

# The Angular Expansion and Distance of the Planetary Nebula BD+30°3639

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## ABSTRACT

The WFPC2 camera aboard the *Hubble Space Telescope* was used to obtain images of the planetary nebula BD+30°3639 at two epochs separated by 5.663 years. The expansion of the nebula in the H $\alpha$  and [N II] bands has been measured using several methods. Detailed expansion maps for both emission lines were constructed from nearly 200 almost independent features. There is good agreement between the (independent) H $\alpha$  and [N II] proper motions. There are clear deviations from uniform radial expansion, with higher expansion rates in regions where the shell is faintest, such as the south-west quadrant.

The Space Telescope Imaging Spectrograph (STIS) was used to obtain echelle spectra in the C II]  $\lambda$ 2326 multiplet and the [O II]  $\lambda$ 2470 doublet, providing well-resolved expansion velocities at two position angles. From the C II] lines, we find that the central velocity split is  $\pm 36.3$  km s $^{-1}$  at a position angle of 99°, and  $\pm 33.5$  km s $^{-1}$  at p.a. 25°. The fainter [O II] doublet does not appear to differ from the C II] multiplet.

To determine the distance of BD+30°3639 by comparison of the angular expansion and the spectroscopically determined radial expansion, we must address the problem of the three dimensional shape of the nebula. We measured the angular expansion along the position of the 99° echelle slit, finding displacements of 4.25 mas yr $^{-1}$  at the shell edge (2'47 from the center). If the nebula were spherical, this would imply a distance of 1.80 kpc. But there is evidence that the nebula is elongated along the line of sight, which suggests that the actual distance

is less. Radio continuum images from 5 and 15 GHz VLA observations provide information on the extent of the radial elongation. We fit the radio brightness variation and the echelle data by approximating the nebula as an ellipsoid, also making use of the ground-based echelle spectra reported by Bryce & Mellema (1999, MNRAS, 309, 731). Our model has an axial ratio of 1.56, is inclined to the line of sight by  $9^\circ.7$ , and exhibits an expansion in the plane of the sky which is  $2/3$  that in the radial direction, leading to a distance of 1.2 kpc. Not all the kinematic data fits this simple model, so the distance must still be regarded as uncertain.

Based on the recent model atmosphere of Crowther et al. (2002), a distance of 1.2 kpc implies a stellar luminosity of  $4250 L_\odot$ . The kinematic age of the nebula,  $\theta/\dot{\theta}$ , varies somewhat from region to region. A good average value is 800 years, while the expansion along the position of the  $99^\circ$  echelle slit gives about 600 years.

*Subject headings:* planetary nebulae: individual (BD+30°3639) — stars: distances

## 1. INTRODUCTION

Understanding the fundamental properties of planetary nebulae (PNe) requires knowledge of their distances. For planetary nebulae, because of the absence of a standard quantity common to all, there is no well-calibrated standard distance scale and distances estimated from different independent methods often disagree with each other. One way to determine the distance of a planetary nebula is to measure its angular expansion during a period of time and combine this with its Doppler expansion velocity. This method has been used for both VLA (Very Large Array) radio maps (Hajian et al. 1993, 1995; Hajian & Terzian 1996; Kawamura & Masson 1996), and optical images from the *Hubble Space Telescope* (*HST*) (Reed et al. 1999).

BD+30°3639 (hereinafter, “BD+30”) is a rapidly evolving, chemically inhomogeneous nebula (Waters et al. 1998). It is one of the few nebulae with detected X-ray flux from its bubble of shocked stellar wind (Kreysing et al. 1992; Arnaud et al. 1996; Guerrero, Chu, & Gruendl 2000; Leahy, Kwok, & Yin 2000; Kastner et al. 2000). This justifies extensive observations of this object. To understand the rapid chemical evolution of BD+30, we need to know the mass and luminosity of its central star, and this requires a reasonably accurate distance. The earliest expansion distance of BD+30 was obtained by Masson (1989) from VLA maps, who gave a result of  $2.8_{-1.2}^{+4.7}$  kpc. Hajian et al. (1993) used different two-epoch

VLA observations, resulting in a distance of  $2.68 \pm 0.81$  kpc. Kawamura & Masson (1996) combined the observations used by Masson (1989) with a new set of data obtained in 1993, and derived the most accurate measurement at that time of  $1.5 \pm 0.4$  kpc.

To obtain a more accurate determination of the distance, and details of its expansion, we used the two-epoch optical observations of *Wide Field Planetary Camera (WFPC2)* on board the *HST* to measure the expansion of this nebula. The total expansion of BD+30 was estimated to be about 15 mas between the first and second epochs, which is about 1/3 of the *Planetary Camera (PC)*  $0''.0455$  pixel. This tiny displacement in diffuse objects can be measured because of the high resolution of *WFPC2* images (Currie et al. 1996), and we can do even better for a number of diffuse knots sharing a common proper motion.

