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PHASE LAGS IN CYGNUS X-1

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ABSTRACT

Coronae of hot electrons are ubiquitous features in models of the inner regions of accreting black holes and nonmagnetic neutron stars. The scattering optical depth of these coronae inferred from observations is often $\tau \sim 3$, so the energy spectrum of the disk in these regions is likely masked by the spectrum of upscattered photons in the corona and some of the disk properties are thus obscured. Observations of the dependence on Fourier frequency of phase or time lags between photons of different energies provide a window onto the disk not available with the energy spectrum. In our picture, the disk emission is modeled as a photon source injected into a Comptonizing corona; Comptonization may also occur in the disk, but for our purposes the disk emission is simply an input to the corona. We show that, contrary to some claims, the functional dependence of lag on Fourier frequency emerges intact from transit through the corona, modulo a multiplicative factor (which may in principle be negative, so that a phase lag can be changed to a phase lead), even if the properties of the corona vary with time. We also show that any frequency dependence of the lag due to variation in the corona itself is only second order in the amplitude of the variation, and cannot exceed the transit time ~ms of the corona; thus, the lags of up to 0.1 s seen in several black hole candidates come from lags in the emission from the disk. Finally, we predict that plots of the time lag versus Fourier frequency in black hole candidates should have a "shelf" of constant lag equal to the coronal lag (~1 ms), with the constant being proportional to $\ln(E_2/E_1)$ for the lag between energies E_2 and E_1. The lack of such a shelf in current observations of several galactic black hole candidates constrains the radii of the coronae to be $R \lesssim 10^8 \tau$ cm.

Subject headings: accretion, accretion disks — black hole physics — radiation mechanisms: nonthermal — stars: coronae — stars: individual (Cygnus X-1) — stars: neutron

1. INTRODUCTION

Most models of the inner regions of accreting black holes and nonmagnetic neutron stars include a corona of hot electrons, which is assumed to upscatter disk photons (see, e.g., Sunyaev & Titarchuk 1980). The ubiquity of these coronae in models is due in part to their success in matching qualitatively the high-energy spectra of many galactic black hole candidates such as Cyg X-1 and GX 339−4 when those candidates are in the X-ray low state, which is characterized by a hard X-ray tail. The early models, such as the Sunyaev-Titarchuk model, employed simplifying assumptions (such as time independence and a uniform temperature) to facilitate a semianalytic treatment. Some subsequent treatments have included effects of a time-varying source (Brainerd & Lamb 1987; Wijers, van Paradijs, & Lewin 1987; Kylafis & Klimis 1987; Bussard et al. 1988; Kylafis & Phinney 1989) or of a time-varying corona (Stollman et al. 1987; Miller & Lamb 1992). These analyses have shown that an optically thick corona decreases the amplitude of luminosity variations coming from a source inside the corona, and that oscillation of the corona itself can produce observable variations.

It has been suggested (Miyamoto et al. 1988) that coronal oscillation is also required to explain the dependence of phase lags between specified photon energies on the Fourier frequency of the analysis. Observations of Cyg X-1 in the low state (Miyamoto et al. 1988, 1992; van der Klis 1994) have revealed that hard X-rays lag soft X-rays by a phase that depends weakly on the frequency of variation, from 0.05 rad at 0.1 Hz to 0.1 rad at 10 Hz. Models of time-independent Comptonization predict that the phase lag should be proportional to the frequency for time lags much greater than the typical transit time of the corona $t_c \sim (R/c)\max[1, \tau^2] \sim 1$ ms (Payne 1980); thus Comptonization cannot by itself explain the lags.

It has also been claimed (Miyamoto et al. 1988) that high-energy gain Comptonization would smear out phase information. If true, this would require, e.g., oscillation in the corona itself to provide the observed lags (Stollman et al. 1987; Miller & Lamb 1992), and we would be robbed of a useful diagnostic of the conditions in the inner disk.

