Compton scattering effects in the spectra of soft gamma-ray repeaters

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The association of all three soft gamma-ray repeaters (SGRs) with supernova remnants has made possible estimates of the distance to and luminosity of the sources of SGRs, which have provided a starting point for detailed modeling. One of the most popular classes of models involves strongly magnetized neutron stars, with surface dipole fields $B \sim 10^{14} - 10^{15}$ G. In these "magnetar" models, many otherwise negligible processes can play an important role. Here we consider the spectral effects of strong-field modifications to Compton scattering, in particular those related to the contribution of vacuum polarization to the dielectric tensor. Vacuum polarization introduces a density-dependent photon frequency, called the second vacuum frequency, at which the normal modes of polarization become nonorthogonal and the mean free path of photons decreases sharply. Monte Carlo simulations of photon propagation through a magnetized plasma show that this effect leads, under a wide range of physical conditions, to a broad absorption-like feature in the energy range $\sim$5 keV—40 keV.

INTRODUCTION

The first soft gamma-ray repeater (SGR) to be discovered, SGR 0526-66 (1,2), was shown soon after its discovery to be positionally coincident with the N49 supernova remnant in the Large Magellanic Cloud (3). More recently, the other two SGRs (SGR 1806-20 and SGR 1900+14) have also been associated with supernova remnants (4–8). It is now generally accepted that SGRs originate from young neutron stars.

For a variety of reasons, summarized in (9), a popular class of models for SGRs assumes that these neutron stars have very strong surface magnetic fields, on the order of $10^{14} - 10^{15}$ G. These models are extremely complicated, and many of their properties depend on microscopic physical processes that are important only in very strong magnetic fields. Here we concentrate on the effects of vacuum polarization and, in particular, on the spectral effects of the second vacuum frequency.

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In strong magnetic fields $B \gtrsim B_c$, where $B_c = 4.414 \times 10^{13}$ G is the magnetic field at which the electron cyclotron energy equals the electron rest mass energy, vacuum polarization effects due to virtual e^+e^- pairs can become significant (see, *e.g.*, (10) for a discussion). In fact, for strong enough fields the vacuum contribution to the dielectric tensor can exceed the plasma contribution (11,12). This affects the photon normal modes and cross sections. Well below the electron cyclotron frequency $\omega_B \equiv eB/m_ec$, the normal modes for photons in a very strong magnetic field are nearly linearly polarized over most photon frequencies and incident angles.

At photon frequencies $\omega \ll \omega_B$, the scattering cross section depends strongly on the polarization of the photon. If the electric field vector of the photon is in the plane formed by the magnetic field and the photon propagation direction $\hat{k}$, the photon is in the parallel mode and the scattering cross section $\sigma_\parallel \sim \sigma_T$, where $\sigma_T = 6.65 \times 10^{-25}$ cm^2 is the Thomson cross section. However, if the electric field vector of the photon is perpendicular to the $\vec{B} - \hat{k}$ plane, the photon is in the perpendicular mode and $\sigma_\perp \sim (\omega/\omega_B)^2 \sigma_T \ll \sigma_T$. This reduction in the cross section is understandable, because in a strong magnetic field it is easier to oscillate an electron along the field than to oscillate it across the field.

There is, however, a small range of photon frequencies in which the vacuum and plasma contributions to the dielectric tensor are comparable to each other and the normal modes become strongly nonorthogonal over a broad range of angles. This frequency, called the second vacuum frequency (the first vacuum frequency is near the cyclotron frequency), depends on the magnetic field and density: $\omega_{c2} \sim n^{1/2}B^{-1}$ for $B \ll B_c$ and $\omega_{c2} \sim n^{1/2}B^{-1/2}$ for $B \gg B_c$. When $\omega \approx \omega_{c2}$, $\sigma_\parallel \approx \sigma_\perp \approx \sigma_T$. Figure 1 shows the dependence of cross section on photon energy for both polarization modes for $\rho = 100$ g cm^{-3} and $\rho = 1000$ g cm^{-3} when $B = 10\,B_c$. This figure shows that at any particular density the cross section is unusually high for a narrow range of frequencies. Conversely, if we assume that most of the flux is transported in the lower cross section mode, for any photon frequency there is a density at which the cross section is high. Thus, the optical depth for a photon is greater than it would be in the absence of the enhanced cross section at ω_{c2}. Because the frequency of the resonance varies with density, in an atmosphere of varying density the vacuum resonance has an effect on the spectrum in a wide range of photon energies.