Much evidence shows that BD+30 is not actually a spherical object (Masson 1989; Bachiller et al. 1991; Shupe et al. 1998; Bryce & Mellema 1999; Bachiller et al. 2000). We thus use an ellipsoidal kinematic model of BD+30 to combine the angular expansion with the radial expansion velocities from spectroscopic observations to find the distance and kinematic age of this nebula.

## 2. Observations

*HST* narrow-band images from our GO programs 5403 and 8116 were used for this study (Table 1); the observations were separated by 5.663 years. For both epochs, images through F656N ( $H\alpha$ ) and F658N ([N II]) narrow-band filters were obtained. In the first epoch there were two images for each band with the same pointing but different exposure times. The images with longer exposure time were saturated in places. For the second epoch, there were three images for each band, with the same exposure time and orientation, but slightly shifted by fractional pixels with respect to each other for the purpose of drizzling. Since the orientation differs for the two epochs, the images had to be aligned to a common center and orientation.

## 3. Data Reduction

The data reduction includes cosmic ray rejection, drizzling, and image alignment. Other adjustments or corrections like geometric distortion correction, saturated pixel masking, were also done during the three main steps.

Because the 2nd epoch  $H\alpha$  and [N II] images were dithered cosmic rays cannot be removed by the commonly used IRAF task *crrej*. However, Mutchler and Fruchter (1997)

and Fruchter & Hook (1998) introduced a method to remove cosmic rays from a group of dithered *HST* images, as well as an IRAF package *drizzle* for this purpose. We first used IRAF task *cross\_driz*, *shiftfind*, and *imshift* to find the relative displacement between images, and align the images. Then, we used *crrej* to combine the aligned images to produce cosmic-ray-free images. Finally, *driz\_cr* was used to create cosmic ray masks, which were used in the drizzling step.

Using the IRAF package *drizzle*, three 2nd epoch  $H\alpha$  images and three 2nd epoch [N II] images with *PC* image resolution  $800\times 800$  were combined into images with resolution  $1600\times 1600$ , respectively. To compare images from the two epochs, the old (the 1st epoch) images were also mapped into the same  $1600\times 1600$  grid by the *drizzle* package, and cosmic rays were removed by the same strategy as for new (the 2nd epoch) images. It is easy to tell that some features are sharper in the new drizzled images than in the old images (which were merely mapped onto finer grids). The drizzled  $H\alpha$  image is shown in Fig. 1.

To perform the comparison between new images and the old ones, images had to be aligned in displacement and position angle. The relative rotational angle and shift between two epochs of images were found by IRAF task *cross\_driz*, *rotfind*, and *shiftfind*, iteratively. The most ideal and simplest situation is that the expansion is isotropic, so that it looks symmetrical. We took this as the 0th order approximation, and did the alignment by trail-and-error strategy with the following measurements to make the measured average expansion along radial lines as symmetrical as possible.

## 4. Expansion Measurement

By blinking the aligned images taken at two different epochs back and forth, the expansion can be seen clearly by eye. The expansion in  $1600\times 1600$  images is estimated about 0.5 to 1 pixel (or 0.25 to 0.5 PC pixels). The actual measurements were performed using several different independent methods.

### 4.1. Methodology

Basically, there are two ways to measure the expansion; one is magnification, which is whole-image-based, the other is to measure the shifts of individual features. If the expansion is spherically symmetric, the new image should be a magnified version of the old image. This method can be applied to the whole image, or along sectors in particular directions from the image center.

Because the old (first epoch) images have many saturated pixels, while the new (second epoch) ones do not, we reduced the new images to fit the old images, which avoids any manipulation of the saturated-pixel masks. The reduction factor was determined by minimization of the square of the difference image rather than by cross-correlation, which we found less sensitive. We estimate the uncertainty at 25% for the global magnification factor and for the magnification factors along radial lines.

However, since the expansion of the nebula may be asymmetrical, the magnification method cannot provide the details of the angular expansion. Thus we made measurements of individual features and took the results of the magnification method as a consistency check. Compared to other PNe such as NGC 6543, BD+30 is more diffuse, so it is not possible to measure small regions of only a few pixels in size. To determine the best size of regions for shift measurements, we tested the effects of square size on some randomly selected features by plotting the curve of measured shifts against the size of the region. We found that when the square size was around 20 pixels (i. e., 10 PC pixels), shifts were the most insensitive to the size of the region. We therefore adopted  $20 \times 20$  pixel squares for the measurements.

The basic compare-and-fit strategy used here is just like what is used to produce a cross-correlated image. First, move one image by, for example,  $\Delta x$  pixels in the  $x$  direction and  $\Delta y$  pixels in the  $y$  direction. Then compare and quantify the difference by summing up the squares of differences of corresponding pixels as in a least-squares fit. This gives the value of the point  $(\Delta x, \Delta y)$  in the least-squares image. Finally, find the minimum of the least-squares image by Gaussian 2-D fitting. This position  $(\Delta x, \Delta y)$  gives the relative shift between two features.

