GRAVITATIONAL RADIATION FROM INTERMEDIATE-MASS BLACK HOLES

M. COLEMAN MILLER

Department of Astronomy, University of Maryland, College Park, MD 20742-2421; miller@astro.umd.edu

ABSTRACT

Recent X-ray observations of galaxies with *ROSAT*, *ASCA*, and *Chandra* have revealed numerous bright off-center point sources that, if isotropic emitters, are likely to be intermediate-mass black holes, with $M \sim 10^2$–$10^4 \ M_\odot$. The origin of these objects is under debate, but observations suggest that a significant number of them currently reside in young high-density stellar clusters. There is also growing evidence that some Galactic globular clusters harbor black holes of similar mass, from observations of stellar kinematics. In such high-density stellar environments, the interactions of intermediate-mass black holes are promising sources of gravitational waves for ground-based and space-based detectors. Here we explore the signal strengths of binaries containing intermediate-mass black holes or stellar-mass black holes in dense stellar clusters. We estimate that a few to tens per year of these objects will be detectable during the last phase of their inspiral with the advanced Laser Interferometer Gravitational-Wave Observatory detector, and up to tens per year will be seen during merger, depending on the spins of the black holes. We also find that if these objects reside in globular clusters, then tens of sources will be detectable with the *Laser Interferometer Space Antenna* from the Galactic globular system in a 5 yr integration, and similar numbers will be detectable from more distant galaxies. The signal strength depends on the eccentricity distribution, but we show that there is promise for strong detection of pericenter precession and Lense-Thirring precession of the orbital plane. We conclude by discussing what could be learned about binaries, dense stellar systems, and strong gravity if such signals are detected.

Subject headings: black hole physics — gravitational waves — stellar dynamics

1. INTRODUCTION

We are entering an era in which numerous experiments will search for astrophysical gravitational radiation. Ground-based detectors such as the Laser Interferometer Gravitational-Wave Observatory (LIGO; Barish 2000), VIRGO (Fidecaro et al. 1997), GEO 600 (Schilling 1998), TAMA (Ando et al. 2002), and others focus on relatively high frequencies $f_{\rm GW} \approx 10$–1000 Hz. These frequencies are appropriate for the final inspiral and merger of binaries with total masses of ~ 1–1000 $M_\odot$. Space-based missions such as the *Laser Interferometer Space Antenna* (*LISA*; see, e.g., Danzmann 2000) will complement this frequency range by focusing on comparatively low frequencies, $f_{\rm GW} = 10^{-4}$–1 Hz. In this range, there are a number of known sources in the form of main-sequence and white dwarf binaries, but there is much more uncertainty about the prevalence and properties of double compact binaries containing neutron stars and black holes. Such binaries are of great interest for many reasons, such as their use as probes of strong gravity and their potential to illuminate other aspects of astronomy, including the evolution of galaxies.

The existence of a new class of double compact binaries is suggested by two recent lines of evidence. First, observations of the kinematics of the central regions of some globular clusters suggest that black holes with $M \gtrsim 100 \ M_\odot$ may exist in their cores (e.g., Gebhardt et al. 2000; D'Amico et al. 2002; Colpi, Possenti, & Gualandris 2002). Second, X-ray observations have revealed variable, unusually high flux point sources in a number of galaxies (e.g., Fabbiano 1989; Fabbiano, Schweizer, & Mackie 1997; Colbert & Mushotzky 1999; Zezas, Georgantopoulos, & Ward 1999; Kaaret et al. 2001; Matsumoto et al. 2001; Fabbiano, Zezas, & Murray 2001; see Colbert & Ptak 2002 for a catalog of

objects). The rapid and strong variability of these sources indicates that they are black holes. If their flux is not strongly beamed, then for their luminosities to be below the Eddington luminosity $L_{\rm E} = 1.3 \times 10^{38}(M/M_\odot)$ ergs s^{-1}, at which radiation forces balance gravity, the masses of the brightest sources must be at least $\sim 10^3 \ M_\odot$ (e.g., Matsumoto et al. 2001). In addition, their off-center positions in their host galaxies indicate that in many cases, the masses cannot be substantially greater than $\sim 10^5 \ M_\odot$; otherwise, dynamical friction would cause the black holes to sink rapidly to the dynamical center (Kaaret et al. 2001; Tremaine, Ostriker, & Spitzer 1975). Recent optical observations suggest that for at least some of the sources, the X-rays may not be beamed strongly, based on recombination emission from surrounding nebulae (Pakull & Mirioni 2002). This implies that the total luminosity is indeed high, so that if the sources are sub-Eddington (however, see Begelman 2002), there may therefore be a significant number of $\sim 10^2$–$10^4 \ M_\odot$ black holes in the universe. Currently, these sources are preferentially in star-forming regions or in starburst galaxies (Matsumoto et al. 2001; Liu & Bregman 2001).

A number of these objects are associated with dense young stellar clusters (e.g., Matsumoto et al. 2001). Combined with the kinematic evidence in globular clusters, this implies that black holes of this sort undergo frequent dynamical encounters. It has been suggested that the ultra-luminous X-ray sources originated from dynamical encounters in globular clusters, then merged with their host galaxies (Miller & Hamilton 2002a, 2002b). Other proposals, if these objects are 10^2–$10^4 \ M_\odot$ black holes instead of being strongly beamed (King et al. 2001; King 2002; Körding, Falcke, & Markoff 2002) or being super-Eddington by a large factor (Begelman 2002), are that they form as a result of a core collapse of a massive young star

cluster (Matsushita et al. 2000; Ebisuzaki et al. 2001) or as remnants of Population III stars (Madau & Rees 2001).

Here we consider the implications for gravitational-wave emission if these sources are indeed intermediate-mass black holes. Independent of their detailed origin, black holes in dense stellar environments are promising sources of gravitational radiation due to the diversity of dynamical interactions that are possible (see also Benacquista 1999, 2002a, 2002b; Benacquista, Portegies Zwart, & Rasio 2001). We focus mainly on black holes in globular clusters, but also address signals that may be evident from young stellar clusters, regardless of whether the black hole formed there or elsewhere. In § 2 we give general arguments about the properties of these sources, based on the ages of the globular clusters and the number and types of secondaries. In § 3 we estimate the rates of encounters, and in § 4 we present calculations of the signal-to-noise ratios (S/Ns) expected for *LISA* from Galactic globular clusters and from globular clusters in the Virgo Cluster, and for coalescence of black holes as observed with the LIGO II detector. In § 5 we discuss what information could be gleaned from a reasonably high S/N detection. We show that both pericenter precession and Lense-Thirring precession are potentially measurable. These effects may allow preliminary mapping of the spacetime around black holes and could even yield an independent measure of the distance to the Virgo Cluster. We present our conclusions and discuss the uncertainties in our estimates in § 6.

2. EXPECTED PROPERTIES OF SOURCES

Here we discuss the expected nature of intermediate-mass black holes if grown in dense clusters. We begin by summarizing the dynamics of dense stellar clusters and then discuss the ways in which intermediate-mass black holes can form and grow. We then derive the approximate distributions of spin parameter and eccentricity that we expect for these sources, with implications for detectability.

2.1. *Cluster Dynamics*

The young super–star clusters now being discovered in many galaxies with active star formation (e.g., Origlia et al. 2001) have typical estimated masses of $\sim 10^6$ $M_\odot$ and half-mass radii of 10 pc (Origlia et al. 2001). The central number density is difficult to estimate, but if we take as a guide the densest young clusters in the vicinity of the Milky Way, the central density could be up to $\sim 10^5$–10^6 pc^{-3} (see, e.g., Massey & Hunter 1998 for observations of the dense R136 region). Clusters with ages of less than a few tens of millions of years have a large number of O and B stars; in clusters older than this, those stars have evolved to compact remnants.

Globular clusters are much older: $\sim 10^{10}$ yr. All stars with initial masses $M_{\rm init} \gtrsim 0.8$ $M_\odot$ have evolved off the main sequence, meaning that the most massive objects present are compact remnants such as black holes, neutron stars, and massive white dwarfs. There are also a small number of ~ 1.5 $M_\odot$ blue stragglers, which are main-sequence stars more massive than the ordinary main-sequence turnoff that may have been rejuvenated by collisions (Lombardi et al. 2002; see, e.g., Hurley & Shara 2002 for a recent discussion of these and other interactions in clusters). A typical globular has $\sim 10^5$–10^6 stars of average mass ~ 0.4 $M_\odot$ and a central number density in the range $n_c = 10^2$–10^6 pc^{-3} (Pryor &

Meylan 1993). Roughly 40% of the globular clusters surrounding the Milky Way have $n_c \gtrsim 10^5$ pc^{-3} (Pryor & Meylan 1993), with a density contrast between the core and the half-mass radius that formally poise them on the edge of core collapse. It is thought that three-body interactions of primordial binaries with field stars heat the cluster, delaying collapse much longer than otherwise possible (e.g., Goodman & Hut 1989; Hut, McMillan, & Romani 1992; Sigurdsson & Phinney 1995).

A productive analogy with thermodynamics, backed up by simulations, has shown that in clusters with stars of different masses, the cluster tries to evolve toward thermodynamic equilibrium, in the sense that the kinetic energies of each of the component stars are drawn from the same distribution (see, e.g., Binney & Tremaine 1987). This means that the typical speed of an object of mass M is proportional to $M^{-1/2}$. This causes more massive objects to sink to the core of the cluster, a process called mass segregation (e.g., Farouki & Salpeter 1982). The typical timescale for such sinking is given by $t \approx t_r(\langle m \rangle / M)$, where for a cluster of N stars and dynamical crossing time t_d, the core relaxation timescale is $t_r \approx t_d N/(8 \ln N)$ (see, e.g., Binney & Tremaine 1987). Typical globular clusters have $t_r \approx 10^7$–10^9 yr (Pryor & Meylan 1993). This means that even small black holes with $M = 10$ $M_\odot$ will sink quickly to the center of a cluster, and if an intermediate-mass black hole with $M = 10^2$–10^4 $M_\odot$ is present, it will rarely be too far from the center, even in a young cluster. However, the short relaxation time in the core and the wandering of black holes of this mass due to dynamical interactions mean that there is rapid refilling of stellar orbits that interact with the black hole. There is therefore no " loss cone " problem, unlike the situation with supermassive black holes in galactic nuclei (Frank & Rees 1976; see Milosavljević & Merritt 2001 for a recent discussion).

