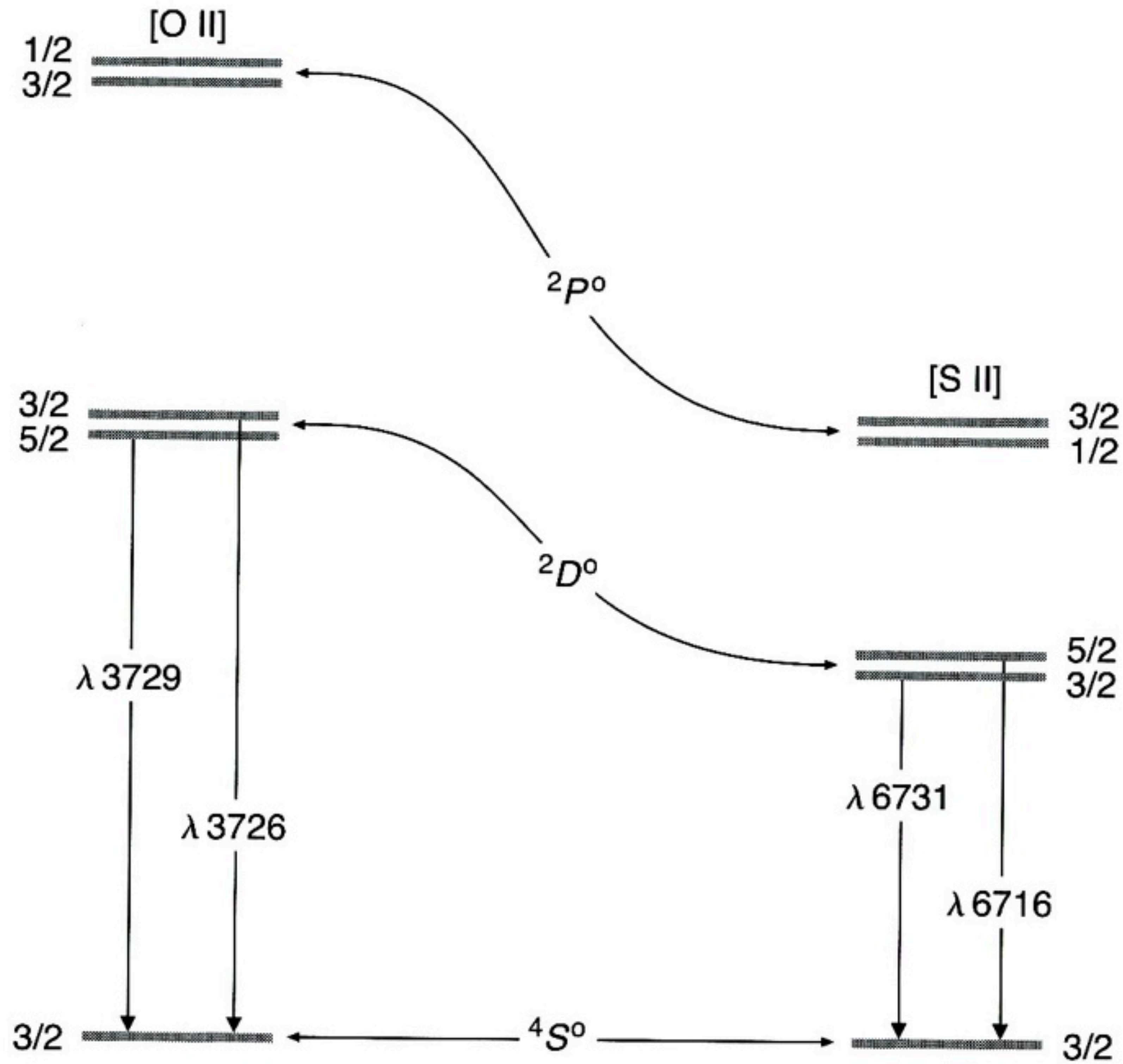

Figure 5.7

Energy-level diagrams of the $2p^3$ ground configuration of [O II] and $3p^3$ ground configuration of [S II].