Care must be exercised in the treatment of the edges of the squares, since almost all regions cut from the original images have non-zero edges (and the edges of saturated pixel masks are also sharp discontinuities). Tapering is not satisfactory, because the regions are very small, as are the shifts; tapering the edges will drag the measured shifts to zero. The method we employed was to treat the 20 pixel square, combined with the saturated pixel mask within the square region, as a window. We first moved the second epoch image, which contained no saturated pixels, then looked at both images through that window, and compared the visible parts by least squares. To simplify the process, IDL routines were developed which can select small regions from images interactively, and calculate the shifts automatically.

## 4.2. Results

We used the magnification method at two levels, one for whole image, another along every radial line at  $10^\circ$  intervals from  $0^\circ$  to  $360^\circ$ . Numerical fitting for the whole image gave magnification factors of 0.99384 for  $H\alpha$  and 0.99361 for  $[N II]$ .

The results from radial line magnification are shown in Fig. 2. As mentioned above, one criterion of alignment was to make the angular distribution of magnification as symmetrical as possible. Even so, we can still notice that the measured expansion shows some structure. The expansion is greatest near the openings in the northeastern and southwestern quadrants. While the distribution of walls and openings is irregular, the smallest expansion seems associated with regions where the shell is brightest. The direction of greatest expansion is not along the longest axis of the nebula’s image; this may be because the nebula is a tri-axial ellipsoid, with none of the axes aligned with the line of sight, so the expansion might not be aligned along the longest axis of the image. This might also explain why some features near the center were measured to have inward displacements.

The shifts of nearly 200 features in the  $H\alpha$  images and the  $[N II]$  images are shown in Fig. 3 and Fig. 4, respectively, which illustrates the details of expansion. Again, except for a few features, the results from the two independent wave-bands show a high degree of consistency (Fig. 5). The agreement between  $H\alpha$  and  $[N II]$  gives us confidence in the results. The discrepancies between the two wave-bands may partly be due to slight misalignment, so that the same coordinates in different images might refer to slightly different regions. In addition, since the ratio of  $[N II]$  to  $H\alpha$  emission varies from point to point within the nebula, there may be real differences in the shifts of features seen in the two wave-bands.

The expansions of the major axis and minor axis of the optical shell are summarized in Table 2. The kinematic age of the nebula, defined as  $\theta/\dot{\theta}$ , varies somewhat with the method of measuring the expansion as well as from region to region. A good average value is 800 years. The magnification factor derived for the whole image gives kinematic ages of 920 and 880 years for  $H\alpha$  and  $[N II]$ , respectively. The magnification factors along sectors give somewhat smaller ages, and the ages along the minor axis tend to be less than along the major axis. Finally, expansions from feature shifts give the smallest kinematic ages, 740-800 years, and are again smallest along the minor axis. As discussed below, we are particularly interested in the expansion along the position of a spectrographic slit at position angle  $99^\circ$ ; the kinematic age along this direction is about 600 years.

## 5. Modeling of BD+30 and Distance Measurement

If we want to derive the distance of the nebula, some kind of kinematic model – implicit or explicit – must be employed to relate the tangential expansion rate to the expansion velocity along the line of sight.

Bryce & Mellema (1999) recently discussed the kinematics of BD+30 based on ground-based optical emission-line spectra. They measured the spectra along two slits roughly aligned to the east-west and north-south directions through the center, which, approximately, are the major and minor axes of the nebula. For the modeling of BD+30, they used a tilted ellipsoidal shell, with a high velocity expanding H<sub>2</sub> ring in the equatorial plane. Furthermore, they found that there is a difference between the low ionization and high ionization regions of this nebula. Compared with low ionization regions, the high ionization regions are smaller, but have much higher velocities. They found the expansion velocity along the line of sight to the central star to be  $28 \pm 1 \text{ km s}^{-1}$  for [N II] profiles and  $35.5 \pm 1 \text{ km s}^{-1}$  for [O III] profiles.

### 5.1. The HST STIS Echelle Spectra

Because the nebula is compact, ground-based long-slit spectra do not have the optimum degree of spatial resolution. As part of our HST program, we obtained STIS spectra with the E230H echelle grating using a  $6'' \times 0''.2$  slit. The grating was set to include the strong C II] multiplet  $\lambda 2324.21, 2325.40, 2326.11, 2327.64$  and  $2328.84$ , as well as the [O II] doublet  $\lambda 2470.97, 2471.09$ . The echelle mode of STIS is available only with the ultraviolet MAMA detector, so substantial dust extinction was unavoidable. Two position angles were observed,  $25^\circ$  and  $99^\circ$ . The  $25^\circ$  orientation suffers badly from extinction, especially to the north, so we rely primarily on the  $99^\circ$  p.a. slit. The slit was positioned so that it passed  $0''.4$  north of the central star to avoid contamination by the stellar continuum. The location of the  $99^\circ$  slit on the H $\alpha$  image is shown in Fig. 7. To increase the S/N ratio, the spectrum was shifted by  $1.53\text{\AA}$  and added to itself to superimpose C II]  $2327.64$  and  $2326.11$ , the two strongest components. The results are shown in Fig. 6. Panel A is the  $25^\circ$  orientation and B1 and B2 the  $99^\circ$  slit. In this figure, the top of the slit is to the east. The [O II] line appears identical to C II], but has lower S/N, so we do not discuss it further.