We show that this is not the case, and that under realistic conditions the shape of the phase dependence on frequency will *not* be altered by Comptonization, though the magnitude and sign of the phase lag may change. This is true even if the Comptonizing corona is itself time dependent. We also show that the time lags introduced by transit through the corona are just the difference between the propagation times for photons of different energy and therefore are not likely to exceed a few milliseconds for coronae around stellar-mass black holes. We demonstrate that, for given corona parameters, the corona-induced delay should scale as $\ln(E_2/E_1)$ for photon energies E_1 and E_2, so that plots of time or phase lag versus frequency can constrain both the temporal structure of the disk emission and the parameters of the Comptonizing corona, especially if simultaneous measurements are made over a wide range of energies.

In § 2 we consider a varying input to a constant corona. We assume that the brightness variation at a given energy and frequency has an amplitude and phase which can be expressed as products of a function of energy and a function of frequency, at least over the energy range where the redistribution function is large. Because the energy redistribution function for Comptonization is fairly peaked (see, e.g., Sunyaev & Titarchuk 1980), this is likely to apply to hot coronae around galactic black hole candidates. With these assumptions, we show that the functional dependence of lag on frequency is maintained through coronal transit. In § 3 we consider a constant input to a varying corona and demonstrate that any dependence of phase lag on frequency requires oscillation at that frequency, and that the only frequency-dependent contributions are second order and higher in the amplitude of the oscillation. As one concrete example of time lags introduced by transit through a corona, we consider the Payne (1980) model of photons diffusing through a cloud of optical depth $\tau \gg 1$ and temperature $kT \gg E$, where E is the photon energy. The results of the section are, however, applicable in other limits as well. We also explore the observability of the time lag due to transit of the corona by combining it with a sample dependence of the time lags from the disk on the Fourier frequency. In § 4 we show that the dependence of phase lag on frequency inherent to the disk will be observed even if the corona oscillates, as long as the time lag in the emission from the disk is much longer than the transit time of the corona and the corona parameters repeat with a period much shorter than the observation period. The results are summarized in § 5.

2. TIME-DEPENDENT INPUT TO A TIME-INDEPENDENT CORONA

Consider a source of photons whose initial brightness variation at energy E and frequency ω is described by

$$I_0(E, t) = A(E, \omega) \sin (\omega t + \phi[E, \omega]) , \quad (1)$$

where $A(E, \omega)$ is the amplitude and $\phi[E, \omega]$ is the phase. At a given frequency ω, the phase is measured relative to some reference energy $E_{\rm ref}$. Passage through a Comptonizing corona will alter this brightness variation by both an intrinsic delay $\delta t(E', E)$, which depends on the initial energy E' and the final energy E but not on ω, and by the redistribution function $R(E, E')$, defined such that the probability that a photon of initial energy E' ends up with an energy between E and $E + dE$ is $P(E) = R(E, E')dE$. The time variation of brightness at energy E after transit of the corona is

$$\begin{aligned} I(E, t) = &\int_0^\infty \int_0^\infty dE'\, d\omega A(E', \omega) \sin \{\omega[t - \delta t(E', E)] \\ &+ \phi[E', \omega]\} R(E, E') \\ = &\int_0^\infty d\omega \Big\{ \sin \omega t \int_0^\infty dE' A(E', \omega) \cos (\phi[E', \omega] \\ &- \omega \delta t[E', E]) R(E, E') \\ &+ \cos \omega t \int_0^\infty dE' A(E', \omega) \sin (\phi[E', \omega] \\ &- \omega \delta t[E', E]) R(E, E') \Big\} . \end{aligned} \quad (2)$$

The phase Φ at energy E and frequency ω' after passage through the corona is given implicitly by

$$\sin (\Phi[E, \omega']) = \frac{a(E, \omega')}{b(E, \omega')\sqrt{1 + [a(E, \omega')/b(E, \omega')]^2}} , \quad (3)$$

where

$$\begin{aligned} a(E, \omega') &= \frac{1}{\pi} \int_{-\infty}^{\infty} I(E, t) \cos \omega' t\, dt \\ b(E, \omega') &= \frac{1}{\pi} \int_{-\infty}^{\infty} I(E, t) \sin \omega' t\, dt . \end{aligned} \quad (4)$$