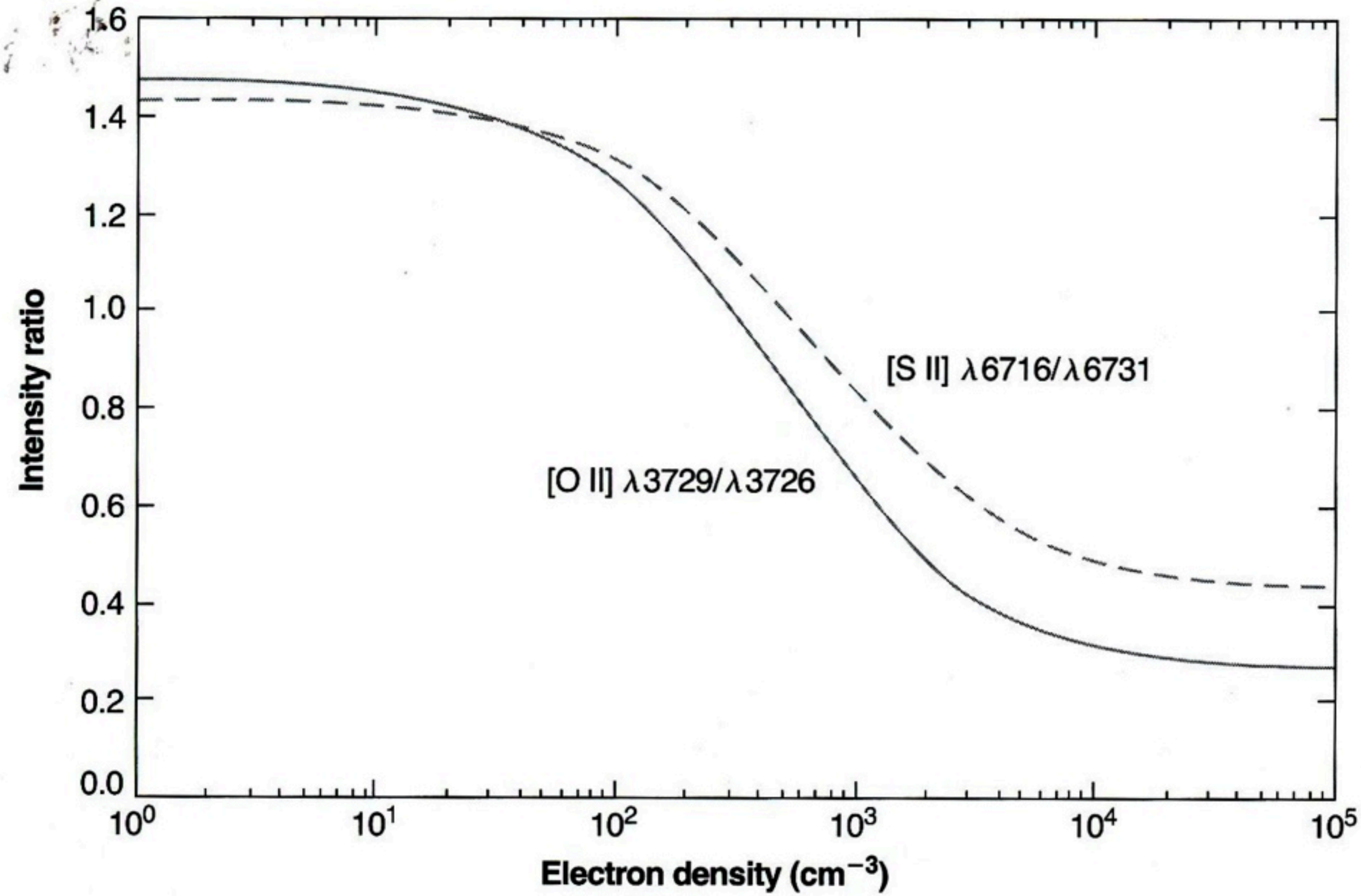

Figure 5.8

Calculated variation of [O II] (*solid line*) and [S II] (*dashed line*) intensity ratios as functions of n_e at T = 10,000 K. At other temperatures the plotted curves are very nearly correct if the horizontal scale is taken to be $n_e(10^4/T)^{1/2}$.