We see that the line has well-defined red and blue components, which yield an accurate expansion velocity. If the nebula were a uniformly expanding ellipsoid, the echelle profile would be an ellipse – tilted, if the ellipsoid major axis were inclined to the line of sight. While our profile is roughly elliptical, there are clearly local irregularities in the expansion.

Panel (B1) shows an ellipse which fits the data reasonably well. The vertical extent of this ellipse is  $4''.94$  and the horizontal splitting (at the center) is  $\pm 36.25 \text{ km s}^{-1}$ . The ellipse is tilted to the right by  $5^\circ$  from vertical. While the splitting seems to be well determined, the tilt of the ellipse is not secure.

But basically, it is a tilted ellipse, which indicates that a tilted ellipsoidal shell is an appropriate model of the nebula. The lower edge is fainter due to local dust extinction (Harrington et al. 1997). Since C II, like N II, is a lower stage of ionization, we were surprised that the expansion velocity of this line was close to the result Bryce & Mellema (1999) found, not for [N II], but for the high ionization [O III] line. Perhaps our higher spatial resolution gives a more accurate velocity, while the lower ground-based resolution fills in the profile center, lowering the average velocity. Another consideration is that, although the C<sup>+</sup> ions will avoid the highly ionized central regions, the excitation level of this UV line is high, so the emission will be biased toward the hottest parts of the nebula.

Because the expansion velocity was only measured along the STIS slit, we made measurements of the astrometric shifts of features along that cut. According to our kinematic model, the tangential expansion is along radial directions, and is linearly proportional to the distance from the center. Under these conditions, projections of expansion velocities on any axis are proportional to the corresponding projections of distances on that axis. So for point  $(x, y)$  of the image where the center is at  $(0, 0)$ , its expansion velocity  $(v_x, v_y)$  satisfy  $v_x \propto x$  and  $v_y \propto y$ , and it does not matter which direction you specify as  $x$  direction. Therefore, we extracted the components of the shifts along the slit for each feature along the cut, and plotting them against the distances to the center of the slit, fitted them to a straight line, as shown in Fig. 7. The least-squares line goes through the center, which is a good demonstration of symmetry. Obviously, the error of the central point will be large, since its shift should be perpendicular to the slit. A fit excluding the center three points, which are likely to have large errors, gave almost the same line. In this way, the expansion rate at the edge of the nebula, which corresponds to the upper and lower edge of the spectral ellipse, was measured to be  $4.25 \pm 0.32 \text{ mas yr}^{-1}$  at  $2''.47$  away from the center.

If the nebula were a uniformly expanding spherical shell, then the above tangential expansion, combined with the  $36.25 \text{ km s}^{-1}$  velocity along the same cut, implies a distance of  $1.80 \text{ kpc}$ . There is evidence, however, that the nebula is not spherical. In the next section, we will attempt to develop a better kinematic model.

## 5.2. Shape Derived from the Surface Brightness Variation

BD+30 is optically thick to ionizing radiation – it is surrounded by a neutral halo (Harrington et al. 1997) – but the photoionized shell is also geometrically thin (perhaps because of the pressure of the  $3 \times 10^6$  K X-ray emitting gas which fills the interior of the shell). Under these conditions, the variation in the surface brightness across the nebula contains information about the three dimensional structure of the nebular shell. Since it would be difficult to correct for the internal extinction due to dust which affects the optical images, we have used the 5 and 15 GHz VLA radio maps described in Harrington et al. (1997) for this analysis. A similar approach was previously used by Masson (1989), who modeled the apparent EW elongation and surface brightness variations of BD+30 in terms of an inclined prolate ellipsoid tilted from our line of sight along the EW direction, consistent with a limited kinematic information available at that time.

The relevant equations are developed in the Appendix. The constancy of flux in pie-shaped sectors radiating from the central star supports the idea of complete absorption of the ionizing radiation. The ratio of the central surface brightness to the total flux indicates that the nebula is elongated along the line-of-sight: the central intensity is substantially less than would be the case for a sphere. Unfortunately, the angular expansion vanishes near the center of the nebula; what we need is information on the shape of nebula where we have measured the angular expansion. We can, however, find the shape of the shell along a chosen wedge from the surface brightness distribution (eq. A3). We integrated the surface brightness in  $25^\circ$  sectors chosen to overlap the region cut by our  $99^\circ$  STIS slit. The results are shown in Fig. 8. The values near the central star are very noisy due to the low signal at the wedge apex. Also, near the edge of the nebula, the finite shell thickness invalidates the assumptions of the method. But overall, the shell seems to be well fit by an ellipse with an axial ratio of 1.5 to 1 along our line-of-sight, as indicated by the dotted line in Fig. 8. This fit also closely matches the shell distance of  $3''.7$  in the direction of the central star – the triangle in Fig. 8 – found from the central intensity (eq. A1).

