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OBSERVING IMBH-IMBH BINARY COALESCENCES VIA GRAVITATIONAL RADIATION

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ABSTRACT

Recent numerical simulations have suggested the possibility of forming double intermediate-mass black holes (IMBHs) via the collisional runaway scenario in young dense star clusters. The two IMBHs that formed would exchange into a common binary shortly after their birth and quickly inspiral and merge. Since space-borne gravitational wave (GW) observatories such as *LISA* will be able to see the late phases of their inspiral out to several gigaparsecs, and LIGO will be able to see the merger and ringdown out to similar distances, they represent potentially significant GW sources. In this Letter we estimate the rate at which *LISA* and LIGO will see their inspiral and merger in young star clusters, and we discuss the information that can be extracted from the observations. We find that *LISA* will likely see tens of IMBH-IMBH inspirals per year, while advanced LIGO could see ∼10 merger and ringdown events per year, with both rates strongly dependent on the distribution of cluster masses and densities.

Subject headings: black hole physics — gravitational waves — stellar dynamics

1. INTRODUCTION

Observations suggesting the existence of intermediate-mass black holes (IMBHs) have mounted in recent years. Ultraluminous X-ray sources—point X-ray sources with inferred luminosities $\gtrsim 10^{39}$ ergs s^{-1}—may be explained by sub-Eddington accretion onto BHs more massive than the maximum of $\sim 10\ M_\odot$ expected from stellar core collapse (Miller & Colbert 2004). Similarly, the cuspy velocity dispersion profiles in the centers of the globular clusters M15 and G1 may also be explained by the dynamical influence of a central IMBH (van der Marel et al. 2002; Gerssen et al. 2002; Gebhardt et al. 2005), although this conclusion remains somewhat controversial (Baumgardt et al. 2003).

The most likely formation scenario for an IMBH is the collapse of a very massive star (VMS), which was formed early in the lifetime of a young star cluster via a runaway sequence of physical collisions of massive main-sequence stars (Portegies Zwart et al. 1999; Ebisuzaki et al. 2001; Portegies Zwart & McMillan 2002; Gürkan et al. 2004). This scenario has been studied in detail for star clusters without primordial binaries, with recent work showing that runaway growth of a VMS to $\sim 10^3\ M_\odot$ occurs generically in clusters with deep core collapse times shorter than the ∼3 Myr main-sequence lifetime of the most massive stars (Freitag et al. 2006).

Due to the computational cost of simulating the more realistic case of star clusters with primordial binaries, it is only recently that such simulations have been performed (Portegies Zwart et al. 2004; Gürkan et al. 2006). The work of Gürkan et al. (2006) was the first to systematically study the influence of primordial binaries on the runaway growth process. They showed that stellar collisions during binary scattering interactions offer an alternate channel for runaway growth, with the main result that clusters with binary fractions larger than ≈10% generically produce *two* VMSs, provided that the cluster is sufficiently dense and/or centrally concentrated to trigger the runaway earlier than ∼3 Myr in the absence of primordial binaries. Observations and recent numerical calculations suggest that star clusters may be born with large binary fractions ($\gtrsim$30%; Hut et al. 1992; Ivanova et al. 2005), implying that *all* sufficiently dense and massive star clusters could form multiple VMSs.