For an observing time much longer than the periods of interest (as in the *Ginga* observations of Cyg X-1; Miyamoto et al. 1988),

$$\frac{a(E, \omega')}{b(E, \omega')} = \frac{\int_0^\infty dE' A(E', \omega') \sin (\phi[E', \omega'] - \omega' \delta t[E', E]) R(E, E')}{\int_0^\infty dE' A(E', \omega') \cos (\phi[E', \omega'] - \omega' \delta t[E', E]) R(E, E')} . \quad (5)$$

The phase lags in Cyg X-1 are ≈ 0.1 radians between 0.1 s and 20 s, so $\phi \ll 1$ is a good assumption. The delay time δt is of the order of the propagation time across the corona, which for a 10^8 cm corona is 3 ms, compared to 16 ms for 0.1 radians at 1 Hz. We therefore make the approximation that $\phi \ll 1$, so that $\sin \phi \approx \phi$ and $\cos \phi \approx 1$, and expand to first order in $\omega' \delta t/\phi$; because δt does not depend on the frequency, the first-order correction term becomes less significant for lower frequencies. If we assume further that the initial phase and amplitude dependence on frequency is independent of amplitude, so that

$$\phi(E, \omega) = e(E) f(\omega) \quad \text{and} \quad A(E, \omega) = g(E) h(\omega) ,$$

then

$$\begin{aligned} \frac{a(E, \omega')}{b(E, \omega')} &\approx \frac{\int_0^\infty dE' g(E') [f(\omega') e(E') - \omega' \delta t(E', E)] R(E, E')}{\int_0^\infty dE' g(E') R(E, E')} \\ &\equiv G(E) f(\omega') \left[1 + \mathcal{O}\left(\frac{\omega' \bar{\delta} t}{\phi} \right) \right] \ll 1 , \end{aligned} \quad (6)$$

where $G(E)$ is some function of energy, $\bar{\delta} t$ is the appropriately averaged delay time, and the inequality holds because of the assumption $\phi \ll 1$. The second term in parentheses effectively adds a component that goes linearly with frequency and (depending on the size of the corona) may be significant at 10 Hz; this term corresponds to the "shelf" discussed in § 3. For simplicity, however, we will assume in the rest of this section that the term is small and may be ignored.

With this simplification, if before scattering the phase difference between different energies was

$$\Delta \phi = f(\omega)[e(E_1) - e(E_2)] , \quad (7)$$

after scattering the difference is

$$\Delta \Phi = f(\omega)[G(E_1) - G(E_2)] . \quad (8)$$

The dependence on ω is the same, though the magnitude may have changed. Therefore, if the prescattering dependence of amplitude and phase on frequency is independent of energy, and if the phase lags are small, then the form of the variation with frequency of the phase lag between given energies is unchanged by Comptonization. For the functional dependence of phase lag on frequency to change, the photons must have been produced in regions with either large phase differences or qualitatively different dependences of phase on Fourier frequency.

If the phase lag is nearly independent of the frequency, as it is in the low states of Cyg X-1, GX 339−4, and GS 2023+338 (Miyamoto et al. 1992), where the lag changes by only a factor of 2 from a frequency of 0.1 Hz–10 Hz, the lag need not even be small to retain frequency independence. Moreover, the redistribution function for Comptonization is fairly peaked (Sunyaev & Titarchuk 1980). If the redistribution function $R(E, E')$ can be characterized by a narrow peak of minimum energy E_0 and width ϵE, where $\epsilon \ll 1$, then

$$\frac{a(E, \omega')}{b(E, \omega')} \approx \frac{\sin(\phi[E_0, \omega'])}{\cos(\phi[E_0, \omega'])} + \mathcal{O}\left(\epsilon \frac{d \ln|\tan \phi[E, \omega']|}{d \ln E}\bigg|_{E_0}\right), \quad (9)$$

so that $\Phi(E, \omega') = \phi(E_0, \omega') + \mathcal{O}(\epsilon)$. Thus, to change the functional form of the frequency dependence, the phase must vary more rapidly with energy than the redistribution function does.