In obtaining the shape from eq. A3 we made the assumption that the front and back surfaces of the shell contribute equally to the surface brightness. But if we look at the echelle profile, we see that there is a great asymmetry between the front (blue-shifted) and back (red-shifted) branches. We might attribute this to dust extinction, and indeed we feel that the relative weakness of the lower (west) part of the profile is likely due to dust. But this cannot explain the relative brightness of the upper, red-shifted side of the line, because this radiation comes from the *back* side of the nebula: this radiation must suffer at least as much extinction as the weaker blue side.

A slit placed across a uniformly expanding shell in the form of a tilted ellipsoid will

result in a spectral line in the form of a tilted ellipse. (The tilt angle of the major axis in a plot of the line profile will vary depending upon the relative scales of chosen for the x-axis and y-axis. The ratio of the x displacement of the top of the ellipse to the width of the ellipse on the x-axis is, however, an invariant.) Because the ellipse that best fits our echelle profile seems tilted, we considered the appearance of a tilted ellipsoidal shell. Equation A7 gives the front and back surface brightnesses, which we found, for the parameters relevant here, to differ by up to a factor of two. We can sum the front and back contributions and apply eq. A3 to the result, to simulate our analysis of the VLA data assuming symmetry. We find that an ellipsoid of axial ratio 1.56, tilted by  $10^\circ$ , closely resembles an untilted ellipsoid of axial ratio 1.5. (Such a solution is shown on Fig. 8 as a dashed line.)

Since the top of our echelle spectrum seems tilted redward, this would imply an ellipsoid with the near side tilted down (westward). Unfortunately, with such a tilt, it is the front (approaching, blue-shifted) side which is brighter at the top (east) end of the slit, just the opposite of what is observed. In an attempt to clarify this situation, we examined the spectra presented by Bryce & Mellema (1999), who reproduce two spectra taken with an east-west slit (close to the orientation of our  $99^\circ$  slit), one in the [O III]  $\lambda 5007$  line and the other in the relatively weak [N II]  $\lambda 5754$  line. The [O III] profile is clearly tilted with the east side blueward, *opposite the apparent tilt of our C II] line*. The [N II] profile also seems to be tilted with the east side blueward, but determination of the tilt is difficult in this case because the parts of the profile corresponding to high velocities are very faint compared to the bright parts at lower velocities. We find that if we apply the same analysis as above to the STIS profile, this time fitting the VLA surface brightness and the tilt of the Bryce & Mellema (1999) [O III] profile, we get a good fit with an ellipsoid of axial ratio 1.56, with the near end tilted up (eastward) by  $9.7^\circ$ . This is the same direction of tilt shown in Fig. 4 of Bryce & Mellema (1999). We were guided by the C II] profile in setting the central velocity and spatial extent of this fit, since the relatively low resolution of the [O III] profile provided less constraint. Fig. 9 shows this predicted profile superimposed on the Bryce & Mellema (1999) data.

While no simple ellipsoidal model seems able to fit these complex data, we feel that this is the best compromise, since it (a) fits the VLA surface brightness, (b) produces a front/back brightness ratio in the same sense as that seen in both the C II] and [O III] profiles, (c) fits the central velocity seen in the C II] profile, and (d) fits the direction and magnitude of the tilt seen in the [O III] line. The problem is that this model predicts a tilt in the opposite sense to that seen in the C II] line remains. In view of the low S/N of this profile, however, its tilt is uncertain. Also, the fact that the ellipsoidal models are only slightly inclined to the line of sight means that irregularities in the shape of the shell can easily change the tilt of the line profiles.

The ultimate goal of our kinematic model is to relate the tangential expansion, as determined by the linear fit in Fig. 7, to the Doppler splitting seen in the echelle spectrum. The model discussed above – a tilted ellipsoid with a 1.56 axial ratio – has a tangential to radial velocity ratio of 0.667. This reduces our estimate of the distance to BD+30 to 1.20 kpc. The error attached to this distance is hard to estimate. The formal error in the linear fit to the astrometric shifts along the slit is less than 10%, and the error in the expansion velocity smaller yet. This implies an error of about 10% in the distance, but *only if we have an accurate kinematic model*.

It is clear that the shape and kinematics have an important but somewhat uncertain impact on our derived distance, almost surely reducing the value significantly below the spherical value, but by an amount that must remain somewhat uncertain. Progress might be made by a program of extensive kinematic mapping with multiple narrow slits – preferably in the infrared to minimize the effects of extinction by internal dust.

We should also bear in mind that all the foregoing analysis is based on the assumption that the expansion of the nebula is radial, and with a velocity proportional to the distance from the center – such an expansion will preserve the shape of the nebula. While this is a reasonable and conservative assumption, it is not hard to imagine that there may be more material around the waist of the nebula (the plane perpendicular to the major axis) which may impede expansion in that direction. Such a situation would reduce the derived distance to the nebula still further.

