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RELIABILITY OF MAGNETIC INCLINATION ANGLE DETERMINATIONS FOR PULSARS

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ABSTRACT

We compare the recent estimates of the inclination angle α between the rotation and magnetic axes of 56 pulsars made by both Lyne & Manchester and Rankin. Their results agree reasonably well when α is $\lesssim 40°$; however, there is no correlation between the two estimates of α if either estimate exceeds 40°. The correlation is better for pulsars with beams having more complicated core structure. Nevertheless, the differences between the two sets of estimates are large enough that use of these estimates to investigate pulsar physics is questionable. We discuss the method for determining α based on the Radhakrishnan & Cooke single-vector model, emphasizing its sensitivity to measurement errors. This method complements the approaches of Rankin and Lyne & Manchester and is preferable when accurate polarization data are available.

Subject headings: pulsars: general — stars: neutron

1. INTRODUCTION

The value and time development of the inclination angle α between pulsar rotation and magnetic axes are important for understanding pulsars. Knowledge of the initial distribution of α would provide valuable information about how pulsar magnetic fields are formed. In many pulsar models, the rotational energy loss rate and radio luminosity depend on α. Measurements of α have also been used to investigate possible alignment (or counteralignment) or decay of pulsar magnetic fields (Kundt 1981; Kuz'min, Dagkesamanskaya, & Pugachëv 1984; Candy & Blair 1983, 1986; Blair & Candy 1989; Bhattacharya 1989; Bhattacharya & van den Heuvel 1991; see Srinivasan 1989 and Lamb 1991 for recent reviews). Thus, accurate determinations of α for a large population of pulsars would, when combined with measurements of other pulsar properties, provide valuable clues to the physics and evolution of pulsars.

Recently Lyne & Manchester (1988, hereafter LM88) and Rankin (1990, hereafter R90) have reported values of the magnetic inclination angle α of more than 100 pulsars by combining polarization measurements with the assumption that the intrinsic beam width depends only on the pulse period. Because the value of α is so important in understanding the physics of pulsars, we have investigated the consistency of their inclination angle estimates for the 56 pulsars common to both studies. We find that the mean difference between the values of α reported by LM88 and R90 is nearly 15°. The reported α values agree better for pulsars classified by LM88 as core-dominated than for pulsars listed as cone-dominated by LM88. The agreement is also better for pulsars with complex profiles (as categorized by emission types S_t, T, or M in the classification scheme of R90). Agreement is poor for pulsars determined by either study to have a large value of α; in fact, the correlation between the values of α found in the two studies is actually *negative* for those pulsars determined by either study to have $\alpha > 75°$. Until the origin of these discrepancies is understood and the values of α confirmed, the use of the reported α values in analyses of pulsar properties is questionable.

We describe briefly an alternative method of deriving α which, though not broadly applicable now, may be of use in the future. This method is complementary to the approaches used by LM88 and R90.

2. COMPARISON OF INCLINATION ANGLE DETERMINATIONS

2.1. *Method of Lyne & Manchester*

Lyne & Manchester (1988) divided their pulsars into several classes: those dominated by conal emission, those with both core and cone emission, and those dominated by core emission. they also listed pulsars with partial cone emission and several pulsars whose classification was uncertain. Their classifications were not meant to indicate intrinsic properties of the radio emission, but rather the parts of the beam that the line of sight happened to intersect. Henceforth, when we refer to "cone-dominated" or "core-dominated" pulsars, we mean pulsars classified as such by LM88.

To estimate the inclination angle, LM 88 examined the duty cycles of a subset of the pulsars. The subset consisted of those pulsars showing complete conal emission, which is to say that there was evidence that a cut across the entire pulsar beam was seen, so that the duty cycle can be used to estimate the intrinsic beam width.