As a result of mass segregation, even though the absolute number of compact objects is much less than that of main-sequence stars, their number density in the core can be comparable to or in excess of that of main-sequence stars for core densities $\gtrsim 10^5$ pc^{-3} (e.g., Sigurdsson & Phinney 1995). In addition, since binaries typically have more mass than single stars, they also tend to sink to the center. This means that three-body interactions of binaries with single stars, and possibly binary-binary interactions, can be dynamically important.

Qualitatively, three-body interactions can be understood using Heggie's rule (Heggie 1975) that hard binaries harden and soft binaries soften. That is, if the initial binary has a binding energy greater than the typical kinetic energy of a field star (i.e., the binary is hard), then a typical encounter with a field star causes the binary to harden further. A soft binary is instead softened gradually until it becomes unbound. Exchanges can also occur in which the interloper object becomes a member of the final binary. Numerous simulations show that in a strong encounter, the most likely occurrence is that the final binary is composed of the two most massive of the three stars that originally interacted (e.g., Sigurdsson & Phinney 1993; Heggie, Hut, & McMillan 1996).

There has long been interest in whether hardening from three-body encounters might bring a pair of black holes close enough together that they would merge via gravitational radiation while still in the cluster. However, each hardening delivers a recoil kick to both the binary and the

single star, and if the recoil of the binary exceeds the ~ 50 km s^{-1} escape speed from the core of a typical dense cluster (Webbink 1985), then the binary is ejected before it can merge. Several simulations (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000) have shown that if attention is restricted to Newtonian three-body encounters of 10 $M_\odot$ black holes, then few if any mergers will occur inside the cluster. There may, however, be a number of mergers that happen outside the cluster after recoil (Portegies Zwart & McMillan 2000).

Recently, Miller & Hamilton (2002a) showed that a merger may happen inside the cluster if a black hole starts out with a greater mass, $M \sim 30$–$50 \ M_\odot$. In this case, recoil is much less significant, and gravitational radiation predominates. They proposed this as a mechanism for the formation and growth of intermediate-mass black holes. Even if only lighter black holes form initially, secular resonances in hierarchical triple systems formed in binary-binary encounters (the Kozai resonance; Kozai 1962; Lidov & Ziglin 1976) may increase the eccentricity of the inner binary enough that it can merge by gravitational radiation without being ejected by three-body recoil (Miller & Hamilton 2002b). It is also possible that in some three-body encounters, two of the black holes will pass close enough to each other to coalesce rapidly. This increases optimism that dense clusters may be the seat of relatively frequent mergers of compact objects. There are, however, a number of questions still remaining. One of these has to do with recoil due to asymmetric emission of gravitational radiation during inspiral. Calculations have yet to be done for strong gravity, but various analyses using Newtonian and post-Newtonian formalisms suggest that the recoil velocity could be from a few up to $\sim 10^3$ km s^{-1}, depending on the mass ratio and how close the black holes get before the final plunge (Peres 1962; Bekenstein 1973; Fitchett 1983; Fitchett & Detweiler 1984; Redmount & Rees 1989; Wiseman 1992). For slowly rotating black holes with mass ratios $M/m \gtrsim 10$, the prime focus of this paper, the best estimates are that the recoil speed will be well below the 50 km s^{-1} escape speed from the core (Fitchett 1983; Fitchett & Detweiler 1984; Wiseman 1992). We therefore assume that if an intermediate-mass black hole forms in a dense cluster, it will remain in the core through multiple mergers.

2.2. *Spin Parameter*

Black holes that have grown by the accumulation of smaller objects in a dense stellar cluster are likely to have a different rotation parameter from either stellar-mass or supermassive black holes. Stellar-mass black holes acquire some spin at birth. Subsequent accretion adds relatively little mass or angular momentum to the black hole, either because not much mass is available (as in low-mass X-ray binaries, in which the companion is usually less than 10% as massive as the black hole; e.g., Verbunt 1993) or because the duration of the accretion phase is short (as in high-mass X-ray binaries, with durations $T < 10^7$ yr, implying a total mass transfer of less than 0.1 $M_\odot$; Verbunt 1993). As a result, the spin parameter $j \equiv cJ/GM^2$, where c is the speed of light, G is Newton's gravitational constant, and J is the angular momentum, of stellar-mass black holes is expected to be close to its birth value. Compelling evidence of significant spin in stellar-mass black holes has emerged from the study of quasi-periodic brightness oscillations (Strohmayer

2001a, 2001b; Miller et al. 2001), and spectral studies are also suggestive of high spin (from line profiles [Miller et al. 2002] and earlier more model dependent fits to continuum emission [Zhang, Cui, & Chen 1997]). For supermassive black holes, the spin parameter depends on what fraction of its mass was provided by an accretion disk with a fixed orientation. If the fraction is close to unity, then the spin parameter could also be close to unity. Some evidence of such rapidly spinning supermassive black holes is emerging in the form of extremely broad Fe Kα lines from a few active galactic nuclei (e.g., Iwasawa et al. 1996; Dabrowski et al. 1997; Wilms et al. 2001).

In contrast, black holes grown by the capture of stars or compact objects in dense stellar clusters undergo a damped random walk in the evolution of their spins after the memory of the initial spin is lost. The damping is because retrograde orbits become unstable at a larger specific angular momentum than do prograde orbits, so it is easier to decrease than to increase the spin parameter of the black hole. Assuming a roughly isotropic distribution of stellar velocities in the cluster core, the final inspiral and deposition of angular momentum happens at random angles to the previous spin axis of the black hole. There is evidence of net rotation in many globular clusters (e.g., Barmby, Holland, & Huchra 2002), so encounters may not be precisely isotropic, but this is likely to be a small effect because relaxation will drive the core distribution toward isotropy (e.g., Einsel & Spurzem 1999; Kim et al. 2002 for Fokker-Planck treatments of the evolution of rotating systems).

If the angular momentum of a large black hole is much greater than the orbital angular momentum of a small black hole that falls into it, then the total angular momentum is roughly constant, so that $j \sim M^{-2}$ (Blandford & Hughes 2002). In contrast, if the angular momentum of the larger black hole is sufficiently small, then the orbital angular momentum of the smaller black hole can make a significant difference. Following Blandford & Hughes (2002), let us define the orbital inclination ι using

$$\cos \iota = \frac{L_z}{\sqrt{L_z^2 + Q}} \ , \tag{1}$$

where L_z is the angular momentum parallel to the spin axis of the more massive black hole and Q is the Carter constant. If the initial spin angular momentum of the massive black hole is J, then merger with a black hole of mass $m \ll M$ with an orbital inclination ι and specific orbital angular momentum $u_\phi(\iota)$ will produce a black hole of mass $M + m$ and angular momentum

$$J' = \sqrt{\left[J + mu_\phi(\mu)\cos\iota\right]^2 + (1 - \cos^2\iota)m^2 u_\phi^2(\iota)}$$
$$= \sqrt{J^2 + 2mJu_\phi(\iota)\cos\iota + m^2 u_\phi^2(\iota)} \ . \tag{2}$$

This assumes that the amount of mass-energy and angular momentum radiated away during the merger is small. Blandford & Hughes (2002) show that to good accuracy, the angular momentum of a test particle in the last stable circular orbit at angle ι to the prograde equatorial direction is approximately

$$|u_{\phi,\mathrm{LSO}}(\iota)| \approx |L_{\mathrm{ret}}| + \frac{1}{2}(\cos\iota + 1)\left(L_{\mathrm{pro}} - |L_{\mathrm{ret}}|\right) , \tag{3}$$

where L_{pro} and L_{ret} are, respectively, the specific angular

momentum of a prograde equatorial and retrograde equatorial particle at the last stable orbit. When $j \ll 1$, this reduces to

$$|u_{\phi,\mathrm{LSO}}| \approx \sqrt{12} M \left(\frac{G}{c}\right)\left[1 - \frac{1}{2}\left(\frac{2}{3}\right)^{2/3} j \cos \iota\right] . \quad (4)$$

Assume that the probability distribution of inclinations is $Q(\cos \iota)$ (e.g., $Q = \frac{1}{2}$ for an isotropic distribution). If the probability distribution for j given M, $P(j|M)$, is stationary in form, then we have

$$P(j|M + m) = \int_{-1}^{1} P[j + \delta(\cos \iota)|M]Q(\cos \iota)\, d(\cos \iota) , \quad (5)$$

where $\delta(\cos \iota)$ is the change in dimensionless angular momentum caused by the accretion of an object of mass m at an orbital inclination ι; that is, such an object changes the dimensionless angular momentum from $j + \delta(\cos \iota)$ to j.

Approximate solutions to this equation can be sought in the form $P(j|M) \approx \exp[-(j - \beta)^2/2\sigma^2]$. For isotropic encounters, a fit to the numerical results displayed in Figure 1 gives $\beta \approx (2m/M)^{1/2}$ and $\sigma \approx (m/2M)^{1/2}$. Figure 1 shows the distribution of spin parameters for a number of simulated black holes that started at $M_0 = 5m$ and accreted objects of mass m (e.g., 50 and 10 $M_\odot$). For intermediate-mass black holes, the expected dimensionless angular momentum is on the order of a few tenths, approximately independent of the initial spin if $M \geq 5m$.