The VMSs formed will undergo core-collapse supernovae and likely become IMBHs on a timescale of ∼4 Myr after cluster formation (the lifetime of a VMS is extended slightly by collisional rejuvenation; see, e.g., Freitag et al. 2006). After their separate formation, the two IMBHs will quickly exchange into a common binary via dynamical interactions. The IMBH-IMBH binary (IMBHB) will then shrink via dynamical friction due to the cluster stars, on a timescale $\sim t_r \langle m \rangle / M_{\rm IMBH} \lesssim 10$ Myr, where t_r is the core relaxation time, $\langle m \rangle$ is the local average stellar mass, and $\langle m \rangle / M_{\rm IMBH} \lesssim 10^{-2}$. Note that since t_r scales inversely with $\langle m \rangle$ for fixed core velocity dispersion and mass density, the dynamical friction timescale is independent of $\langle m \rangle$ (Binney & Tremaine 1987). The IMBHB will then shrink further via dynamical encounters with cluster stars (Quinlan 1996; Yu & Tremaine 2003; Miller 2005), until it merges quickly via gravitational radiation, on a timescale ≈ 1 Myr $(\sigma_c/20\ {\rm km\ s^{-1}})^3 (\rho_c/10^5\ M_\odot\ {\rm pc^{-3}})^{-1} (M_{\rm IMBH}/10^3\ M_\odot)^{-1}$, where σ_c is the cluster core velocity dispersion and ρ_c is the core mass density (Quinlan 1996, eqs. [29] and [30]). This timescale has also been confirmed by numerical scattering calculations (K. Gültekin 2006, private communication).

Only the more massive IMBHBs merge in the *Laser Interferometer Space Antenna* (*LISA*) band of 10^{-4} to 1 Hz (redshifted binary mass $M_z \equiv (1+z)M \gtrsim 4 \times 10^3\ M_\odot$, where M is the total binary mass). Figure 1 shows the final gravitational wave (GW) frequency f_f (the frequency at the innermost stable circular orbit if within the *LISA* frequency range [large M_z], otherwise the maximum *LISA* frequency of ≈1 Hz [small M_z], as in Will 2004) and the frequency 1 yr prior, f_i, as a function of redshifted mass M_z, for the reduced mass parameters $\eta = 0.25$ (equal-mass binary) and $\eta = 0.1$ (mass ratio 0.13; see, e.g., Will 2004). For a wide range in M_z, the late stages of inspiral clearly span the *LISA* "sweet spot" (roughly a decade centered on $10^{-2.2}$ Hz), implying that *LISA* could easily detect the chirp signal, enabling a measurement of the masses of the binary members. Such an observation would be direct evidence for an IMBH.

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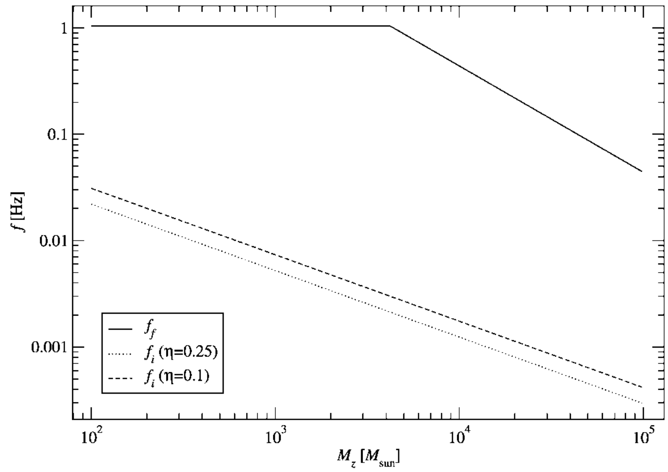

FIG. 1.—Final GW frequency f_f (see text), and the frequency 1 yr prior, f_i, for an IMBHB with total mass M and reduced mass parameter η, as a function of redshifted binary mass M_z, for $\eta = 0.25$ (equal-mass binary) and $\eta = 0.1$ (mass ratio 0.13). (The final frequency is roughly independent of η.)