For the standard angle definitions shown in Figure 1, one can show that

$$M \equiv \left(\frac{d\psi}{d\phi}\right)_{\max} = \frac{\sin \alpha}{\sin \beta}, \tag{1}$$

where $\beta = |\zeta - \alpha|$ is the "impact parameter." If, as LM88 assume, the emission beam is circular, then

$$\sin^2(\rho/2) = \sin^2(\Delta\phi/2) \sin \alpha \sin \zeta + \sin^2(\beta/2), \tag{2}$$

where $2\Delta\phi$ is the total pulsar width in longitude and ρ is the half width of the beam. Both M and $\Delta\phi$ are directly observable. By measuring M and $\Delta\phi$ for the pulsars in their sample and using equations (1) and (2), LM88 found that the dependence of ρ on the pulse period P can be described reasonably well by the expression

$$\rho = 6\overset{\circ}{.}5\, P^{-1/3}, \tag{3}$$

where P is the pulse period measured in seconds. They estimated α by inserting the measured values P, M, and $\Delta\phi$ into equations (1), (2), and (3).

Using the method of LM88 to estimate α requires that complete conal emission be observed. Their method tacitly assumes that ρ is determined solely by the rotation period, which may

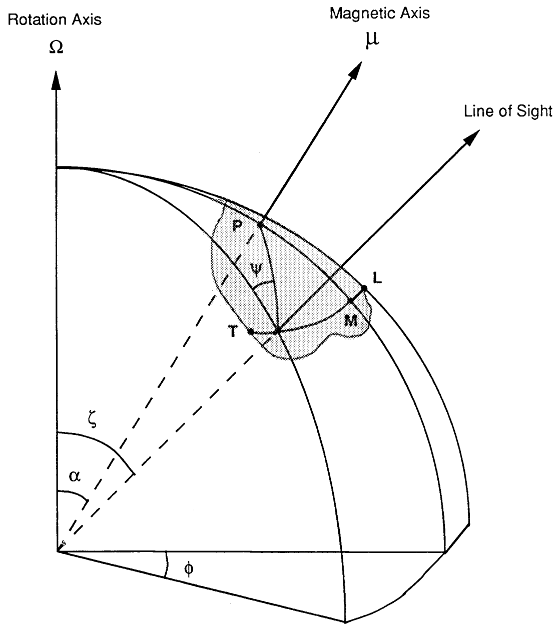

FIG. 1.—Diagram of pulsar geometry showing the angle α between the rotation and magnetic axes, and the angle ζ between the rotation axis and the line of sight; $\beta = |\zeta - \alpha|$ is the minimum angle between the magnetic axis and line of sight. The azimuthal and polarization angles are ϕ and ψ, respectively. The shaded area, which we have drawn of arbitrary shape, represents the emission region. the line of sight is assumed to move from the leading edge **L** to the trailing edge **T**, crossing the meridian at the point **M**.

not be correct. Furthermore, the assumption of a circular emission beam has been called into question (see, e.g., Biggs 1990). R90 has expressed concern that the estimates of β may be disturbed by core components, which could affect the polarization and hence M.

2.2. *Method of Rankin*

Unlike LM88, R90 groups pulsars by what she believes to be their intrinsic emission attributes, and not by the pattern that happens to intersect our line of sight. Rankin classifies pulsars according to their core emission as core single stars (class S_t), triple stars (class T), or multiple stars (class M). When we refer to, for example, "S_t" pulsars, we will mean those pulsars placed by R90 into the core-single class. It is important to make a clear distinction between the classifications of R90 and LM88, because their respective categories do not have meaning to the two studies jointly.