These principles are illustrated in Figure 1, which plots the phase lag versus frequency before and after transit through a Compton corona with $\tau = 3$ and $kT = 50$ keV for different values of the phase. This figure demonstrates that, for amplitudes and phases separable into the product of a function of energy and a function of frequency, if the phases are small, the functional dependence of phase lag on frequency survives through Comptonization (modulo a multiplicative factor), regardless of the dependence of phase on energy. Expressed mathematically, the function $F(\omega) \equiv d \ln|\Delta\phi|/d\omega$ is unchanged by the Comptonization. If the phases are not small, the dependence on frequency is still retained if the phase changes slowly enough with energy. Note that in either case the multiplicative factor can in principle be negative, as shown in Figure 1*c*, where a phase lag is changed to a phase lead; however, in practice the sign of the phase lag is unlikely to be changed by Comptonization.

3. TIME-INDEPENDENT INPUT TO A TIME-DEPENDENT CORONA

Consider now the converse of the previous situation, in which a time-independent source of photons is processed

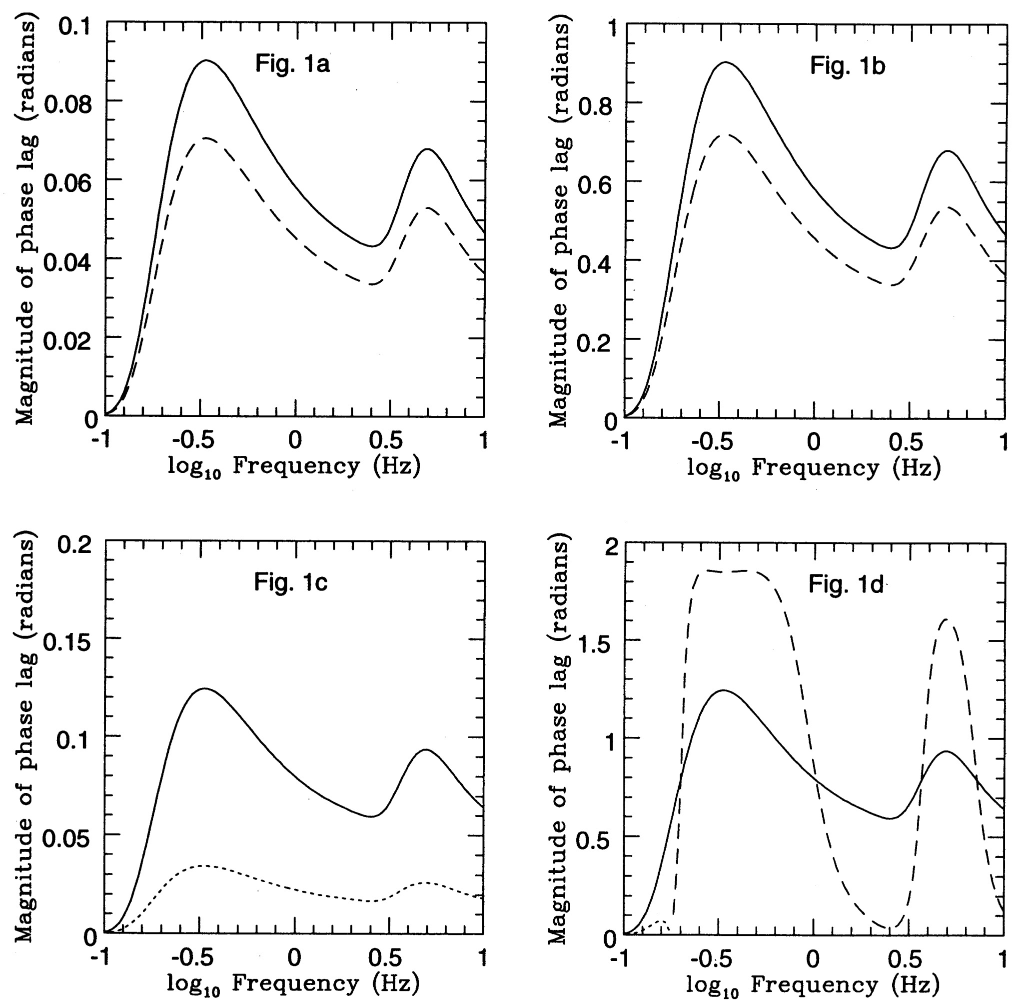

FIG. 1.—Phase lags vs. frequency of variation. Fig. 1*a* compares the phase lag of 5 keV X-rays relative to 2.5 keV X-rays vs. frequency before (*solid line*) and after (*dashed line*) transit through a corona of optical depth $\tau = 3$ and temperature $kT = 50$ keV, assuming an amplitude of the form $A \sim E^3/(\exp[E/2.5 \text{ keV}] - 1)$. The initial phase-frequency relation is two-humped and intended to reproduce qualitatively the dependence found (Miyamoto et al. 1992) in the low state of several galactic black hole candidates, and the energy dependence is $e(E) \sim \ln(E/1 \text{ keV})$, similar to that found for the quasi-periodic oscillation (QPO) in GS 1124−68 (van der Klis 1994; Takizawa et al. 1994). Fig. 1*b* is the same as Fig. 1*a*, except that the phase lags are a factor of 10 greater. In Fig. 1*c*, the initial energy dependence of the phase is changed to $e(E) \sim \sin(E/500 \text{ eV})$, and the other parameters are the same as in Fig. 1*a*; the phase lag after transit is represented by a dotted line to indicate that the phase lag has been changed to a