Plan of Lecture
The Formation of Structure

How did the universe become lumpy?

Initial conditions.

Growth of perturbations in a static universe.

Growth in the expanding universe.

Rotation and gas physics.

The evolution of galaxies.

The Problem

From microwave background, perturbations were $\sim 10^{-5}$ of average density.

"Linear" regime.

Now, in accessible universe, *factor* of $\sim 10^{45}$!

Very nonlinear indeed!

How do you get here from there?

Within this general topic, we can ask how structures such as stars, galaxies, and clusters formed.

Did big or little form first?

How important is gravity compared to other effects?

Cooling, rotation, etc.

Static Spacetime

Set up an idealized problem.

Space is infinite.

Constant density ρ_0 everywhere.

Consider only gravity.

What happens? Nothing! A given particle is attracted equally in all directions.

Now suppose that in a sphere of radius R, there is a constant extra density $\rho_0 \delta$, $\delta \ll 1$.

What happens?

Dense region contracts further.

Let's analyze this more carefully.

Gravitational Collapse

Consider particle on edge of density perturbation.

Gravity on it is sum of background (constant density), plus perturbation.

Background gives no net force.

Only perturbation matters!

Extra mass is $M = \frac{4}{3}\pi R^3 \rho_0 \delta$.

Acceleration is $a = GM/R^2 = \frac{4}{3}\pi G R \rho_0 \delta$.

How long does it take to shrink by $R\delta$?

$\frac{1}{2}at^2 = R\delta \Rightarrow t \sim 1/\sqrt{G\rho_0}$.

Density contrast doubles in time $\sim (G\rho_0)^{-1/2}$.

Collapse on Static Background

Suppose spacetime is not expanding.

At redshift z, critical density is
$9.2 \times 10^{-30}(1+z)^3$ g cm^{-3}.

Close to this in early universe.

Time scale to collapse is $\sim 10^{10}(1+z)^{-3/2}$ yr.

Very short at $z \sim 1000$!

This would form structure easily, within millions of years.

"Static background" is valid for, e.g., molecular cloud.

But universe itself is expanding.

Collapse on Expanding Background

Now suppose that there is a slight density enhancement but the universe is expanding.

As long as $\delta \ll 1$, the density decreases.

Turns out that $\delta \propto a$, where a is the "scale factor" of the universe.

$a \propto (1 + z)^{-1}$.

Assumes matter domination.

Growth of perturbations is much slower!

Led to predictions of initial δ.

First structure expected $z \sim 10 - 20$.

Misconception: Universal Expansion

Let's address a common misconception.

The universe has expanded by $\sim$50% since formation of solar system.

Has solar system expanded?

Consider the Earth:

Orbital speed ≈ 30 km s^{-1}.

"Expansion speed"

$H_0 \times (1 \text{ AU}) \approx 3.4 \times 10^{-10}$ **km s^{-1}.**

Tiny perturbation!

Everything experiences Hubble flow, but dense concentrations of matter feel much stronger local effects.

Nonlinear Collapse

Now suppose we've reached a point where $\delta \sim 1$.

Overall expansion unimportant.

Collapse is then a free fall.

Consider first "dark matter": no nongravitational interactions.

Collapse can't proceed indefinitely!

When random motions become comparable to orbital speeds, quasi-equilibrium is reached.

"Violent relaxation"!

If initially little motion, final radius is $\approx 1/2$ initial radius at $\delta \sim 1$.

What happens next?

Gas Physics

If all matter were noninteracting elementary particles, nothing would happen.

Achieve relaxed configuration.

Sit there indefinitely!

However, normal matter interacts differently.

Gas collides, radiates.

Cooling causes sinking in potential.

Therefore, normal matter can achieve much higher densities.

When density is high enough, stars can form.

The First Stars

How would the first star in a large volume differ from today's stars?

No metal enrichment: only H and He (negligible Li, Be).

Relatively little cooling.

Therefore, Jeans mass (critical for collapse) remains high.

Also, winds and pulsations are less effective (driven by metal line opacities).

First stars could be very massive, $10^{2-3}\,M_\odot$.

Conceivably, might see effects in cluster metal abundances.

From Small to Large

In the standard picture of primordial fluctuations, larger δ on smaller scales.

The first nonlinear structure therefore forms on small scales.

However, temperature prevents structure on too small a scale.

Compromise: $\sim 10^5\,M_\odot$.

Globular clusters?

Small fluctuations are strongest on big fluctuations.

Therefore, hierarchical structure formation.

Small structure forms early.

Big structure forms late.

Small halos merge to big.

Consequences and Predictions

One prediction relates to the number of “small” halos to “big” halos.

MW should have hundreds.

Oops!

Why so few dwarf galaxies? Maybe if too small, first star blows away all gas; just BH and dark matter!

Would still have DM halos.

Possibly seen with lensing!

If BH are formed at centers of all halos, would expect many mergers of BH when halos merge.

See gravitational waves!

Rotation and Magnetic Fields

Most primordial rotation dissipates in expansion.

However, during collapse, rotation might be interesting in some cases.

Spirals if rotating?

Ellipticals if not?

The origin of magnetic fields is also a frontier subject.

Very little in early universe.

Can amplify small fields.

Origin of original fields?

Summary

As far as we know, structure started small and got bigger.

Normal matter can cool, hence became more compactly distributed than dark matter.

But associated with DM!

The first stars might have been a lot different than stars are now...

Challenge: how would star formation change after the first star formed?