R90 reported a relation between P and the angular width of the core ρ_c, based on interpulsars whose core widths could be measured with reasonable accuracy and which she believed have $\alpha = 90°$. From six such pulsars, she found the relation $\rho_c = 2.^\circ45\ P^{-1/2}$ where as before P is in seconds. To estimate α for other pulsars, she used equation (2) with $\beta \ll \rho_c$ and $\beta \ll \alpha$, that is,

$$2\Delta\phi \approx 2.^\circ45\ P^{-1/2}/\sin\alpha\ . \quad (4)$$

Measurement of the core duty cycle and P then yields α. R90 used the small-β approximation, which eliminates the dependence on β, because (as mentioned above) she believes that estimates of β derived from polarization measurements may be corrupted by core emission. R90 also emphasized that if, as she considered likely, the angular intensity distribution of the core emission is a bivariate Gaussian in latitude and longitude, then the FWHM of the core emission is strictly independent of β. However, the low-β approximation has the disadvantage that if the emission is not of a special form (such as a bivariate Gaussian), and if β is not small compared to ρ_c or α, then the value of α, then the value of α derived from equation (4) is incorrect. Specifically, if the line of sight cuts across the pulsar beam, at a nonnegligible β, the estimated duty cycle will be too low and the estimate of α too high. The method of R90, like that of LM88 also makes the assumption that the intrinsic beam width depends only on the rotation period. If other factors are important, the values of α derived using this assumption are unreliable.

2.3. *Discussion of Results*

The inclination angles found by LM88 and R90 are compared in Figure 2. All 56 pulsars plotted are listed by LM88 as complete conal emitters. The two estimates of α agree reasonably well for small values of α but become increasingly discordant as either estimate of α increases. The agreement is significantly better for core-dominated pulsars (filled shapes) than for cone-dominated pulsars (open shapes).

The two sets of α estimates show significant systematic differences. These are summarized in Table 1. The average absolute value of the difference between the inclinations reported by R90 and LM88 is 14.°6. The average difference for pulsars classifed by LM88 as core-dominated is 7.°9, whereas for cone-dominated pulsars it is 21.°3. The average difference for pulsars classified by R90 as core-single (S_t) is 20.°1, whereas for core-triples (T) the difference is 14.°1 and for core-multiples (M), 4.°8. As discussed in § 2.2, the assumption by R90 that the impact

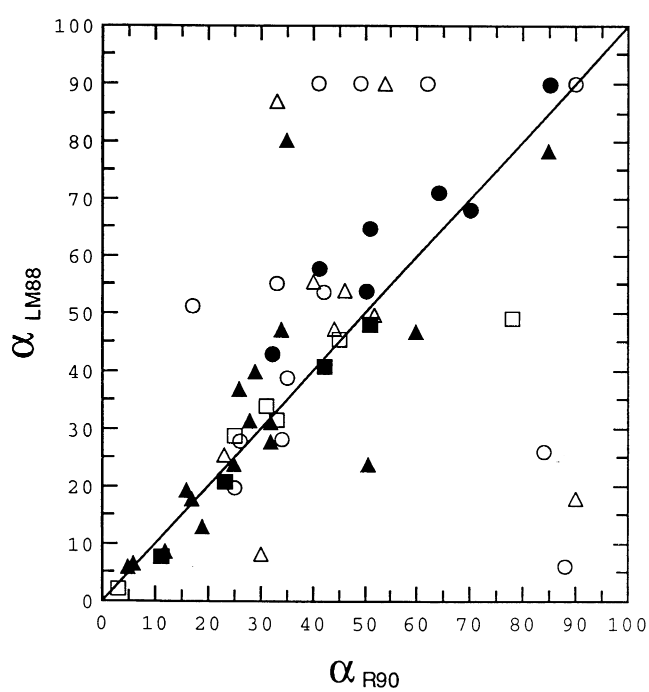

FIG. 2.—Inclination angle estimates of Rankin (1990) α_{R90} vs. the estimates of Lyne & Manchester (1988) α_{LM88} for pulsars with complete conal emission. Core-dominated pulsars are represented by filled shapes and cone-dominated by open shapes. Pulsars of the Rankin S_t class are plotted as circles (either filled or open), T class pulsars are plotted as triangles, and M class pulsars are plotted as squares. Solid line is the line $\alpha_{LM88} = \alpha_{R90}$.