phase lead. This energy dependence is unrealistic but is chosen to illustrate a phase that changes rapidly with energy, and to show that *in principle* the sign of the lag may be changed by Comptonization, though in practice this is unlikely. Fig. 1*d* is identical to Fig. 1*c* but with the phase lags increased by a factor of 10. The dotted line from ~0.1 Hz to 0.2 Hz indicates a phase lead, and the dashed line from ~0.2 Hz to 10 Hz indicates a phase lag.

through an oscillating corona. Numerical simulations (Stollman et al. 1987) have shown that in this case, as in the case of a time-independent corona, the main cause of delay between high- and low-energy photons is the time t_{max} needed to reach the maximum brightness at energy $E \ll kT$ (where T is the electron temperature) from an instantaneous input of photons of energy $E_0 \ll E$. For an electron scattering optical depth $\tau \gg 1$ and a Compton parameter $y \equiv (4kT/m_e c^2)\tau^2 < 3\alpha \ln E/E_0$, where m_e is the electron mass and $\alpha = \pi^2/3$ for a sphere, Payne (1980) found that for photons diffusing through a corona of uniform density and temperature

$$t_{max} \approx \frac{R}{\tau c} \frac{m_e c^2}{kT} \ln\left(\frac{E}{E_0}\right) \frac{1}{2} \left(\frac{9}{4} + \frac{4\alpha}{y}\right)^{-1/2} , \quad (10)$$

where R is the radius of the corona and this expression has been rewritten slightly. For parameters believed typical of accreting stellar-mass black holes, $t_{max} \sim$ 1–10 ms. The numerical value of t_{max} depends on the specific model, but the $\ln(E/E_0)$ dependence is more general, and comes from the assumption that, for $kT \gg E$, each scattering produces a secular increase in the photon energy (Kompaneets 1957; Payne 1980).

If the coronal properties are now allowed to oscillate, t_{max} will change accordingly, with its behavior dependent on what variable oscillates and the value of y. For example, if only τ varies, then if $y \ll 1$, $t_{max} \sim \tau$, whereas if $y \gg 1$, $t_{max} \sim$ const. If only T varies, then for $y \ll 1$, $t_{max} \sim T^{-1/2}$ and for $y \gg 1$, $t_{max} \sim T^{-1}$. Similarly, one could postulate a variation changing only the number density, or one where the physical size of the corona oscillates. Each type of variation changes t_{max} differently.