ENERGY RANGE OF THE VACUUM FEATURE

As described in, e.g., (10), the scattering cross section depends on the parameter

$$b \approx \frac{\sin^2\theta}{2\cos\theta} \frac{\omega_B}{\omega} \left[1 - \frac{\alpha}{2\pi} \left(\frac{\omega}{\omega_p} \right)^2 (\eta_1 - \eta_2) \right] , \qquad (1)$$
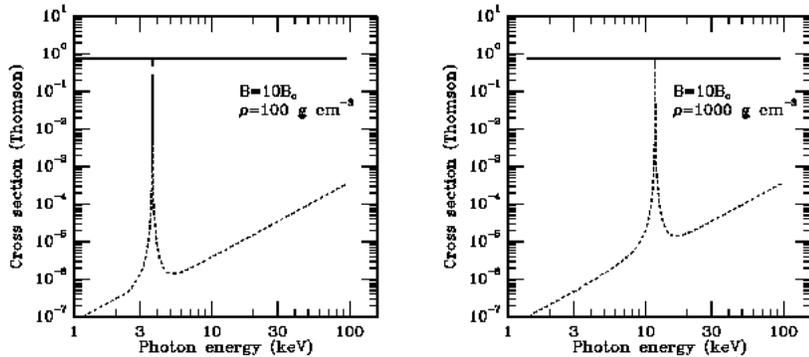
FIG. 1. Cross section in units of the Thomson cross section versus photon energy for the perpendicular (dotted line) and parallel (solid line) polarization modes, for two different densities. The plasma is assumed to consist of fully ionized hydrogen, and the photon propagation angle is $\theta = 60°$.

where θ is the angle of propagation with respect to the magnetic field, α is the fine structure constant, ω_p is the plasma frequency, and η_1 and η_2 are functions of the magnetic field related to the indices of refraction of the two normal modes (13). When $|b| \gg 1$ the scattering is non-resonant and the perpendicular mode has $\sigma \sim (\omega/\omega_B)^2 \sigma_T$. When $|b| \ll 1$ the scattering is resonant and the cross section in either mode is $\sigma \sim \sigma_T \sin^2 \theta$. The second vacuum frequency is

$$\omega_{c2} = \omega_p \left(15\pi/\alpha\right)^{1/2} \left(B_c/B\right) \qquad \text{for } B \ll B_c \text{ , and} \tag{2}$$

$$\omega_{c2} = \omega_p \left(3\pi/\alpha\right)^{1/2} \left(B_c/B\right)^{1/2} \qquad \text{for } B \gg B_c \text{ .} \tag{3}$$

Because $\omega_{c2} \sim n^{1/2}$, as radiation propagates from regions of high density to regions of low density the frequency of the vacuum resonance changes. In effect, the resonance acts as a sliding high opacity barrier. Qualitatively, we expect that the resonance will lower the flux at a given frequency if a) the optical depth through the resonance is greater than unity, and b) the optical depth to the surface without the resonance is less than unity. These requirements give us, respectively, the minimum and maximum photon energies at which the vacuum resonance strongly affects the spectrum.

As we show in (14), for $B \ll B_c$ and an atmosphere of scale height ℓ, this simple analytical treatment gives minimum and maximum photon energies

$$\hbar\omega_{\min} \approx 6 \left(B/B_c\right)^{-1/3} \left(\ell/10 \text{ cm}\right)^{-1/3} \text{ keV , } \tag{4}$$

$$\hbar\omega_{\max} \approx 30 \left(\ell/10 \text{ cm}\right)^{-1/4} \left(\sin\theta\right)^{-1} \text{ keV ; } \tag{5}$$

if instead $B \gg B_c$,

$$\hbar\omega_{\min} \approx 3 \, (\ell/10 \text{ cm})^{-1/3} \text{ keV} , \tag{6}$$

$$\hbar\omega_{\max} \approx 20 \, (\ell/10 \text{ cm})^{-1/4} \, (B/B_c)^{1/4} \, (\sin\theta)^{-1} \text{ keV} . \tag{7}$$

The scalings with magnetic field and scale height are confirmed by our numerical results (see (14) and the following section): typically $\hbar\omega_{\min} \approx 5$ keV and $\hbar\omega_{\max} \approx 40$ keV, if the radiation is produced deep in the atmosphere.