2.3. Eccentricity

The expected eccentricity for a black hole binary depends on the way in which it was formed and on the orbital frequency at which it is observed. We discuss each of the evolu-

FIG. 1.—Simulated distribution of spin parameters $j = a/M = cJ/GM^2$ for a black hole of initial mass $5m$ that accretes objects of mass m up to a total of $10m$ (solid line: initial spin $j = 0$; dotted line: initial spin $j = 0.9$), $30m$ (short-dashed line: initial spin $j = 0$), or $100m$ (long-dashed line: initial spin $j = 0$). Orbital inclinations ι of encounters are uniformly distributed in $\cos \iota$.

tionary paths in turn, but in brief we find that at the high frequencies observable with ground-based detectors, the orbits will have circularized to high accuracy, whereas at the lower frequencies of space-based detectors, the eccentricity is expected to be significant for the majority of the detectable period.

First, consider formation via three-body encounters. As long as the time to merger is longer than the time to the next encounter, the eccentricity will be reset by each interaction. During this period, there will be relatively little gravitational radiation emitted, as the merger time is typically long. At some point, however, an encounter will leave the binary with a relatively short merger time, and thus the binary will go into an uninterrupted two-body inspiral. The precise distribution of eccentricities at the start of this phase depends on the details of the three-body interactions, but preliminary results (K. Gultekin & M. C. Miller 2002, in preparation) suggest that immediately after the last three-body encounter, the eccentricity tends to be fairly high: $e > 0.9$. This is because the time to merger decreases rapidly with increasing eccentricity, so a chance fluctuation up to high eccentricity allows a binary to merge before the next encounter. For moderate to high eccentricities, the gravitational radiation merger time is

$$\tau_{\mathrm{merge}} \approx 3 \times 10^{17} \left(\frac{M_\odot^3}{\mu M^2}\right)\left(\frac{a}{1\ \mathrm{AU}}\right)^4 \left(1 - e^2\right)^{7/2}\ \mathrm{yr} \quad (6)$$

(Peters 1964), where M is the total mass and μ is the reduced mass of the binary. For $e \approx 1$, this is $\tau_{\mathrm{merge}} \approx 3 \times 10^{18}(a/1\ \mathrm{AU})^{1/2}(r_p/1\ \mathrm{AU})^{7/2}$ yr, where $r_p = a(1 - e)$ is the pericenter distance. For a typical merger time of a few hundred thousand to a few million years after the last encounter, the pericenter distance is $r_p \sim$ a few $\times\ 10^{10}$ cm, which is hundreds to thousands of gravitational radii for an intermediate-mass black hole. Although the full period of an orbit could be relatively high (10^4–10^5 s), the majority of the power is emitted at a frequency of the order of the angular frequency at pericenter (see § 4.1), which could be several times higher than the fundamental orbital frequency. The peak frequency of gravitational waves is then typically on the order of 10^{-4} to 10^{-3} Hz for the majority of the inspiral. This is what would most likely be seen from sources in our Galactic globular system, and hence the eccentricity would be expected to be high, perhaps typically $e > 0.9$.

In later stages of inspiral, however, the circularizing influence of gravitational radiation reduces the eccentricity substantially. A simple way to estimate the eccentricity uses the constant

$$ae^{-12/19}\left(1 - e^2\right)\left(1 + \frac{121e^2}{304}\right)^{-870/2299} \quad (7)$$

found by Peters (1964) for the lowest order quadrupole radiation (see Glampedakis & Kennefick 2002; Glampedakis, Hughes, & Kennefick 2002 for a discussion of how constant this expression is when higher order terms are included). Disregarding factors of the order of unity, this means that $r_p e^{-12/19}$ is roughly constant. Thus, r_p remains approximately fixed as long as e is close to unity, after which point it drops significantly. In the first stage, where $e \approx 1 \Rightarrow a \gg r_p$, $a \propto \tau_{\mathrm{merge}}^2$, so $1 - e = r_p/a \propto \tau_{\mathrm{merge}}^{-2}$. Note that in this stage, the emission is strongly dominated by the time near pericenter. Thus, gravitational waves are emitted for a few cycles
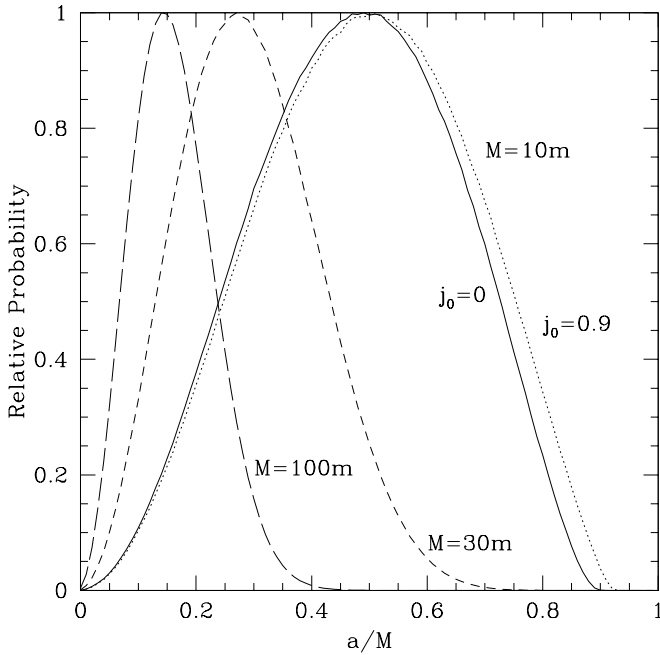
near closest approach, and then little emission occurs until the next pericenter passage. As a result, the peak frequency of the gravitational-wave emission is approximately the orbital frequency at pericenter, which is nearly constant during this phase of inspiral. In the second stage, where $a \approx r_p$ and thus $r_p \propto \tau_{\mathrm{merge}}^{1/4}$, we have $e \propto \tau_{\mathrm{merge}}^{19/48} \approx \tau_{\mathrm{merge}}^{0.4}$. Here the frequency of gravitational waves is roughly twice the mean orbital frequency, or $f_{\mathrm{GW}} \propto r_p^{-3/2}$, so $e \propto f_{\mathrm{GW}}^{-19/18} \approx f_{\mathrm{GW}}^{-1}$.

In practice, the transition between relatively rapid change in eccentricity and relatively slow change happens at surprisingly high eccentricity. For example, suppose the binary initially has a merger time of 10^6 yr and an eccentricity of 0.9. At 10^5 yr from merger, the eccentricity would be $e = 0.55$, at 10^4 yr from merger $e = 0.25$, and at 10^3 yr $e = 0.1$. If the initial eccentricity is instead 0.99, then at 10^5 yr $e = 0.83$, at 10^4 yr $e = 0.46$, and at 10^3 yr $e = 0.2$. The net result is that almost all the sources of this type in the *LISA* band are likely to have appreciable eccentricity. In contrast, high-frequency sources detectable by ground-based instruments will have much lower eccentricities, typically $e \sim 0.01$. This means that templates based on circular orbits will be adequate for many purposes (see Martel & Poisson 1999 for a discussion of the amount of S/N lost by nonoptimal signal processing).

A second formation scenario involves Kozai resonances in hierarchical triple systems formed by binary-binary encounters (Miller & Hamilton 2002b). Here the eccentricities could be much higher and the pericenter distances much closer than in the three-body scenario. However, the eccentricities will certainly not be higher than what would produce a merger within a single orbital period of the inner binary, because the eccentricity is increased gradually, and such a short merger time would terminate the increase. For a merger time of ~ 1 yr and two 10 $M_\odot$ black holes, this implies a pericenter distance of $\approx 10^8$ cm. Even in this very extreme case, by the time the binary reaches the $\sim 10^7$ cm separation necessary to bring the frequency into the LIGO II detector band, the eccentricity would be $e < 0.03$. In more realistic circumstances, the pericenter distance would be more than 10^9 cm, and the binary would again be nearly circular when it entered the frequency range of ground-based detectors.

The third formation scenario involves direct two-body capture by emission of gravitational radiation. If this happens with a black hole of mass M and one of mass $m \ll M$, then the distance of closest approach needed to cause a binary of initial relative velocity v to become bound is (see Quinlan & Shapiro 1987)

$$r_p = \left(\frac{85\sqrt{2}\pi}{12} \right) p^{2/7} G M^{2/7} m^{2/7} (M + m)^{3/7} c^{-10/7} v_\infty^{-4/7}$$
$$\approx 7 \times 10^9 M_{100}^{5/7} m_{10}^{2/7} v_{\infty,6}^{-4/7} \text{ cm} , \qquad (8)$$

where $m = 10 m_{10}$ $M_\odot$ and $M = 100 M_{100}$ $M_\odot$. Here $v_{\infty,6}$ is the speed at infinity in units of 10^6 cm s^{-1}. For stellar-mass or intermediate-mass black holes, this distance is so much larger than the radius $R \sim 6GM/c^2 \approx 10^8 M_{100}$ cm of the last stable orbit that, again, when the frequency is high enough to detect with ground-based instruments, the orbit will have circularized to $e < 0.01$. Note that because $r_p \propto M^{5/7}$, whereas the horizon radius $R_h \propto M$ for supermassive black holes, there is a point at which in order to become bound the smaller object would have to pass inside

the horizon, so that a direct capture occurs. This happens at $M \gtrsim 2 \times 10^8 (v_\infty/100 \text{ km s}^{-1})^{-2}$ $M_\odot$, and this means that above this mass, one will not observe multiple orbits. If two-body capture happens during a three-body interaction, the criterion is that the capture must occur before a slight perturbation from the third object can deflect the binary from its highly eccentric orbit. This means that the merger time must be less than 0.1–1 yr. For such short merger times, there is a significant probability that the pericenter distance will be comparable to the radius of the last stable orbit. This is in part because in the gravitational focusing regime, the cross section for the closest approach r_p scales as r_p and not r_p^2. However, we expect these to form a minor fraction of the coalescences observed with ground-based detectors.

In summary, all of the scenarios above predict that when a binary is orbiting with high frequency, its eccentricity will be small enough for the use of circular orbit templates. At the frequencies detectable with *LISA*, however, we expect significant eccentricities to be typical.