TABLE 1

RESULTS OF RANK-ORDER CORRELATION TEST

Sample	Number	$\lvert\alpha_{R90} - \alpha_{LM88}\rvert_{ave}$	$(\alpha_{R90} - \alpha_{LM88})_{ave}$	$\mathscr{P}(r_S)$	r_S
All	56	14°6	−1°9	5.46×10^{-7}	0.612
Core-dominated	28	7.9	−2.5	2.53×10^{-10}	0.889
Cone-dominated	28	21.3	−1.4	1.05×10^{-1}	0.313
S_t class	20	20.1	−4.8	1.20×10^{-1}	0.359
T class	26	14.1	−1.8	1.07×10^{-4}	0.687
M class	10	4.8	3.4	9.31×10^{-8}	0.988
α_{LM88} or $\alpha_{R90} > 75°$	13	38.9	−0.8	1.32×10^{-2}	−0.665
α_{LM88} or $\alpha_{R90} > 40°$	32	22.4	−3.8	9.27×10^{-1}	−0.017

NOTES.—r_S is the Spearman rank-order correlation coefficient. $\mathscr{P}(r_S)$ is the probability that uncorrelated data would result in a value of r_S greater than that determined from the data.

parameter β is negligible should, if β is not always small, lead to overestimates of α. However, the estimates of R90 are, on average, smaller than the estimates of LM88 (the average of the signed difference $\alpha_{R90} - \alpha_{LM88}$ is $-1^\circ\!.9$).

In order to quantify the comparison of α_{LM88} with α_{R90}, we performed the Spearman rank-order correlation test (see, e.g., Press et al. 1986) for the full data set and for various subsets of the data. The results are listed in Table 1. The Spearman rank-order correlation coefficient r_S provides a measure of the correlation and ranges from $r_S = 1$ (perfect correlation) to $r_S = -1$ (perfect anticorrelation). More useful information is obtained by looking at $\mathscr{P}(r_S)$, the probability that the Spearman rank-order correlation coefficient for two uncorrelated data sets would be greater (in magnitude) than the value of r_S found.

The statistical tests confirm the trends evident in Figure 2. There is an overall positive correlation between α_{LM88} and α_{R90}, and it is unlikely ($\mathscr{P} < 10^{-6}$) that their values are unrelated. There is good agreement between α_{R90} and α_{LM88} for pulsars classified as core-dominated by LM88 ($\mathscr{P} < 10^{-9}$), whereas for cone-dominated pulsars there is only a weak correlation ($\mathscr{P} \approx 10^{-1}$). The correlation increases with the complexity of the core emission ($\mathscr{P} \approx 10^{-1}$, 10^{-4}, and 10^{-9} for pulsars in the R90 classes S_t, T, and M, respectively). For whatever reason, the two methods produce much better agreement for pulsars with core-dominated beam emission and more complicated pulse structure.

Figure 2 shows that for those pulsars determined to have large values of α in either study, the two estimates are very unlikely to agree. In fact, for $\alpha > 40°$ there is zero correlation, while for $\alpha \gtrsim 75°$ there is actually a small *negative* correlation between α_{LM88} and α_{R90}. This discrepancy is particularly serious for studies of alignment, because the statistics of pulsar alignment are influenced strongly by pulsars with large values of α. We therefore caution against using conclusions from the LM88 or R90 α-distributions in such studies.

3. SINGLE-VECTOR MODEL DETERMINATIONS OF α

Soon after the discovery of pulsars, Radhakrishnan & Cooke (1969) proposed an emission model in which the polarization is determined by the direction of the magnetic field (assumed to be axisymmetric) at the point of emission. While this phenomenological model has been very successful in accounting for the pulse profiles of many pulsars, it may fail for pulsars with significant core components, and may actually be misleading even if the polarization curve appears smooth (see, e.g., R90). Nonetheless, fits to the Radhakrishnan & Cooke model can yield alternate estimates of α, provided that accurate polarization measurements are available (see, e.g., Narayan & Vivekenand 1982; LM88; Phillips 1990). Moreover, these fits provide independent estimates of α, since the assumption of single-vector polarization is different from the assumptions of R90 and of LM88.