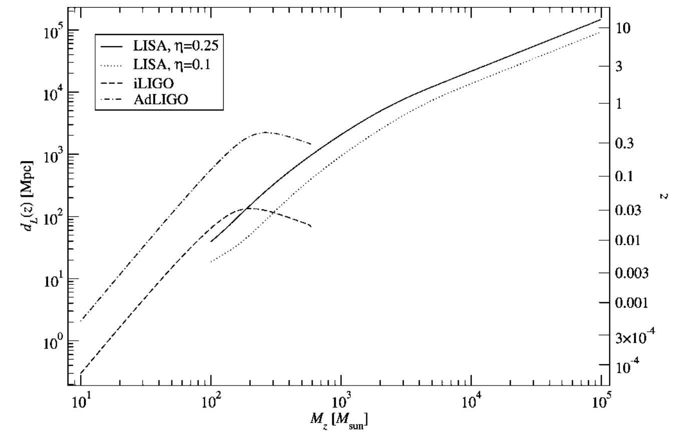

FIG. 2.—Luminosity distance, $d_L(z)$, to which an IMBHB of total mass M and reduced mass parameter η can be seen via its inspiral with *LISA* with S/N = 10 for a 1 yr integration, and via its merger and ringdown with S/N = 8 for iLIGO and adLIGO, as a function of the redshifted mass M_z. The corresponding redshift (calculated using the *Wilkinson Microwave Anisotropy Probe* year 3 cosmological parameters, as discussed in the text) is shown on the right vertical axis.

In § 2 we estimate the rate at which *LISA* will observe inspiral of IMBHBs in young star clusters. In § 3 we estimate the rate at which the Laser Interferometer Gravitational-Wave Observatory (LIGO) will observe their merger and ringdown. Finally, in § 4 we discuss the observational consequences.

2. ESTIMATING THE *LISA* DETECTION RATE

We first need to know the distance to which *LISA* can see IMBHB inspirals. Following the techniques in Will (2004) and Flanagan & Hughes (1998), we adopt the latest *LISA* sensitivity curve (Larson et al. 2000),[5] including confusion noise from Galactic white dwarf binaries (Bender & Hils 1997), and calculate the maximum luminosity distance, $d_L(z)$, to which an IMBHB of total mass M and reduced mass parameter η can be seen with S/N = 10 for a 1 yr integration. The results are shown in Figure 2 as a function of M_z, for $\eta = 0.25$ and $\eta = 0.1$. Note that the results of Gürkan et al. (2006) show that the masses of the IMBHs never differ by more than a factor of a few ($\eta \gtrsim 0.15$). Thus, *LISA* will be able to see typical IMBHBs ($M \sim 10^3\ M_\odot$) out to a few gigaparsecs.

With this information in hand, we first make a crude estimate of the *LISA* event rate. Following Miller (2002), we write for the total rate

$$R \equiv \frac{dN_{\rm event}}{dt} = \left(\int_0^{z_{\rm max}} \frac{dV_c}{dz}\,dz\right) \frac{dN_{\rm cl}}{dV}\, g\, \frac{1}{t_U}. \qquad (1)$$

The first factor, $\int_0^{z_{\rm max}} (dV_c/dz)\,dz$, is the integrated comoving volume of space in which *LISA* is sensitive to the events. The second factor, $dN_{\rm cl}/dV$, is the number density of star clusters sufficiently massive to form IMBHBs. Since the *globular* clusters we currently see were likely at least a few times more massive at formation (Joshi et al. 2001), we set this factor to the current density of globular clusters in the local universe, $dN_{\rm cl}/dV \approx 8\ h^3\ {\rm Mpc}^{-3}$ (Portegies Zwart & McMillan 2000). The third factor, g, is the fraction of sufficiently massive clusters that have a large enough binary fraction and initial central density to produce IMBHBs. Since initial cluster structural parameters are largely unknown, we treat g as a parameter. The fourth factor is the event rate per IMBHB-producing cluster, taken to be one divided by the age of the universe, since only one IMBHB is formed per cluster over its lifetime. We adopt a ΛCDM cosmology, with parameters $\Omega_M = 0.24$, $\Omega_\Lambda = 0.76$, and $h = 0.73$, for which $t_U = 13.8$ Gyr (Spergel et al. 2006). Putting this together for $d_L = 4.9$ Gpc ($z_{\rm max} = 0.79$), the distance to which *LISA* can see IMBHBs with $M = 2 \times 10^3\ M_\odot$, equation (1) gives $R \approx 1(g/0.1)\ {\rm yr}^{-1}$.