3. RATE ESTIMATES

What is the rate at which intermediate-mass black holes are expected to merge with less massive compact objects? The actual rate in a given cluster is set by the frequency of encounters, which depends on number densities and effective cross sections. However, if this rate is too high, then it is self-limiting. One upper limit is set by the supply of smaller objects; this supply cannot be exhausted much faster than the current age of the cluster. Another upper limit is set by the mass of the large black hole; the characteristic growth time of the black hole cannot be much shorter than the current cluster age. We now consider these in turn.

3.1. *Encounter Rates*

The encounter rate includes contributions from two-body capture by gravitational radiation and from three-body encounters. Consider first two-body interactions. If the smaller objects have number densities $10^6 n_6$ pc^{-3}, and if they are in thermal equilibrium with ~ 0.4 $M_\odot$ main-sequence stars with a velocity dispersion of $10^6 v_{\mathrm{ms},6}$ cm s^{-1} [so that the velocity dispersion of the black holes is $\approx 10^6 (0.4/10 m_{10})^{1/2} v_{\mathrm{ms},6}$ cm s$^{-1} = 2 \times 10^5 m_{10}^{-1/2} v_{\mathrm{ms},6}$ cm s^{-1}], then the rate of two-body capture by a given large black hole is

$$\nu_{\mathrm{enc}} = \langle n \sigma v_\infty \rangle \approx 2 \times 10^{-8} n_6 m_{10}^{11/7} M_{100}^{12/7} v_{\mathrm{ms},6}^{-11/7} \text{ yr}^{-1} \quad (9)$$

(e.g., Quinlan & Shapiro 1987; Miller & Hamilton 2002a). The typical rate can therefore be $\sim 10^{-10}$ to 10^{-6} yr^{-1}, depending on the cluster parameters and black hole masses.

Now consider three-body interactions. If the large black hole is initially solitary, it will acquire a companion when it interacts strongly with an existing binary, where " strongly " means roughly a closest approach less than the semimajor axis a of the binary. The interactions are dominated by gravitational focusing, so the cross section for a close approach within distance a is roughly $\pi a (2GM/v^2)$, where $v = 10^6 v_6$ cm s^{-1} is the velocity at infinity. For a moderately hard binary with $a = 1$ AU (and thus an orbital velocity of 30 km s^{-1}), this cross section is about $2 \times 10^{30} M_{100} v_6^{-2}$ cm^2. If 10% of stars are in binaries, their number density is about $3 \times 10^{-51} n_6$ cm^{-3}, so the interaction time is $t = 1/(n\sigma v) = 1/[(3 \times 10^{-51} n_6)(2 \times 10^{30} M_{100} v_6^{-2})(10^6 v_6)] =$

$5 \times 10^{12} v_6 M_{100}^{-1} n_6^{-1}$ s. If the relative velocity of binaries with large black holes is somewhat smaller than the main-sequence velocity dispersion (due to the higher mass of binaries), this is typically $\sim 5 \times 10^6 M_{100}^{-1} n_6^{-1}$ yr.

Once the massive black hole is in a binary, the question is how long it will take for subsequent interactions to harden it so that it merges because of gravitational radiation. Since there is not a significant loss cone for objects of this mass, the black hole can now interact with all the stars (density $3 \times 10^{-50} n_6$ cm^{-3}), so at least at first the interaction rate is about 1 per $5 \times 10^5 M_{100}^{-1} n_6^{-1}$ yr. The fractional hardening per interaction is small because of the large mass ratio, but the eccentricity can increase substantially (Quinlan 1996). Depending on how the eccentricity evolves, merger within a few hundred interactions is likely to be typical. For a 10^3 $M_\odot$ black hole, this will give rates of $\sim 10^{-7} n_6$ mergers per year, with lower rates for less massive black holes. Thus, the overall rate of interactions is likely to be dominated by three-body encounters, but the merger rate could have a significant contribution from two-body contributions as well, because such an interaction leads to a direct merger.

3.2. *Supply of Objects*

A model-independent upper limit on the rate of mergers is set by the requirement that the characteristic timescale on which the smaller objects are removed from the population cannot be much shorter than the age of the cluster; otherwise, the population would be reduced rapidly. For example, suppose the central black hole has a mass of 10^2 $M_\odot$ and we consider its interactions with neutron stars, of which we assume there are 10^3 currently in the cluster (and at most a few hundred in the core). For a 10^{10} yr old cluster, the interaction rate is therefore limited to not much more than 1 per 10^7 yr, and could be less. Of course, if other processes are more important, then the rate could be much less than these values, but the rate cannot be much more. For example, if the process of merging with one neutron star has along the way caused the ejection of 100 neutron stars (e.g., through three-body recoil), the maximum rate drops to 1 per 10^9 yr. The rate is therefore $R \lesssim R_{\mathrm{supply}} = N_s/T_0$, where N_s is the current number of small objects of a given type in the cluster and T_0 is the current cluster age.

3.3. *Growth Rate of Large Black Holes*

Similarly, the timescale of increase in the mass of the central black hole cannot be much shorter than the cluster age; otherwise, the massive black hole would acquire mass rapidly and grow until it exhausted its fuel. In the example above, if neutron stars are accreted faster than 1 per 10^8 yr, the black hole mass will grow well past its current 10^2 $M_\odot$ in a Hubble time. Thus, $R \lesssim R_{\mathrm{grow}} = M/(mT_0)$, where m is the mass of the objects accreted.

This provides a guide to the most common expected interactions between an intermediate-mass black hole and other compact objects. Because exchange interactions tend to favor more massive objects, the companion to an intermediate-mass black hole is likely to be among the more massive objects present in abundance in the core of the cluster. If there are enough stellar-mass black holes (say, tens to hundreds), then in dense clusters these may dominate the actual merger rate, even if most of the encounters are with other types of stars. If the number of stellar-mass black holes is smaller, then neutron stars (with perhaps $>10^3$ in a

cluster; Grindlay et al. 2001) or massive white dwarfs (which could constitute several percent of the number of stars, or up to $\sim 10^4$ in a cluster) may dominate the interactions. As we see in § 4, mergers between two black holes will provide most of the signal for high-frequency ground-based detectors, whereas the more frequent interactions between a neutron star or white dwarf and an intermediate-mass black hole will likely provide most of the signal for lower frequency space-based instruments. Note that for frequencies in the majority of the *LISA* band, white dwarfs act as point masses. Tidal disruption occurs at the Roche separation $R_{\mathrm{R}} \sim R_{\mathrm{WD}}(M/m)^{1/3}$, where $R_{\mathrm{WD}} \sim 10^9$ cm is the radius of the white dwarf. This implies a gravitational-wave frequency of $(GM/R_{\mathrm{R}}^3)^{1/2}/\pi \approx 0.1$ Hz, independent of the mass of the black hole. Below this frequency, white dwarfs can make a clean contribution to gravitational-wave signals in the *LISA* band.

3.4. *Overall Rate of Encounters*

The instantaneous rate of mergers is governed by R_{enc}, but as the supply is decreased and the black hole mass is increased, the encounter rate can change in response. In addition to simply decreasing the number of smaller objects with which the large black hole interacts, three-body effects inject energy into the population, which tends to increase the scale height of that population. The compact objects that interact with the black hole settle back via interactions with normal stars, but as the black hole mass increases, the rate of energy injection per object increases. This is primarily because the fractional change in binding energy per encounter scales roughly as m/M (Quinlan 1996). The binding energy released before subsequent encounters actually eject the interloper object therefore scales roughly as M. Thus, if the initial encounter rate is high, it will decrease until it approximately matches the characteristic time of depletion and of mass increase.

For ground-based detectors, which can see at most just the last few seconds of a merger, it is only the overall rate that matters. In contrast, for space-based detectors, one must also know the distribution of time to merger for a given class of system. To estimate this, suppose that for a given type of secondary, one can establish the typical time $\bar{\tau}$ between mergers. If the probability of a given system being a time $\tau < \tau_{\mathrm{merge}}$ away from merger is given by a Poisson distribution, then $P(\tau < \tau_{\mathrm{merge}}) \propto 1 - \exp(-\tau_{\mathrm{merge}}/\bar{\tau})$. For example, consider a 10^3 $M_\odot$ black hole interacting with a population of 10^3 neutron stars of mass 1.5 $M_\odot$. The maximum rate of interaction is roughly 1 per 10^7 yr; if this rate is realized in a particular cluster, the probability of that cluster having a system less than 10^6 yr away from merger is 10%. The implied S/N for *LISA*, however, depends on the eccentricity of the system, which we examine in § 4.

Note that a young dense cluster of age $\sim 10^8$ yr has much less serious constraints placed on interaction rates by supply limits and mass growth timescales. However, the maximum rate is still limited by the encounter frequency itself. Within $\sim 10^8$–10^9 yr, a massive young cluster may be able to evolve to high density in the core, without having yet ejected many compact objects by three-body processes (Portegies Zwart & McMillan 2000). In principle, therefore, some young clusters could have interaction rates an order of magnitude larger than those for globular clusters. To be conservative, however, we assume that the overall rate of mergers or

inspirals from young clusters is at most comparable to that for globular clusters.