## 6. Discussion

We have used pairs of HST WFPC images separated by 5.663 years to produce detailed maps of the angular expansion of BD+30. Images in both  $H\alpha$  and  $[N II]$  show similar patterns of expansion. Near the northeastern corner, the direction deviations of expansion flow are very regular, and the magnitudes of expansion are also close to each other. If we look at the opening near the northeastern corner, the movements of the material in the inner part tends to move toward the opening, not always along the radial direction. Because measurements were performed on small independent regions, this cannot be a systematic error. It appears that the material in the inner part is trying to go out through the opening, which has relatively low density, instead of moving along a radial direction. This probably shows that the inner material has a higher velocity than the outer shell. The west parts of the image do not have very regular patterns over a large scale, but we can still notice some systematic movements. For example, the expansions along the southwestern direction going outward are almost all along one direction with same magnitude, and there is another convergence

center at the southwestern part of the image. The velocities are particularly large at the outer edge of the southwest quadrant. It is interesting that the largest proper motions are approximately along the axis of the bipolar outflow as delineated by CO "bullets" imaged by Bachiller et al. (2000), rather than along the (EW) major axis. Apparently, measurements of the angular expansion can reveal the "true" bipolar axis, consistent with ground-based kinematics.

Overall, our measurements are more secure than earlier results, since the high resolution of these images allows us to measure the displacements of small, discrete features – in contrast to the radio work, which could only measure the global expansion. A comparison of proper motions listed in Table 2 with the radio expansion measured by Kawamura & Masson (1996) shows a good agreement, but both measurements are significantly larger than the radio expansion determined by Hajian et al. (1993). The difference between these two radio proper motion measurements arises from a correction used by Kawamura & Masson (1996) to account for the decreasing surface brightness of BD+30 as it expands at an assumed constant luminosity. The present measurements confirm the validity of this correction, which must be applied if an optically thick nebula is observed with an instrument whose spatial resolution is comparable to the size of the object. While the HST spatial resolution is superior with respect to the existing radio observations, a rather remarkable agreement with measurements of Kawamura & Masson (1996) demonstrate that ground based radio observations can provide reliable proper motions for compact PNe.

Although our STIS spectra have rather poor S/N, their high spatial resolution gives us a precise measure of the expansion of the brightest part of the nebular shell in a well defined location. This velocity data can be combined with the astrometric shifts along the same cut to derive a distance. Here, the greatest uncertainty is the shape and kinematics of the nebula: how elongated is the shell and how uniform and radial is the expansion? We have used the radio surface brightness to address the shape, but we have had to assume uniform radial expansion – even though our maps of angular expansion show clear non-radial motions in some sectors. Still, we feel that our resulting distance determination of 1.2 kpc is better justified than previous distance estimates, and offers an improvement over the most recent value of  $1.5 \pm 0.4$  kpc found by Kawamura & Masson (1996) from radio expansion measurements combined with the ground-based kinematic data.

At a distance of 1.2 kpc, the luminosity of the central star of BD+30 is equal to  $4250 L_{\odot}$ , following the recent determination of Crowther et al. (2002) based on modeling of the stellar spectrum. (This is 60% lower than the luminosity based on the earlier model of Leuenhagen, Hamann, & Jeffrey (1996).) Most of the stellar radiation is absorbed within the nebula, primarily by dust in the neutral halo (the infrared luminosity is  $2000 L_{\odot}$ ) and in

the photoionized shell (nebular models indicate that  $\sim 800 L_{\odot}$  is required to account for the observed radio continuum). The mostly neutral nebular mass is equal to  $0.13 M_{\odot}$ , based on the [C II]  $158 \mu\text{m}$  line luminosity derived from the Infrared Space Observatory (ISO) measurements of this line flux (Liu et al. 2001) and the 1.2 kpc distance. In terms of its mass and luminosity, BD+30 is not exceptional among Galactic PNe. But its central Wolf-Rayet WC star is hydrogen poor and ISO observations revealed the presence of crystalline silicate dust in this carbon-rich nebula (Waters et al. 1998), signaling a recent change from the oxygen-to carbon-dominated chemistry. The origin of these rapid abundance variations is not understood at present. One promising hypothesis involves a final thermal pulse (Waters et al. 1998; Herwig 2001) which occurred at most a few thousand yr ago, shortly before BD+30 left the Asymptotic Giant Branch phase of evolution. With the fairly well determined distance, perhaps this bright and well-observed PN can provide a stringent test of various hypotheses invoked to explain the origin of WC stars and the presence of abundance inhomogeneities in PNe.