Writing down a generalized form of the rate integral in equation (1) is straightforward. Since the time between cluster formation and IMBHB merger is $\ll t_U$, we assume that the merger is coincident with cluster formation. Thus, the rate integral is

$$R \equiv \frac{dN_{\rm event}}{dt_o} = \int_0^{z_{\rm max}} \frac{d^2M_{\rm SF}}{dV_c dt_e}\, g_{\rm cl}\, g\, \frac{dt_e}{dt_o}\, \frac{dV_c}{dz} \times \int_{M_{\rm cl,\,min}(z)}^{M_{\rm cl,\,max}} \frac{d^2N_{\rm cl}}{dM_{\rm SF,\,cl}\,dM_{\rm cl}}\, dM_{\rm cl}\, dz. \qquad (2)$$

Here $R \equiv dN_{\rm event}/dt_o$ is the event rate observed at $z = 0$ by *LISA*, $d^2M_{\rm SF}/dV_c dt_e$ is the star formation rate (SFR) in mass per unit of comoving volume per unit of local time, $g_{\rm cl}$ is the fraction of star-forming mass that goes into star clusters more massive than $10^{3.5}\ M_\odot$ (generally a function of z), g is as above, and $d^2N_{\rm cl}/dM_{\rm SF,\,cl}dM_{\rm cl}$ is the distribution function of clusters over individual cluster mass $M_{\rm cl}$ and total star-forming mass in clusters $M_{\rm SF,\,cl}$. Finally, dt_e/dt_o is simply $(1+z)^{-1}$, and dV_c/dz is the rate of change of comoving volume with redshift, which is a function of cosmological parameters (Hogg 1999). Note that we set $z_{\rm max} = 5$, since this is roughly the limit to which the cosmic SFR can be traced. Thus, the integral in equation (2) should be considered a mild lower limit to the true rate. We now discuss each element in equation (2) in more detail.

Following Porciani & Madau (2001), we adopt three different choices for the SFR:

$$\left(\frac{d^2M}{dV_c dt}\right)_{{\rm SF}i} = C_i h_{65} F(z) G_i(z)\ M_\odot\ {\rm yr}^{-1}\ {\rm Mpc}^{-3}, \qquad (3)$$

where $i = 1$, 2, and 3 denote the different rates, C_i is a constant,

[5] The Online Sensitivity Curve Generator is located at http://www.srl.caltech.edu/~shane/sensitivity.

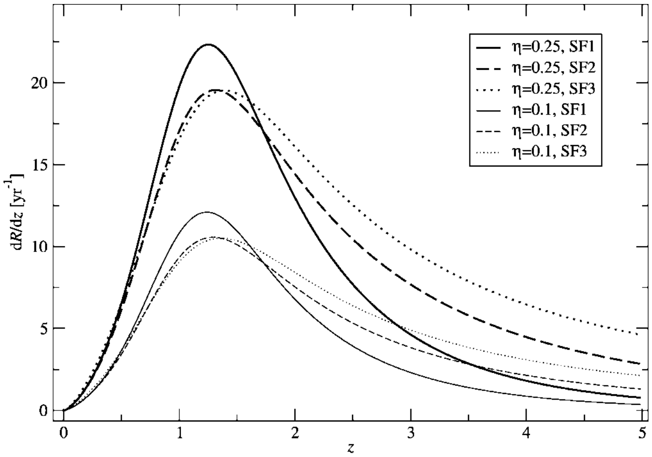

FIG. 3.—Integrand of the rate integral in eq. (2) for the three different SFRs in eq. (3), for $\eta = 0.25$ and $\eta = 0.1$.

$G_i(z)$ is a function of z, $h_{65} = h/0.65$, and $F(z) = [\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda]^{1/2}/(1+z)^{3/2}$. The first is from Madau & Pozzetti (2000), with $C_1 = 0.3$ and $G_1(z) = e^{3.4z}/(e^{3.8z} + 45)$, which peaks between $z = 1$ and 2 and decreases at larger redshift. The second is from Steidel et al. (1999), with $C_2 = 0.15$ and $G_2(z) = e^{3.4z}/(e^{3.4z} + 22)$, which is roughly constant for $z \gtrsim 2$. The third is from Blain et al. (1999), with $C_3 = 0.2$ and $G_3(z) = e^{3.05z-0.4}/(e^{2.93z} + 15)$, which increases above $z \approx 2$.