From the spherical triangles in Figure 1, one may derive the relation

$$\tan \psi = \frac{\sin \alpha \sin \phi}{\sin \zeta \cos \alpha - \cos \zeta \sin \alpha \cos \phi}, \tag{5}$$

where ψ and ϕ are the differences in values of the polarization angle and longitudinal angle, respectively, from those at the meridian (defined by the plane passing through the rotation and magnetic axes). From the polarization curve $\psi(\phi)$, the angles α and ζ can be estimated by a least-squares fit to equation (5). However, these estimates are very sensitive to measurement errors. The sensitivity can be seen by considering $\cos \zeta$, which using equations (2) and (5) can be written

$$\cos \zeta = \left(\frac{\sin \phi}{\tan \psi} - \frac{1}{M}\right)(1 - \cos \phi)^{-1}. \tag{6}$$

Fractional errors of ϵ_ϕ, ϵ_ψ, and ϵ_M in the measured values of ϕ, ψ, and M result in a relative error in $\cos \zeta$ of

$$\begin{aligned}\frac{\Delta(\cos \zeta)}{\cos \zeta} = {} & \epsilon_\phi\left[\frac{\phi \cos \phi}{\tan \psi \cos \zeta(1 - \cos \phi)} - \frac{\phi \sin \phi}{1 - \cos \phi}\right] \\ & - \epsilon_\psi\left[\frac{\psi \sin \phi}{\sin^2\psi \cos \zeta(1 - \cos \phi)}\right] \\ & + \epsilon_M\left[\frac{\sin \phi}{\tan \psi \cos \zeta(1 - \cos \phi)} - 1\right].\end{aligned} \tag{7}$$

The presence of $(1 - \cos \phi)$ in the denominators of the coefficients amplifies any errors. For example, if $\phi = 20°$, $\alpha = 60°$, and $\beta = 5°$ errors of 1% in ψ, ϕ, and M cause errors of, respectively, 37%, 15%, and 16% in $\cos \zeta$. The magnitudes of the errors are roughly proportional to ϕ^{-1}. Thus, if ϕ is instead 40°, the errors in $\cos \zeta$ would be reduced to 19%, 3%, and 4%. Systematic errors in ϕ and ψ tend to cancel each other out, since their contributions to $\Delta(\cos \zeta)$ are of opposite sign.

Equation (5) has three independent variables; α, ζ and ϕ. However, since $\zeta = \alpha + \beta$ and $M = \sin \alpha / \sin \beta$, if M can be determined independently of α and ζ, then only two variables need be considered. Near the meridian, where $\tan \psi \approx \psi$,

$\sin \phi \approx \phi$, and $\cos \phi \approx 1 - (\phi^2/2)$, equation (5) reduces to

$$\psi \approx \phi M\left(1 - M \frac{\phi^2}{2} \cos \zeta\right), \quad \text{and} \tag{8}$$

$$\frac{d\psi}{d\phi} \approx M - \frac{3}{2} M^2 \phi^2 \cos \zeta . \tag{9}$$

Since M is typically less than 30, the polarization is almost exactly a linear function of the longitude for $\phi < 1°$. Therefore, independent estimation of M should be possible. As an alternative to simple least-squares fitting, one could use the observed polarization curve $\psi(\phi)$ to estimate M, equation (6) to estimate ζ, and then equation (1) to estimate α.

Since few pulsars have measurements of polarization angle over more than 20° in longitude, and since the uncertainty in the measured polarization angles is $\gtrsim 5°$ for some pulsars, the error magnification inherent in this method renders it unsuitable at present for many pulsars. However, where accurate polarization measurements can be made, fits to the single-vector model should yield reasonable estimates of α. The single-vector model works best for pulsars without the core components that may influence polarization measurements. Therefore, fits to this model are complementary to the R90 and LM88 methods, which are most consistent for pulsars with core components.

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