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## A. Appendix

To the extent that the temperature in the nebular gas is constant, and if we neglect the secondary effects of diffuse radiation produced within the gas, the emission in a hydrogen recombination line or in the free-free continuum will depend only upon the flux of stellar ionizing radiation that is absorbed in the volume of gas. Consider a nebular shell which is optically thick to ionizing radiation but which is geometrically thin. Then, the emission from the surface of the shell will simply be proportional to the solid angle of the surface element as seen from the central star. Consider an element of the shell  $dS$  which is inclined so that the angle between the normal to the surface and our line-of-sight is  $\beta$ . Let  $\alpha$  be the angle between the radial vector from the star and the normal to  $dS$ . Then  $dS$  will appear to have a surface brightness  $\Sigma$  of

$$\Sigma = \frac{F}{4\pi} \frac{\cos \alpha}{\cos \beta} \left[ \frac{D}{r} \right]^2 \quad (\text{A1})$$

where  $F$  is the total flux from the whole nebula,  $r$  is the distance of  $dS$  from the central

star, and  $D$  is the distance from the nebula to the observer. Near the central star, where  $\cos(\alpha) = \cos(\beta)$ , the ratio of  $\Sigma$  to  $F$  provides the distance of the surface from the star (in angular units:  $(r/D)$  radians). One complication is that we see both the front and back sides and must separate these contributions or assume symmetry. From the central surface brightness of the radio frequency images, it is apparent that the nebula is elongated along the line-of-sight, since the central intensity is substantially less than would be the case for a sphere. We find  $(r/D)$  to be at least  $3''.4$ ; extrapolation to zero radius of the flux in rings about the center yields  $3''.7$ .

Consider a spherical coordinate system whose origin is the central star, with the z-axis directed towards the observer. Call the azimuthal angle  $\phi$  and the angle between the z-axis and the vector to a point on the surface  $\theta$ . The flux in any pie-shaped section of the image, bounded by angles  $\phi_1$  and  $\phi_2$ , must be  $F(\phi_2 - \phi_1)/2\pi$ , because that part of the image arises from the absorption of the fraction of the stellar flux emitted into the volume bounded by the planes  $\phi = \phi_1$  and  $\phi = \phi_2$ . In fact, this provides a check on our assumption that the nebula is optically thick and that diffuse nebular radiation is not important: equal pie-shaped segments of the radio image should have the same flux. This check is satisfied to within 5%, except for a sector at p.a.  $\sim 60^\circ$ , where there is deficit of 11%. (We use the radio images because the H $\alpha$  images are affected by dust absorption/scattering.) Unfortunately, the angular expansion vanishes near the center of the nebula; what we want is information on the shape of nebula where we have measured the angular expansion.

Let  $p$  be the (projected) angular distance of a bit of the surface  $dS$  from the star. Then the flux in the image segment between  $[p, p + dp]$  and  $[\phi, \phi + d\phi]$  is equal to the product  $\Sigma(p, \phi) p dp d\phi$ . There are two contributions to  $\Sigma$ : radiation from the front and back surfaces. We assume that we can separate them or that they are equal. So  $\Sigma$  here refers just to the front or back surface. The emitting piece of surface is located at the (unknown) angle  $\theta$  from the star. The flux emitted by the surface is  $F d\Omega/4\pi$ , where  $d\Omega$  is the solid angle subtended by the surface as seen from the star.  $d\Omega = \sin(\theta) d\phi d\theta$ , so that the flux is  $[F/4\pi] \sin(\theta) d\phi d\theta$ . By equating fluxes we can write

$$\frac{F}{4\pi} \sin(\theta) d\phi d\theta = \Sigma(p, \phi) p dp d\phi \quad (\text{A2})$$

Consider a thin segment of the nebula between  $\phi$  and  $\phi + d\phi$ . Integrate the expression above from an angle  $\theta$ , corresponding to some impact parameter  $p$ , to  $\theta = \pi/2$ , corresponding to  $p_0$ . ( $\theta = \pi/2$  is the plane passing through the star and perpendicular to the line-of-sight. If the nebula is expanding radially, then this is the plane with zero radial velocity.) We find that

$$\cos \theta = \frac{4\pi}{F} \int_p^{p_0} \Sigma(p, \phi) p dp \quad (\text{A3})$$

The expression on the right hand side is just the integrated flux from the azimuthal segment (from back or front shell only), divided by  $F/4\pi$ . We use this relation to find  $\theta(p)$ . Now  $p = r \sin(\theta)$  and  $z = r \cos(\theta)$ , so  $z = p/\tan(\theta)$ . The p-z plot is the cross-section of the nebula in the  $\phi$ -plane.

As a simple example, let the nebula be an ellipsoid of revolution about the z axis with the minor axis  $b$  in the plane of the sky and the major axis (z-axis)  $a$  along the line of sight. Then the radius vector from the star to a point on the surface is

$$r = \frac{ab}{\sqrt{b^2 \cos^2(\theta) + a^2 \sin^2(\theta)}} \quad (\text{A4})$$

The angle between the z-axis and the normal to the ellipsoid surface is

$$\beta = \arctan[(a/b)^2 \tan(\theta)] \quad (\text{A5})$$

Then from equation (1), we can show that the surface brightness is

$$\Sigma = \frac{a^2}{r^4 \cos(\theta)} \quad (\text{A6})$$

Now,  $p = r \sin(\theta)$  and  $\frac{dp}{d\theta} = \frac{r^3 \cos(\theta)}{a^2}$ , so that  $\Sigma p dp = \sin(\theta) d\theta$ , and equation (3) is satisfied.