4. S/N STATISTICS

4.1. *Gravitational-Wave Amplitudes*

Suppose that a binary of eccentricity e and orbital period T is a distance r from us and that in a coordinate frame in which the center of mass of the binary is at the origin, we are in a spherical polar direction (θ, ϕ), where the orbit of the binary is in the plane $\theta = \pi/2$. Following the notation and development of Pierro et al. (2001), the dimensionless metric perturbations in the $\times$ and $+$ polarizations are

$$h_\times = \frac{\cos\theta}{\sqrt{2}} \left[2h_{xy}\cos 2\phi - (h_{xx} - h_{yy})\sin 2\phi \right] ,$$

$$h_+ = \frac{1}{\sqrt{2}} \left\{ \frac{3+\cos 2\theta}{4} \left[2h_{xy}\sin 2\phi + (h_{xx} - h_{yy})\cos 2\phi \right] \right.$$
$$\left. - \frac{1-\cos 2\theta}{4}(h_{xx} + h_{yy}) \right\} . \quad (10)$$

In the adiabatic approximation (i.e., the assumption that the orbital parameters do not change significantly in a single orbit), the metric components are expressible as sums over harmonics (Peters & Mathews 1963; Peters 1964; Pierro et al. 2001):

$$h_{xy} = \sum_{n=1}^{\infty} h_{xy}^{(n)} \sin\left(n\frac{2\pi}{T}t\right) ,$$

$$h_{xx} \pm h_{yy} = \sum_{n=1}^{\infty} h_{x\pm y}^{(n)} \cos\left(n\frac{2\pi}{T}t\right) , \quad (11)$$

where the harmonic components are

$$h_{xy}^{(n)} = h_0 n(1 - e^2)^{1/2}[J_{n-2}(ne) + J_{n+2}(ne) - 2J_n(ne)] ,$$

$$h_{x-y}^{(n)} = 2h_0 n\left\{ J_{n-2}(ne) - J_{n+2}(ne) \right.$$
$$- 2e[J_{n-1}(ne) - J_{n+1}(ne)] + \frac{2}{n}J_n(ne) \right\} ,$$

$$h_{x+y}^{(n)} = -4h_0 J_n(ne) . \quad (12)$$

Here the J_n are Bessel functions, and the prefactor is

$$h_0 = \frac{cT}{4\pi r}\frac{1 - \Delta^2}{\chi^{5/3}} , \quad (13)$$

where $\Delta \equiv |M - m|/(M + m)$ is the fractional mass difference and $\chi \equiv cT/[2\pi G(M + m)/c^2]$. For a circular orbit, the power is all at the second harmonic ($f_{GW} = 2f_{bin}$), and if $M \gg m$, the angle-averaged dimensionless strain is

$$h \approx 7 \times 10^{-21}\left(\frac{f_{GW}}{10^{-4}\ \text{Hz}}\right)^{2/3} M_{100}^{2/3}m_{10}\left(\frac{10\ \text{kpc}}{r}\right) . \quad (14)$$

The harmonic of peak amplitude is $N_{\max}(e) \propto (1 - e^2)^{-3/2}$ (see Pierro et al. 2001).

4.2. *Initial Inspiral*

The majority of the inspiral is only detectable with low-frequency instruments such as *LISA*. S/N estimates for

LISA are complicated by the expectation of a significant background of astrophysical sources, especially white dwarf binaries (Phinney 2001). As the detector acquires data and frequency resolution becomes better due to the longer observational baseline, individual sources can be identified. They can therefore be removed from the data stream, so that the unresolved background will effectively diminish in strength. However, until that point, the true S/N is less than what would be computed from the instrumental curve alone, particularly for $f_{GW} < 10^{-3}$ Hz, where the white dwarf background is expected to make its strongest contribution. There are also numerous other sources, especially coalescing supermassive black holes, that may produce a strong astrophysical background over much of the *LISA* frequency range (e.g., Phinney 2001; Hughes 2002). As with the white dwarf sources, in the course of the lifetime of the experiment, such individual sources can be accounted for in analysis, so that the effective sensitivity of *LISA* to other sources will improve with time. For the estimates in this section, we consider only the expected instrumental noise curve (kindly provided by R. Stebbins). We also consider only the far-field inspiral. For inspiral effects at smaller distances, including the effects of black hole spin, see, e.g., Finn & Thorne (2000) and Hughes (2001).

As is evident from § 4.1, the detectability of a signal depends strongly on its frequency distribution and amplitude because of the strong frequency dependence of the detector sensitivity (e.g., Danzmann 2000). In particular, the harmonic content of the signal is important; orbits with high eccentricities have important contributions at frequencies many times the orbital frequency. For example, an orbital frequency of 10^{-5} Hz is far below the most sensitive range of *LISA*, but for $e = 0.99$, most of the power is in the 10^{-3} to 10^{-2} Hz range, in which the S/N = 5 threshold in a 1 yr observation is a dimensionless strain of $h_5 \approx 10^{-23}$.

If the time to merger is more than about 10^6 yr, then the S/N from a source at a fixed distance in a year's integration with *LISA* depends on the merger time but is insensitive to other parameters such as the individual masses or the eccentricity. To show this, note that $h^2 f^2 \propto L$, where $L \propto (GMm/a)/\tau_{\text{merge}}$ is the luminosity and f is the frequency, and that at the low-frequency end, the strain that gives an S/N of 5 in a 1 yr integration is $h_5 \propto f^{-5/2}$ for frequencies less than $\sim 10^{-3}$ Hz (R. Stebbins 2002, private communication). If $M \gg m$ and $1 - e \ll 1$, then the merger time is $\tau_{\text{merge}} \propto a^4 M^{-2}m^{-1}(1 - e^2)^{7/2}$ and the frequency of peak emission is $f_{\text{peak}} \propto (1 - e^2)^{-3/2}M^{1/2}a^{-3/2}$. For simplicity, assume that all the power in the signal is concentrated at f_{peak}. Then

$$\frac{\text{S}}{\text{N}} \propto \frac{h}{h_5} \propto M^{-1/8}m^{-3/16}(1 - e^2)^{5/32}\tau_{\text{merge}}^{-19/16} . \quad (15)$$

The weak dependence on the masses and eccentricity arises because of competing effects. For example, at a fixed merger time, a binary with a high eccentricity has a large semimajor axis (and thus a low fundamental frequency), but the frequency peak is at a high harmonic. A binary with a low eccentricity has a small semimajor axis and thus a high fundamental frequency, but the frequency peak is at a low harmonic. In both cases, the frequency peak is about the same. At merger times $\tau_{\text{merge}} \lesssim 10^6$ yr, however, the S/N is larger for more circular binaries because the frequency peak is greater than or about the optimal frequency, $\sim 3 \times 10^{-3}$ Hz

for *LISA*, and the S/N is greater when the signal is concentrated into a few frequency bins.

The total S/N is the square root of the sum of the squares of the S/Ns for each of the harmonics. To compute the S/N, we therefore need to (1) pick an eccentricity and merger time, (2) calculate the frequency distribution and dimensionless strain amplitudes for the harmonics, (3) take the ratio with the expected *LISA* sensitivity at each frequency, and (4) sum these in quadrature for the final S/N. The computation of a cumulative S/N plot thus requires assumptions about the distribution of eccentricities for a given merger time, although from the calculation above, this only makes a significant difference if $\tau_{\rm merge} \lesssim 10^6$ yr.

As a sample calculation, let us assume that for a large merger time, the probability distribution of eccentricities is the thermal distribution $P(e) = 2e$ that emerges from close interactions of three equal-mass objects (Heggie 1975). We expect that this will underestimate the eccentricity for large merger times, at which the eccentricity may be pushed to higher values by many small interactions (Quinlan 1996). At merger times much less than the typical time between encounters, the eccentricity will decrease because of gravitational radiation. As mentioned earlier, Peters (1964) showed that to lowest order, the eccentricity and semi-major axis evolve so as to keep the expression $ae^{-12/19}(1 - e^2)(1 + 121e^2/304)^{-870/2299}$ constant. Combining this with the expression for merger time, we take as an approximation that when $\tau_{\rm merge} < \tau_{\rm enc}$, the eccentricity is diminished by $e \approx e_0(\tau_{\rm merge}/\tau_{\rm enc})^{0.4}$. We assume that the

merger time is given by a Poisson distribution, $P(t < t_0) \propto 1 - \exp(-t_0/\bar{\tau})$, where $\bar{\tau}$ is the typical time between mergers as defined above. Given the limits of growth of the black hole and consumption of the secondaries, we use $\bar{\tau} = 10^7$ yr for $M = 10^3\ M_\odot$ and $m = 1\ M_\odot$. This is the minimum value of $\bar{\tau}$ allowed, so if the time is greater, the contours are moved to the left in Figure 2. In all cases, we fix the distance to the globular at 10 kpc, a representative distance to the globular clusters around our Galaxy (actual distances vary from 2.2 to 122 kpc, with roughly 55% being less than 10 kpc distant; see Harris 1996).

If these curves are roughly representative, it suggests that globular clusters of at least moderate density will have a significant probability of containing a high-S/N gravitational-wave source, especially if $10^3\ M_\odot$ black holes are common. In a 10 yr *LISA* integration, the S/N would be greater than 10 for ~90% of the Galactic globular clusters that have 10^3 $M_\odot$ black holes and a significant supply of 1 $M_\odot$ compact objects. This could be several tens of globular clusters around our Galaxy. Over 10 yr, the strongest of the sources could have an accumulated S/N of several hundred. There is also a chance to detect inspiraling intermediate-mass black hole binaries at greater distances with *LISA*. For example, the Virgo Cluster, at a distance ≈16 Mpc (e.g., Graham et al. 1999), contains ~10^3 galaxies and ~10^5 globular clusters (using the mass fraction $M_{\rm gc}/M_{\rm stars} \approx 2 \times 10^{-3}$ of McLaughlin 1999). From the model above, we expect there to be a few binaries within ~10^3 yr of merging, which would therefore have S/Ns of more than 5–10 in a 1 yr *LISA* integration. As we discuss in § 5, such binaries are likely to have appreciable eccentricities and detectable frame dragging, so that there are a number of interesting strong gravity effects that can be probed.

4.3. *Final Inspiral and Merger*

As discussed in § 2, coalescing binary black holes in dense stellar clusters may contain intermediate-mass black holes or may consist of two stellar-mass black holes (via the Kozai mechanism or due to a close pass in a two- or three-body interaction). We first treat intermediate-mass black holes, then discuss the possible rates for stellar-mass black hole coalescence.