Nonetheless, regardless of how the corona varies, if the variation timescale is much greater than t_{max}, then the phase at a given energy and frequency is unchanged to first order in the amplitude of the variation. To show this, call the varying combination of parameters x, and assume it changes in time like

$$x = x_0 + x_1 \sin \omega t , \quad (11)$$

where $\omega t_{max} \ll 1$ (it is straightforward to extend this treatment to a sum of oscillations). The distribution of transit times for photons of initial energy E_0 and final energy $E \gg E_0$ is well approximated by a δ-function (Kylafis & Klimis 1987). Thus, at a given energy E, the brightness as a function of time depends on the value of x a time t_{max} earlier:

$$I(t) = I\{x[t - t_{max}(x)]\} , \quad (12)$$

where

$$x[t - t_{max}(x)] = x_0 + x_1 \sin \omega(t - t_{max}) \approx x_0 + x_1(\sin \omega t - \omega t_{max} \cos \omega t) . \quad (13)$$

To second order,

$$I(t) = I(x_0) + x_1(\sin \omega t - \omega t_{max} \cos \omega t) \left.\frac{\partial I}{\partial x}\right|_{x0} + \frac{1}{2} x_1^2(\sin^2 \omega t - 2\omega t_{max} \sin \omega t \cos \omega t + \omega^2 t_{max}^2 \cos^2 \omega t) \left.\frac{\partial^2 I}{\partial x^2}\right|_{x0} , \quad (14)$$

where

$$t_{max} = t_{max}(x_0) + x_1[\sin \omega t - \omega t_{max}(x_0) \cos \omega t] \left.\frac{\partial t_{max}}{\partial x}\right|_{x0} . \quad (15)$$

As in the previous section, the phase depends on $a(E, \omega)/b(E, \omega)$, where $a(E, \omega)$ and $b(E, \omega)$ are defined in equation (4). In the limit of infinitely long integration times,

$$\int_{-\infty}^{\infty} C(\omega t) \sin \omega t \, dt = \int_{-\infty}^{\infty} C(\omega t) \cos \omega t \, dt = 0 , \quad (16)$$

if $C(\omega t)$ is a product of an even number of sines and cosines of ωt. Thus, in the above expression for I, all terms proportional to $\sin^2 \omega t$, $\cos^2 \omega t$, or $\sin \omega t \cos \omega t$ integrate to 0. Dropping these terms, we find finally that

$$\frac{a(E, \omega)}{b(E, \omega)} \approx -\omega t_{max}(x_0) + \mathcal{O}(x_1^2) . \quad (17)$$

Thus any change in phase lags or leads between hard and soft X-rays is second order in the oscillation amplitude. The maximum phase lag at a given energy and Fourier frequency is just $\omega[t_{max}(E_1) - t_{max}(E_2)]$, just proportional to the difference in propagation times through the corona. Observations of Cyg X-1 and other galactic black hole candidates show that at low frequencies ($\sim$0.1 Hz), the time delay between different energies can be as great as 0.1 s. For this to be caused by delays in the corona, the corona would have to be extremely large, with a radius $R \sim 10^9$ cm (see Lamb 1988 for further discussion of this point). Note in particular that increasing the optical depth τ to large values will not result in a greatly increased delay, because although the *average* time of propagation of a photon through the corona will go like τ^2 in the optically thick limit, a photon emerging with a *given* energy E has experienced a fixed number of scatterings, which does not increase with τ. Thus, though the brightness at a given E can change substantially with τ, the time delay does not.