Measuring the fraction of star-forming mass in clusters is difficult for anywhere but the local universe. Similarly, while we know reasonably well the initial cluster conditions required to form an IMBHB (Gürkan et al. 2006), we know much less well the distribution of cluster properties at birth. We therefore treat $g_{\rm cl}$ and g as parameters, taking $g_{\rm cl} = 0.1$ and $g = 0.1$ somewhat arbitrarily as canonical values.

Assuming that the spectrum of cluster masses is neither a function of cosmic epoch nor the total star-forming mass available for clusters, the factor $d^2N_{\rm cl}/dM_{\rm SF,\,cl}dM_{\rm cl}$ can be separated as

$$\frac{d^2N_{\rm cl}}{dM_{\rm SF,\,cl}dM_{\rm cl}} = \frac{f(M_{\rm cl})}{\int M_{\rm cl} f(M_{\rm cl}) dM_{\rm cl}}, \qquad (4)$$

where $f(M_{\rm cl})$ is the (normalized) distribution function of cluster masses. For this we adopt the power-law form observed for young star clusters in the Antennae, which is thought to be universal: $f(M_{\rm cl}) \propto M_{\rm cl}^{-2}$ (Zhang & Fall 1999), with a lower limit of $10^{3.5}\ M_\odot$ and an upper limit of $10^7\ M_\odot$.

It is the limit $M_{\rm cl,\,min}(z)$ in equation (2) that encodes all information about the detectability of an IMBHB inspiral by *LISA*. Specifically, the redshift to which *LISA* can see the inspiral is a function of the binary mass, which is itself a function of the host cluster mass. Adopting an efficiency factor $f_{\rm GC}$ for the fraction of cluster mass going into the IMBHB, this relationship is inverted to obtain $M_{\rm cl,\,min}(z)$. Recent numerical work shows that the efficiency factor is $f_{\rm GC} \approx 2 \times 10^{-3}$, independent of cluster initial conditions (Gürkan et al. 2004), which we take as our canonical value. At low redshift, $M_{\rm cl,\,min}(z)$ is clamped at the value $M_{\rm cl} = 200\ M_\odot/f_{\rm GC}$, set by adopting the definition that an IMBH have mass $\geq 10^2\ M_\odot$. At high redshift ($z > 5$, so not relevant to our calculation), $M_{\rm cl,\,min}(z)$ is clamped at the value of $10^7\ M_\odot$ from the cluster mass function; in other words, no cluster is sufficiently massive to produce an IMBHB massive enough to be observable by *LISA*, so the integral is zero.

We numerically integrated equation (2) for the different SFRs in equation (3), for S/N $= 10$ and an integration time of 1 yr, to find that the rate is

$$R(\eta = 0.25) \approx 40\text{–}50 \left(\frac{g_{\rm cl}}{0.1}\right)\left(\frac{g}{0.1}\right)\ {\rm yr}^{-1}, \qquad (5)$$

with the spread in the coefficient from the different SFRs. The coefficient decreases to 20–25 for $\eta = 0.1$. The rate is dominated by clusters in the mass range 10^6–$10^{6.5}\ M_\odot$ [IMBHB mass $(2\text{–}6) \times 10^3\ M_\odot$], with more than half the contribution to the rate coming from this mass range, for both $\eta = 0.1$ and 0.25, and for all three SFRs in equation (3) (except SF3 for $\eta = 0.1$). Note that equation (2) is only strictly valid when the source is visible by the instrument for less than the integration time. This turns out not to be precisely correct. A typical IMBHB with mass $M = f_{\rm GC}10^{6.25}\ M_\odot$ takes roughly 4 years to cross the *LISA* band from the edge of the white dwarf confusion knee at ≈2 mHz to the upper edge of the band at ≈1 Hz. Thus, the rate presented in equation (5) is an underestimate by of order a factor of a few.