By the time that a binary with an intermediate-mass black hole is in the frequency range of a ground-based instrument, it will have shrunk through a large enough factor in pericenter distance that the orbit will be nearly circular. As discussed by Flanagan & Hughes (1998a, 1998b) and Cutler & Thorne (2002), the remaining coalescence can then be conveniently divided into inspiral (in which analytical calculations are adequate), merger (in which numerical simulations are mandatory because of strong gravity effects), and ringdown (in which analytic theory exists). The boundary between inspiral and merger is somewhat arbitrary, but it can be said to be roughly where the inward radial speed increases rapidly (see Ori & Thorne 2000). Flanagan & Hughes (1998a) conservatively assume that the inspiral is ended when the orbital frequency becomes $0.02c^3/(GM)$; we discuss below the effects of spin on this number.

It is likely that radiation of significant energy and angular momentum in a merger phase requires that the total angular momentum of the system exceed the Kerr threshold for the total mass of the system (S. Hughes 2002, private communication). From the work of Pfeiffer, Teukolsky, & Cook
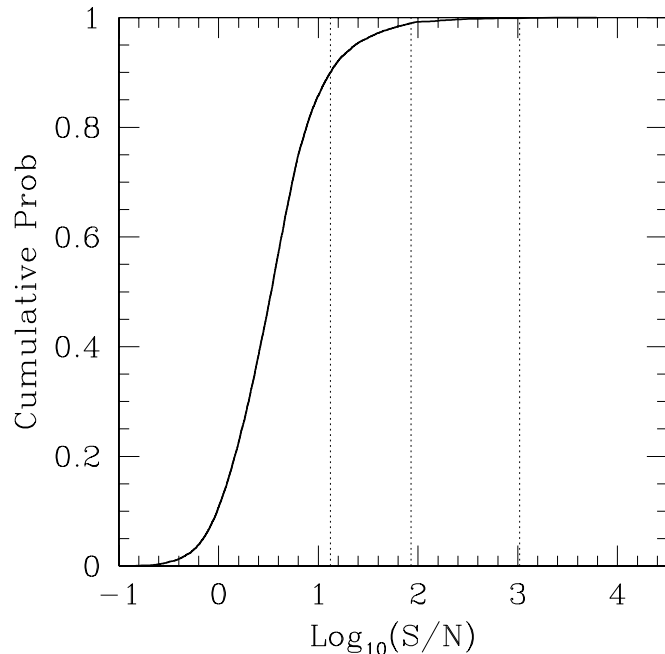

Fig. 2.—Sample cumulative probability distribution of a *LISA* 1 yr angle-averaged S/N for binaries consisting of a $10^3\ M_\odot$ black hole and a 1 $M_\odot$ compact companion (either a neutron star or a massive white dwarf), at a distance of 10 kpc from us. See text for other parameters. From this graph, we see that, e.g., 90% of events will have S/N > 1, 15% will have S/N > 10, and so on. The vertical dotted lines are at cumulative probabilities of 90% (S/N > 13), 99% (S/N > 85), and 99.9% (S/N > 1050). If ~20% of the globular clusters around the Milky Way have black holes of this mass and central densities $\gtrsim 10^5$ pc^{-3}, this suggests that several sources will have 1 yr *LISA* S/Ns above 10, with an expected maximum of ~20–100.

(2000), the orbital angular momentum at the last quasi-circular orbit between two black holes (with total mass M and reduced mass μ) is $\approx (2.5–3.5)\mu M G/c$, depending on the initial spin magnitudes and orientations. Even for equal masses, $\mu = M/4$, this is not enough by itself to exceed the Kerr maximum of $M^2 G/c$, and hence the black holes must be prograde and spinning fairly rapidly for there to be a hang-up phase before final merger (S. Hughes 2002, private communication). For the higher mass ratio inspirals that are the main focus here, it is probably impossible to exceed the Kerr threshold unless the larger black hole is already close to maximally rotating. It is therefore likely that there will be no merger radiation per se, but that any excess energy or angular momentum would be emitted in an extended ring-down phase. However, in order to be complete, we include estimates of the detectability assuming that a merger-like phase releases an energy that we scale to a fraction $0.1(4\mu/M)^2 = \epsilon_m(4\mu/M)^2$ of the total mass-energy of the system (following the estimate by Flanagan & Hughes 1998a). Similarly, the ring-down energy is scaled to a fraction $0.03(4\mu/M)^2 = \epsilon_r(4\mu/M)^2$ of the total mass-energy.

For a given M and μ, there is a luminosity distance out to which each of these phases can be detected with a given instrument with an S/N of at least 10. From Flanagan & Hughes (1998a), for the advanced LIGO instrument these luminosity distances for a source at redshift z are

$$D_i \approx 3(1+z)^{-1/2} M_{100}^{-1/2} \left(\frac{4\mu}{M}\right)^{1/2} \text{ Gpc}$$

$$\approx 1.8(1+z)^{-1/2} \mu_{10}^{1/2} M_{100}^{-1} \text{ Gpc} ,$$

$$D_m \approx 7.6(1+z) \left(\frac{\epsilon_m}{0.1}\right)^{1/2} M_{100} \left(\frac{4\mu}{M}\right) \text{ Gpc}$$

$$\approx 3(1+z) \left(\frac{\epsilon_m}{0.1}\right)^{1/2} \mu_{10} \text{ Gpc} ,$$

$$D_r \approx 0.85(1+z)^{5/2} \left(\frac{\epsilon_r}{0.03}\right)^{1/2} M_{100}^{5/2} \left(\frac{4\mu}{M}\right) \text{ Gpc}$$

$$\approx 0.3(1+z)^{5/2} \mu_{10} M_{100}^{3/2} \left(\frac{\epsilon_r}{0.03}\right)^{1/2} \text{ Gpc} . \quad (16)$$

The actual formulae are more complicated, but these expressions are reasonably accurate over the 50–1000 $M_\odot$ range of interest (except for ring-down, which scales as M above 240 $M_\odot$ and as $M^{-1/2}$ above 620 $M_\odot$). The rate of detection at S/N > 10 is given by

$$R = \int \frac{4\pi}{3} D(M)^3 \nu(M) n_{gc} f(M) \, dM . \quad (17)$$

Note that for $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} \approx 0.7$, the redshift is $z = 0.13$ at a distance of 2 Gpc and $z = 0.4$ at a distance of 3 Gpc, so cosmological corrections will usually be small. Here $n_{gc} \approx 8h^3 \text{ Mpc}^{-3}$ is the number density of globular clusters in the local universe (as estimated by Portegies Zwart & McMillan 2000) and $\nu(M)$ is the rate at which smaller objects merge with black holes of mass M in a given cluster. The mass distribution of large black holes in clusters is $dN/dM = f(M)$, where $\int f(M) \, dM = f_{tot} < 1$ is the total fraction of globular clusters that have intermediate-mass black holes. As a specific example, let $\nu(M) = 10^{-9}\mu_{10}^{-1}(M/100 \, M_\odot) \text{ yr}^{-1}$ and $f(M) = [f_{tot}/\ln(M_{max}/M_{min})]M^{-1}$ for M between M_{min} and M_{max} and 0 otherwise. For $M_{max} \gg M_{min}$, the

approximate rates are then

$$R_i \approx 10h^3 \left(\frac{f_{tot}}{0.1}\right) \mu_{10}^{1/2} M_{min,100}^{-2} \left(\ln \frac{M_{max}}{M_{min}}\right)^{-1} \text{ yr}^{-1} ,$$

$$R_m \approx 60h^3 \left(\frac{f_{tot}}{0.1}\right) \mu_{10}^2 M_{max,100}$$

$$\times \left(\ln \frac{M_{max}}{M_{min}}\right)^{-1} \left(\frac{\epsilon_m}{0.1}\right)^{3/2} \text{ yr}^{-1} ,$$

$$R_r \approx 0.07h^3 \left(\frac{f_{tot}}{0.1}\right) \mu_{10}^2 M_{max,100}^{5/2}$$

$$\times \left(\ln \frac{M_{max}}{M_{min}}\right)^{-1} \left(\frac{\epsilon_r}{0.03}\right)^{3/2} \text{ yr}^{-1} . \quad (18)$$

For example, if $M_{min} = 50 \, M_\odot$, $M_{max} = 300 \, M_\odot$, $h = 0.7$, $\mu_{10} = 1$, $\epsilon_m = 0.1$, and $\epsilon_r = 0.03$, then $R_i \approx 8(f_{tot}/0.1)$ yr^{-1}, $R_m \approx 34(f_{tot}/0.1)$ yr^{-1}, and $R_r \approx 0.2(f_{tot}/0.1)$ yr^{-1}. At a different S/N threshold SNR, these rates should be multiplied by roughly $(10/\text{SNR})^3$, modulo cosmological corrections. Note that the rapid increase in number with diminishing S/N means that for the purposes of statistical analysis, these will dominate the constraints possible with these data. Similar answers are obtained for more general power laws, $f(M) \propto M^{-p}$.

Note that the division of numbers between inspiral and merger is based on the conservative assumption that inspiral continues only to the frequency $0.02M^{-1}$ (Flanagan & Hughes 1998a), which is the frequency at the innermost stable circular orbit for a test particle around a nonrotating black hole. As shown in § 2.2, the random walk process of accretion of smaller black holes is likely to produce spin parameters of the order of a few tenths. For example, a 100 $M_\odot$ black hole that has grown from 50 $M_\odot$ by accretion of 10 $M_\odot$ black holes has a mean spin parameter $j \approx 0.5$ (see Fig. 1). At this spin, the frequency is increased by a factor of 1.7 for equatorial prograde orbits, implying an increase of up to a factor of several in the detection rate, because the detection sensitivity increases with frequency in this range. Further analysis of this effect will be important.