If the ellipsoid is inclined to the x-axis by an angle  $i$ , the surface brightness becomes

$$\Sigma = \frac{(ab)^2}{r^4} \frac{1}{b^2 \cos(\theta) \cos(i) - a^2 \sin(\theta) \sin(i)} \quad (\text{A7})$$

In this expression,  $\Sigma$  is negative for values of  $\theta$  corresponding to the back side of the shell. Even for modest  $i$  and  $a/b$ , the effects of  $\beta$  and  $r$  may combine to produce large front-to-back shell brightness ratios.

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Fig. 1.— Drizzled  $H\alpha$  image of BD+30. The intensity scale is linear. North is up and east is to the left. In this figure, the field of view is  $9''.10 \times 9''.10$ .

Fig. 2.— Line magnification measurements of BD+30 in  $H\alpha$ . The numbers are the magnification factor minus 1 along the corresponding radial directions. In this and the following figures, the field of view is  $6''.37 \times 6''.37$ .

Fig. 3.— Expansions of individual features in the BD+30  $H\alpha$  image. All numbers are in units of  $\text{mas yr}^{-1}$ .

Fig. 4.— Expansions of individual features in the BD+30 [N II] image. All numbers are in units of  $\text{mas yr}^{-1}$ .

Fig. 5.— The comparison of the expansions of individual features in the  $H\alpha$  band and in the [N II] band. Solid lines represent the  $H\alpha$  shifts, and dashed lines are for [N II]. White regions in the figure are masked saturated pixels.

Fig. 6.— The C II]  $\lambda 2327$  line profiles of BD+30, obtained with the HST STIS. Panel (A) is with the slit at position angle  $25^\circ$  east of north; north is at the top. Panels (B1) and (B2) show the spectrum taken with position angle of  $99^\circ$ ; east is at the top. In panels (A) and (B1) we have overplotted the best fitting ellipses. The bright dots mark the centers of the ellipses. Panel (B2) shows the same data as (B1) without the distraction of the ellipse. For both positions the slit was shifted  $0''.4$  from the central star, although a trace of continuum at the top may be a broad stellar line scattered by nebular dust. The apparent spur seen in (B) branching from the profile at about 10 o'clock is the trace of another line of the C II] multiplet.

Fig. 7.— The position of slit through which the STIS echelle spectrum (Fig. 6 B) was taken, and the fit to the expansion. The left panel shows the slit orientation and width, and the measured shifts of features along the slit. The right panel shows the fit of shifts along the slit against the distances from the center of slit. The triangles are the shifts and the square marks the origin. The dashed lines at  $\pm 2''.47$  mark the edges of the fitting ellipse shown in Fig. 6 B1.

Fig. 8.— The cross-section of the nebula (arcsec) along p.a.  $99^\circ$ . The vertical axis is in the plane of the sky, the horizontal axis is toward the observer. The + symbols are from equation A 3, where for  $\Sigma$  we used the average surface brightness of the radio images over a  $\phi$  interval of  $25^\circ$ . The triangle is from equation A 1. The dotted line is an ellipse of 1.5 axial ratio. The dashed line is the shape that would be inferred from equation A 3 applied to an ellipse of 1.56 axial ratio, inclined by  $9^\circ.7$ . See text.

Fig. 9.— The white curve is our model fit plotted over the [O III]  $\lambda 5007$  line profile of Bryce & Mellema (1999), Fig. 2(d). This spectrum was taken with an E-W slit in  $1''$  seeing. We have smoothed their original data and reversed the y-axis to place east at the top, as in our STIS echelle spectra (Fig. 6).

Table 1. Observations Used for Expansion Measurement

Filter	Exposure (sec)	HST P.A. (deg)	Date (yyyy-mm-dd)	Proposal
H $\alpha$ F656N	200,300	115 40 58.8	1994-03-06	5403
H $\alpha$ F656N	3 $\times$ 160	249 04 05.9	1999-11-04	8116
[N II] F658N	200,300	115 40 58.8	1994-03-06	5403
[N II] F658N	3 $\times$ 160	249 04 05.9	1999-11-04	8116

Table 2. Expansion Rates of BD+30 in  $\text{mas yr}^{-1}$ . The values from two magnification methods came from the magnification factors times the distance of the edges of the major and minor axes from the center of image. The values from feature measurement were calculated by averaging some values near the edges of major and minor axes.

	$\text{H}\alpha$		[N II]	
	Major Axis	Minor Axis	Major Axis	Minor Axis
Radius	$2''.32$	$1''.84$	$2''.32$	$1''.84$
Magnification ( $\text{mas yr}^{-1}$ )	$2.54 \pm 0.51$	$2.02 \pm 0.40$	$2.64 \pm 0.53$	$2.09 \pm 0.42$
Line Magnification ( $\text{mas yr}^{-1}$ )	$2.82 \pm 0.56$	$2.30 \pm 0.46$	$2.91 \pm 0.58$	$2.40 \pm 0.48$
Feature Shifts ( $\text{mas yr}^{-1}$ )	$2.89 \pm 0.29$	$2.49 \pm 0.25$	$3.07 \pm 0.31$	$2.45 \pm 0.25$

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