Figure 3 shows the integrand of the rate integral in equation (2) for the three different SFRs in equation (3), for $\eta = 0.25$ and 0.1. Most events originate from $z \sim 1$. Unfortunately, neither R nor dR/dz is particularly sensitive to the cosmic SFR, with dR/dz decreasing quickly above $z \approx 2$ even when the SFR is increasing (as in SF3). Thus, observations of IMBHB inspirals will not be very informative about the cosmic SFR. However, they will likely yield a handle on the fraction of star formation that is in compact massive clusters.

3. ESTIMATING THE LIGO DETECTION RATE

Shortly after the two IMBHs merge, the merger product can be well described as a single perturbed black hole, emitting GWs at its quasi-normal frequencies. Largely falling within the initial and advanced LIGO (iLIGO and adLIGO) sensitivity bands, the merger and ringdown waves will likely carry a few percent of the rest mass energy of the hole (see, e.g., Flanagan & Hughes 1998). Numerical simulations suggest that a merging pair of nonspinning equal-mass black holes will emit a fraction $\epsilon \simeq 0.03$ of their rest mass in merger and ringdown GWs, forming a black hole with spin parameter $a \simeq 0.7$ (Baker et al. 2002; Campanelli et al. 2006; Baker et al. 2006). Under these conditions, the ringdown frequency is given by

$$f \approx \frac{c^3}{2\pi G M_z}\left[1 - 0.63(1-a)^{3/10}\right] \approx 180 \left(\frac{M_z}{10^2\ M_\odot}\right)^{-1}\ {\rm Hz} \qquad (6)$$

(see eq. [3.17] of Flanagan & Hughes 1998). We can express the distance to which we are sensitive to ringdown waves at signal-to-noise ratio ρ as

$$d_L(z) = \left(\frac{2\epsilon M_z}{5\pi^2\rho^2 f^2 S(f)}\right)^{1/2}, \qquad (7)$$

where $S(f)$ is the spectral noise density of LIGO. Combining this expression with the concordance cosmological model and iLIGO and adLIGO sensitivity curves, we find the range to which LIGO can detect ringdown shown in Figure 2.

To obtain a conservative estimate for the rate at which iLIGO and adLIGO could detect these mergers with a ringdown-only search, we use equation (1) with a moderately optimistic range of $d_L \approx 100$ Mpc for iLIGO and $d_L = 2$ Gpc for adLIGO. The expected detection rate is then $10^{-4}(g/0.1)\ \mathrm{yr}^{-1}$ and $1(g/0.1)\ \mathrm{yr}^{-1}$ for iLIGO and adLIGO, respectively. More detailed estimates using machinery analogous to equation (2) increase these estimates by roughly an order of magnitude, making the rate for adLIGO $10(g_{\mathrm{cl}}/0.1)(g/0.1)\ \mathrm{yr}^{-1}$.

4. DISCUSSION

It appears likely that *LISA* will see tens of IMBHB inspiral events per year, while adLIGO could see ∼10 merger and ringdown events per year, with both rates strongly dependent on the distribution of cluster masses and densities. Detection of an IMBHB would have profound implications. A match-filtered observation of the inspiral would yield the redshifted masses of the black holes, directly confirming the existence of IMBHs. It would also yield the luminosity distance to the source; with enough observations, constraints could be placed on the cosmic history of star formation in dense, massive clusters. Detection of the ringdown signal from the merger product will also yield its spin, which may provide insight into its formation history.

Typical IMBHBs spend $\gtrsim 10^6$ yr inspiraling through the *LISA* band, with nearly all of that time spent at low frequencies ($\lesssim 10^{-3}$ Hz). In the low-frequency region they will thus appear as a large number of monochromatic sources, possibly contributing to confusion noise and increasing the noise floor (e.g., Farmer & Phinney 2003). A detailed calculation of this is beyond the scope of this Letter. However, we note that if their contribution is similar in magnitude to that of Galactic compact object binaries (Bender & Hils 1997), the rates predicted in equation (5) would decrease by about 20%.

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