For the LISA instrument, black holes in this mass range will not be detectable during merger and ring-down. From Flanagan & Hughes (1998a), the final inspiral could be detected at S/N ≥ 10 in a 1 yr integration out to a luminosity distance

$$D_{i,LISA} \approx 0.2(1+z) M_{100}^{1/2} \mu_{10}^{1/2} \text{ Gpc} . \quad (19)$$

Thus, the expected rate of objects detectable in a 1 yr integration is

$$R_{i,LISA} \approx 0.02h^3 \left(\frac{f_{tot}}{0.1}\right) \mu_{10}^{1/2} M_{max,100}^{3/2}$$

$$\times \left(\ln \frac{M_{max}}{M_{min}}\right)^{-1} \text{ yr}^{-1} . \quad (20)$$

Note, however, that over a longer integration, the rate goes up dramatically because the gravitational-wave amplitude scales with frequency as $h \sim f_{GW}^{2/3}$, whereas in this frequency range, the S/N = 5 threshold of LISA scales as f_{GW}. For example, in a 10 yr integration, LISA would be expected to see several to tens of objects in the last phase of their inspiral. This leads to the interesting possibility that a 50–100 $M_\odot$ black hole coalescing with a 10 $M_\odot$ black hole

may be observed with *LISA* a few years prior to merger; then its final merger could be seen later with ground-based instruments. The waveform observed with *LISA* could be projected to the final merger, so that the time, phase, and other characteristics could be anticipated and detected with ground-based gravitational-wave detectors, and the region could be observed simultaneously with conventional telescopes. Given that the angular resolution of *LISA* would be at best a few degrees for these sources (e.g., Cutler 1998), it may be necessary to employ wide-field monitoring to catch the final merger. We note that detection of any significant electromagnetic radiation during the merger would require a profound revision of our understanding of these systems and possibly of gravitational radiation itself. In a globular cluster, the gas density is too low for there to be meaningful accretion from the interstellar medium. If at one point there were a substantial accretion disk around either compact object, it is likely that it will have been completely disrupted or accreted by the time of the final merger. In addition, gravitational radiation is not thought to couple significantly to electromagnetic fields, so negligible photon luminosity is expected from the final merger. If instead there is an electromagnetic counterpart to a merger of an intermediate-mass black hole with a neutron star or stellar-mass black hole, parts of this picture must be revised.

Let us now turn from interactions of intermediate-mass black holes to interactions among only stellar-mass black holes. As before, there is an upper limit to the rate set by the timescale in which the supply of stellar-mass black holes is used up. For example, if a cluster contains $N_{\rm BH}$ stellar-mass black holes, a strong upper bound on the binary black hole merger rate by any mechanism is $\sim 10^{-10} N_{\rm BH}$ yr^{-1}, assuming that the supply is not exhausted in much less than a Hubble time. The actual rate depends on details. Consider, for example, merger by the Kozai mechanism after a binary-binary encounter (Miller & Hamilton 2002b). The rate is proportional to the square of the fraction of black holes in binaries. If most black holes are in binaries, then the rate could be several tens of percent of the maximum (see Miller & Hamilton 2002b). If the fraction in binaries is f_b, then the rate per cluster is $\sim 10^{-10} N_{\rm BH} f_b^2$ yr^{-1}. We expect that the clusters that contribute most to the rate of stellar black hole coalescences will be those with relatively high central densities, because low-density clusters will have a low interaction rate. Let us parameterize the fraction of clusters that contribute significantly as f, where f is likely to be a few tenths. Note that the denser clusters are also the ones most likely to produce intermediate-mass black holes; hence, it is likely that $f \approx f_{\rm tot}$, but to keep the processes distinct, we use different variables to represent the fractions. The volume rate in the universe for mergers of two stellar-mass black holes via the Kozai mechanism is then $\sim 10^{-2} N_{\rm BH} h^3 f_b^2 (f/0.1)$ Gpc^{-3} yr^{-1}. The other way stellar-mass black holes can coalesce in a cluster is by having a near approach during a three-body encounter. As estimated in § 6, the rate of such coalescences is roughly 10% the rate at which black holes are ejected from the cluster by dynamic recoil. Using the bounding estimates of Portegies Zwart & McMillan (2000; their eqs. [8] and [9]), this implies a rate in the universe of $\sim (1-6) h^3 (f/0.1)$ Gpc^{-3} yr^{-1}. Thus, if $f_b > 0.3$, then the Kozai mechanism dominates; otherwise, direct coalescence in three-body interactions is more important. From Flanagan & Hughes (1998a), the inspiral of a pair of 10 $M_\odot$ black holes could be detected with S/N ≥ 10

out to a distance 1.6 Gpc. This implies a combined rate of $\sim 10 h^3 (f/0.1) \left(10^{-2} N_{\rm BH} f_b^2 + 1 \right)$ yr^{-1} in the advanced LIGO detector for two stellar-mass black holes, compared to a rate of $\sim 40 (f_{\rm tot}/0.1)$ yr^{-1} for mergers of stellar-mass black holes with intermediate-mass black holes.

5. INFORMATION FROM WAVEFORMS

5.1. *Pericenter Precession*

The high expected eccentricities of binaries in the *LISA* band imply that it may be possible to observe precession of the pericenter much deeper in a gravitational well than is possible for known Galactic neutron star binaries. The angle of precession in an orbital period is

$$\Delta\phi = \frac{6\pi GM}{a(1-e^2)c^2} \qquad (21)$$

(e.g., Misner, Thorne, & Wheeler 1973, p. 1110). If $e \approx 1$, this is $\Delta\phi \approx 3\pi GM/(r_p c^2)$. The effect of precession is to split the single frequency of the orbit into a pair, with a separation that can be detected if the observation is for a time $t_{\rm obs} > [r_p/(2GM/c^2)]T$, where as before, T is the orbital time (e.g., Pierro et al. 2001). Combining factors, the required observation time to barely resolve pericenter precession is

$$t_{\rm obs} > 4 m_{10}^{5/8} M_{100}^{-1/4} (1-e^2)^{-35/16} (1-e) \left(\frac{\tau_{\rm merge}}{10^6 \ {\rm yr}} \right)^{5/8} \ {\rm yr} \ . \qquad (22)$$

For example, a 1 $M_\odot$ compact object in an $e = 0.9$ orbit around a 10^3 $M_\odot$ black hole, with a merger time of 10^6 yr, requires roughly 2 yr of observation. When the same binary has a merger time of 10^3 yr (and therefore has $e \ll 1$), only 2 days of observation are required. This suggests that binaries in the Virgo Cluster, which are numerous enough for $\tau_{\rm merge} \lesssim 10^3$ yr to be probable, are good candidates for observation of pericenter precession and that such precession may also be observable in our own Galactic globular system.

5.2. *Lense-Thirring Precession*

The Lense-Thirring precession rate, at which the axis of the orbital plane changes, is approximately $\omega_{\rm LT} = 2jG^2 M^2/c^3 r^3$. Integrated over an orbit of semimajor axis a and eccentricity e, the average rate is $\omega_{\rm LT} = 2jG^2 M^2/[c^3 a^3 (1-e^2)^{3/2}]$. As is evident from the formulae in § 4.1, the signature of such precession would be a periodic change in the relative amplitudes in the two polarizations as the angle θ varies. For precession of more than a radian to occur during an observation time $t_{\rm obs}$ therefore requires

$$t_{\rm obs} > 100 j^{-1} m_{10}^{3/4} M_{100}^{-1/2} (1-e^2)^{-9/8} \left(\frac{\tau_{\rm merge}}{10^6 \ {\rm yr}} \right)^{3/4} \ {\rm yr} \ . \qquad (23)$$

Thus, it will typically be difficult to detect this effect in Galactic globular clusters unless the S/N is so high that precession of $\lesssim 0.1$ radians can be detected. However, a binary in the Virgo Cluster with a 1 $M_\odot$ and a 10^3 $M_\odot$ black hole with $j = 0.1$ that will merge within 10^3 yr will require ≈ 0.3 yr of observation, so this is well within reach.

An even better hope of detecting Lense-Thirring precession lies in characterization of the inspiral waveform with a

ground-based detector such as the advanced LIGO detector. The ratio of Lense-Thirring to orbital frequency is $2j(GM/rc^2)^{3/2}$ for nearly circular orbits, so if the expected tens to hundreds of orbits of inspiral are observed, then the precession of the orbital plane should be evident. The high-inclination orbits expected in this scenario are sensitive to the multipole moments of the mass distribution and may therefore test the no-hair theorem (Ryan 1996), although S/Ns of tens are typically required for significant constraints in a single inspiral (Ryan 1997). Nonetheless, a statistical combination of the constraints from observations of multiple mergers, even at a relatively low S/N, could provide interesting limits. Observations of these orbits may therefore allow initial mapping of spacetime around rotating black holes (particularly in the final merger detectable with ground-based instruments), a job expected to be completed with high precision by *LISA* observations of stellar-mass black holes being consumed by supermassive black holes in galactic centers (Hughes 2001).

5.3. *Decay of Orbit*

For binaries close to the end of their inspiral, the orbital frequency could change enough during ~ 10 yr that the inspiral is detected. An orbit with merger time τ_{merge} will change its frequency by a fractional amount $\delta \approx 0.4(t_{\mathrm{obs}}/\tau_{\mathrm{merge}})$ in an observation of duration t_{obs}. For this to be detected via a one cycle shift during the observation requires that $t_{\mathrm{obs}} \gtrsim T/\delta$, or

$$t_{\mathrm{obs}} > 30 m_{10}^{3/16} M_{100}^{1/8} (1-e^2)^{-21/32} \left(\frac{\tau_{\mathrm{merge}}}{10^6 \text{ yr}}\right)^{11/16} \text{ yr} . \quad (24)$$

With some luck (specifically, $\tau_{\mathrm{merge}} \lesssim 10^5$ yr), this may be observable within the Galactic globular system, but once again the Virgo Cluster binaries are excellent candidates, where at S/Ns greater than 10, the decay of the orbit will be clear in the signal.