MODEL SPECTRA

To synthesize spectra, we use a Monte Carlo code and propagate 100,000 photons through an atmosphere that is exponential in density. The input spectrum is a 10 keV blackbody, and for the spectra shown here the magnetic field is $B = 10 \, B_c$. In Figure 2 we show two of our model spectra, one with a scale height $l = 10$ cm and the other with $l = 1000$ cm. We find (see also (14)) that below ~5 keV the spectral shape is dominated by free-free absorption and induced scattering, whereas above ~5 keV direct Compton scattering is most important. As expected, for photon energies between $\sim \hbar\omega_{\min}$ and $\hbar\omega_{\max}$ the extra opacity created by the vacuum resonance causes a deficit in the spectrum, and the upper range of this deficit decreases with increasing scale height, as predicted by the analytical model above. For $l = 10$ cm an absorption-like feature is seen, whereas for $l = 1000$ cm the feature would be evident as a low-energy falloff. In principle, a transition from a falloff to an absorption-like feature would be expected as the source goes from its bursting phase into an afterglow, and observation of such a spectral transition would be evidence for very strong magnetic fields. Note also that the high-energy continuum is steeper than the input blackbody, because the mean free path of photons to scattering goes as ω^{-2}, which shifts the distribution of escaping photons to lower energies.

It is essential to include the effects of mode switching in these simulations. At temperatures of ~10 keV the energy change of a photon in a single scattering is much greater than the width of the vacuum resonance. Thus, in the >50% of scatterings in the resonance when the photon switches modes, the photon will be Comptonized out of the resonance and may require several thousand scatterings to return to the low cross section mode, during which its fractional energy change can be of order unity. However, the mean free path in the parallel mode is small enough that the net distance traveled in the parallel mode is negligible.

CONCLUSIONS

For magnetic fields and plasma densities like those expected in SGRs, photon scattering is strongly affected by the vacuum resonance. We find that the increased opacity caused by this resonance typically creates a falloff or absorption-like feature in the spectrum at ~ 5 keV to 40 keV. The spectral
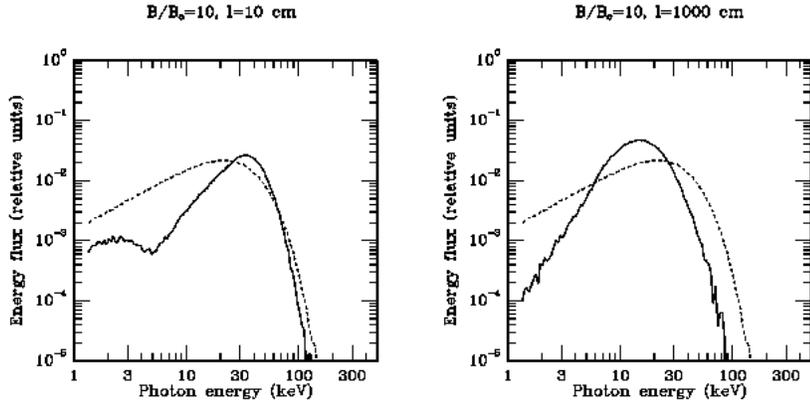
FIG. 2. (left panel) Model spectrum seen at stellar surface (solid line) for $B = 10\,B_c$ and a scale height of $l = 10$ cm. Input spectrum (dotted line) is a 10 keV blackbody. (right panel) Same as left panel, except that $l = 1000$ cm.

energy and shape of the feature depend on quantities such as the magnetic field and scale height of the atmosphere. Since scattering opacity is likely to dominate over absorption opacity in the high-temperature environments of SGRs, the effects of the vacuum resonance must be included in any detailed calculations of radiative transfer through strongly magnetized atmospheres.

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