One can combine the three effects discussed in this section to speculate about astrophysical information that might be available from gravitational radiation. An interesting possibility is that the distance to the Virgo Cluster could be estimated from gravitational-wave signals alone, with an accuracy that is competitive with optical measurements. Alternatively, using optically measured distances as an input, the system would be overdetermined, and detection of pericenter precession and orbital decay would allow strong consistency checks of the underlying formulae. Suppose that *LISA* is operational for 10 yr. Then, from the estimates in § 4, there will be several sources in Virgo that are detectable with *LISA* with S/N > 50 in that 10 yr period. With such a high S/N, the modulation due to the orbit of the Earth will localize the sources to within several degrees, and the membership in Virgo will be based on this association. From the discussion of black hole spin and binary eccentricity in § 2, the typical eccentricity for such binaries would be $e \approx 0.1$, and the typical spin parameter would be $j \approx 0.1$. Thus, pericenter precession, Lense-Thirring precession, and decay of the orbit would all be detectable. The eccentricity would be evident from the waveform. This means that the combinations $(M+m)/a^3$ (from the frequency), $(M+m)/a$ (from pericenter precession), and $a^4/[(M+m)Mm]$ (from decay of the orbit) would all be independently measurable. Combining these, a, m, and M would all be determined. Along with the strength of the

waves at the detector, this would yield an estimate of the distance to the cluster from the gravitational radiation alone (for discussion of globular cluster distance determinations using gravitational waves, see Benacquista 2000). It would also be possible to estimate the spin angular momentum of the larger black hole from Lense-Thirring precession.

6. CONCLUSIONS

Dense stellar clusters are promising locations for sources of gravitational radiation. We estimate that up to several tens of times per year, the advanced LIGO detector will see the coalescence of a small black hole with a larger one (a few to tens during inspiral, and most of the rest during merger, with both numbers dependent on black hole spin). Mergers of two stellar-mass black holes in clusters will likely be detectable at a rate of a few per year. As always in astrophysical scenarios, there are sources of uncertainty. The most major is the question of the number of black holes that are to be expected in a dense cluster, whether in the ~ 10 $M_\odot$ stellar-mass range or the $\sim 10^2$–10^4 $M_\odot$ intermediate-mass range. Perhaps the most easily calculable input to this question is the number of black holes that were originally produced. If the initial mass function for the cluster was the Salpeter function $dN/dM \propto M^{-2.35}$ above 1 $M_\odot$ and flatter below 1 $M_\odot$ (e.g., Meyer et al. 2000), then approximately 0.1% of stars began with $M > 25$ $M_\odot$ and presumably evolved into black holes. If the initial mass function were instead the Scalo (1986) distribution, which drops off more sharply at higher masses, the fraction with $M > 25$ $M_\odot$ would be closer to 0.05% (Portegies Zwart & McMillan 2000). Current dense clusters have $\sim 10^6$ stars and may have had several times more at birth, so the number of stars that evolved into black holes is typically $\sim 10^3$.

There are, however, numerous ways in which black holes may be lost from the cluster. The first is in the supernova that produces the black hole. Neutron stars are known to have significant birth kicks of tens to hundreds of km s^{-1} (e.g., Hansen & Phinney 1997; Fryer & Kalogera 1997). The mechanism for this is still debated (Spruit & Phinney 1998; Kusenko & Segre 1999; Lai, Chernoff, & Cordes 2001), but it is thought that similar kick velocities for black holes are much smaller (e.g., Brandt, Podsiadlowski, & Sigurdsson 1995; Podsiadlowski et al. 2002; Colpi & Wasserman 2002; see Nelemans, Tauris, & van den Heuvel 1999 for a somewhat different perspective), if for no other reason than that black holes are several times more massive than neutron stars, so that a fixed energy or momentum in the kick would lead to reduced speeds. It is therefore plausible that black holes do not receive birth kicks of $\gtrsim 50$ km s^{-1}, in which case they are retained in the cluster.

A second loss mechanism involves three-body recoil. Several simulations have shown that black holes of a fixed mass of ~ 10 $M_\odot$ in binaries with other such black holes tend to be ejected by three-body interactions before they can merge by gravitational radiation (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000). The actual fraction of ejected black holes can depend on the mass function of stars and other variables, but recent estimates suggest that 10% or more of the initial black holes can be retained by their clusters over a Hubble time (Portegies Zwart & McMillan 2000). If more massive black holes are present initially, then Miller & Hamilton (2002a) showed that they are usually not ejected. These can grow by mergers

after multiple three-body encounters, but typically they will eject several to tens of field stars along the way. The majority of the encounters, however, will not be with black holes, so this mechanism is not expected to deplete the black hole supply significantly. There are, in addition, at least two ways in which multibody interactions can produce a merger without accompanying dynamical recoil.

One, discussed by Miller & Hamilton (2002b), is that binary-binary interactions can produce a stable hierarchical triple system, and if the inclination of the orbit of the tertiary to the orbit of the inner binary is in the right range, then a secular Kozai resonance can increase the eccentricity of the inner binary to the extent that it merges before the next encounter, without significant recoil. The impact of this effect depends on the binary fraction and the distribution of inclinations (see Miller & Hamilton 2002b), but this could allow the merger of some tens of percent of the original population of black holes.

The other recoilless possibility involves resonant encounters in three-body interactions, in which the three objects orbit hundreds or thousands of times before resolving into a binary and an unbound single star. If, during these orbits, two black holes pass close enough to each other that losses to gravitational radiation in a single pass cause rapid merger, then again there is no dynamical kick. Various estimates suggest that in an equal-mass three-body encounter, the probability of the closest approach being less than $\epsilon < 1$ times the initial semimajor axis is $\sim\epsilon^{1/2}$ (Hut 1984; McMillan 1986; Sigurdsson & Phinney 1993). Two 10 $M_\odot$ black holes must approach to within $\sim 10^9$ cm to merge in a year, which will happen in a given resonant encounter with a probability of a few tenths of a percent for a semimajor axis of a few astronomical units. If it takes ~ 10 equal-mass encounters to harden a binary to the point of ejection, this suggests that several percent of binaries will merge before ejection in this fashion. Combining all of the above effects, it seems likely that tens of percent of the original black hole population will not be ejected by three-body recoil, leaving a present-day population of hundreds.

The third loss mechanism involves the merger itself. The emission of gravitational waves during inspiral and merger is somewhat asymmetric, leading to recoil. Calculations thus far have focused on the weak-field limit. They suggest that in the post-Newtonian limit, the kick scales as $a_{\rm LSO}^{-4}$ with the separation $a_{\rm LSO}$ at the last orbit before dynamical instability. At the separation $a_{\rm LSO} = 6GM/c^2$ appropriate for test particles around slowly rotating black holes, the kick will be a few km s^{-1} for the mass ratio $M/m = 2.6$ that maximizes the kick (Fitchett 1983; Wiseman 1992). Binaries with large mass ratios or nearly equal masses experience less recoil (for example, by symmetry, equal-mass binaries have no kick). Also, $a_{\rm LSO}$ is greater in a comparable-mass binary than in the test-particle limit (Pfeiffer et al. 2000), which also decreases the kick. It is not clear what level of recoil is to be expected in the merger phase, in which the radial velocity becomes rapid. Strong-field calculations are required to resolve whether this process is dominant (perhaps kicking most merging black holes out of the cluster) or insignificant (if the recoil speeds are much less than the ~ 50 km s^{-1} escape speeds from the core).

Thus, if mergers do not kick black holes out of dense clusters, one can expect at least tens to hundreds of black holes in many current systems. These are expected to reside primarily in the core of the cluster, where they have a greater tendency to interact with the more massive (and hence compact) objects in the cluster. This may explain why no definitive examples of black hole low-mass X-ray binaries are known in the globular cluster systems of the Milky Way or Andromeda; such a population would not undergo mass transfer and would thus be observable only by its gravitational-wave emission. Note, however, that there is a population of more than 10^{39} ergs s^{-1} sources in the globular clusters around a number of elliptical galaxies (Angelini, Loewenstein, & Mushotzky 2001; White, Sarazin, & Kulkarni 2002; Kundu, Maccarone, & Zepf 2002). Possible differences between these systems are an important subject for future study.

What about the fraction of clusters with intermediate-mass black holes? Miller & Hamilton (2002a) estimate that clusters with central densities greater than $\sim 10^5$ pc^{-3} have high enough encounter rates to produce 10^2–10^4 $M_\odot$ black holes. In the Milky Way globular system, this would imply that roughly 40% of globular clusters could host such objects (Pryor & Meylan 1993). However, to be conservative, we have adopted 10% as our fiducial value for the estimates of merger rates. We have also been conservative in assuming that the number density and mass of globular clusters is the same out to 2–3 Gpc as it is in the local universe. As first discussed by Aguilar, Hut, & Ostriker (1988), evaporation and tidal interactions attenuate the globular system of a galaxy. Therefore, it is possible that coalescence rates a few billion years ago were higher by up to a factor of a few than they are now (Portegies Zwart & McMillan 2000), but this is highly uncertain.

The general model described here is one that can be tested and enhanced in ways both observational and theoretical. From the observational standpoint, it is important to continue kinematic work on globular clusters to look for evidence of the velocity and density cusps that are the expected signatures of black holes (Bahcall & Wolf 1976; Frank & Rees 1976; see Gebhardt et al. 2000; D'Amico et al. 2002; Colpi et al. 2002 for recent results). Further characterization of the ultraluminous X-ray sources is also important. For example, if a mass estimate can be obtained via radial velocity measurements of a companion, this will shed new light on the nature of these objects. From the theoretical standpoint, there are several important calculations. These include (1) strong-gravity computations of the recoil speed of a black hole binary after merger, for different mass ratios and spins, (2) comprehensive numerical simulations of three-body interactions with high mass ratios, to represent intermediate-mass black holes, and (3) detailed numerical analysis of binary-binary encounters and the role of the Kozai resonance, among others. Whatever the results of this work, there will be significant new understanding gained on many fronts, including the information that can be obtained from analysis of the waveforms of the gravitational radiation produced by black hole mergers in dense clusters.

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