Models of stellar-mass black holes with 10^9 cm Comptonizing coronae do exist (Miyamoto & Kitamoto 1991), but in this case the delay between X-rays of different energies would be expected to be at least of order the transit time. If, as in the Miyamoto & Kitamoto (1991) model, the coronal optical depth is less than unity (0.5–1 in this case), the time variation would be a superposition of the original, unscattered photons and those photons that interacted with the corona and were therefore delayed. The lag between energies would thus be intermediate between the scattered and unscattered lags; for phase lags $\phi_1, \phi_2 \ll 1$ and relative amplitudes A_1 and A_2, the observed phase difference is $\phi \approx (A_1\phi_1 + A_2\phi_2)/(A_1 + A_2)$. For an optical depth $\tau \sim$ 0.5–1 the scattered and unscattered amplitudes are comparable, so the expected minimum time lag for a 10^9 cm corona is $\sim \frac{1}{2}R/c \approx 15$ ms. This is not the case (the minimum time lags are on the order of 1–2 ms), so for these sources processes inherent to the disk are more likely to produce the observed variation of lag with frequency. However, the time delays at higher frequencies ($\sim$10 Hz), which can be $\sim$2 ms, may well be caused by Comptonization delays. In fact, since typically $t_{max} \sim \ln(E/E_0)$, then if the amplitude of oscillations is small the predicted time delay between photons of energy E_1 and E_2 should go as $\ln(E_2/E_1)$. Since the total time lag is roughly the sum of the lags from the disk and from the corona, this model predicts that the time lag will not drop significantly below Δt_{max}. Thus, time lag versus frequency plots with different pairs of energies should display a

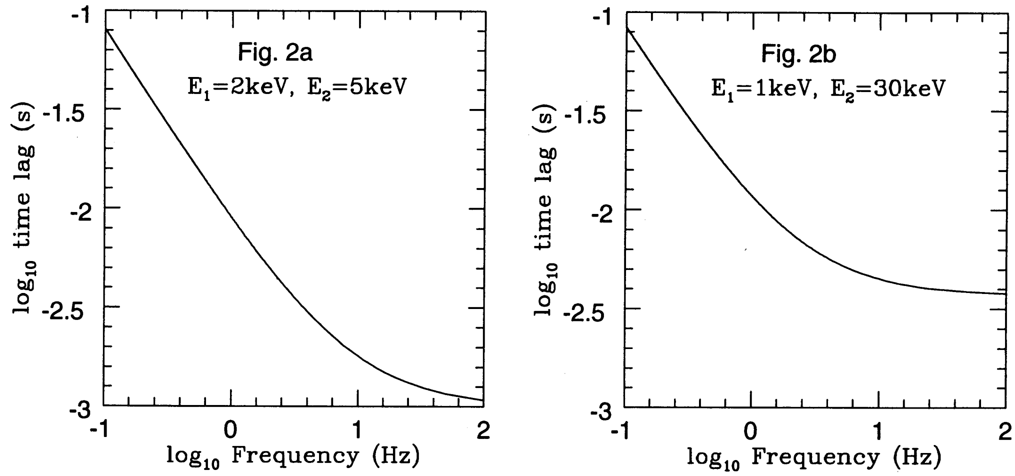

FIG. 2.—Qualitative sketch of the dependence of time lags between energies on the energies considered. In (*a*) the energies are 5 keV and 2 keV, and the corona parameters are chosen so that the delay induced by Comptonization is 1 ms. In addition, it is assumed that the phase lag from the disk is a constant 0.05 radians. As a result, the time lag is a constant at high frequencies, but goes as the reciprocal of the frequency at low frequencies. In (*b*) the corona and disk parameters are the same as in (*a*), but the energies considered are now 30 keV and 1 keV. Because the predicted delay goes as $\ln(E_2/E_1)$, the delay induced by the corona is 4 times that in (*a*), or 4 ms. The resulting "shelf" is at a higher time lag, and exists down to lower frequencies, than in (*a*).

"shelf" below which the lag does not drop, with the value of the lag at the shelf proportional to $\ln(E_2/E_1)$.

For example, let $\tau = 3.1$ and $kT = 36.4$ keV, which is what was derived for GX 339−4 (Grabelsky et al. 1994). For a corona of radius $R = 3 \times 10^7$ cm, the time delay between different energies is then $t_{max} \approx 1 \ln(E_2/E_1)$ ms. For a given pair of energies, this constant time lag will be added to whatever time lag came from the disk. In Figure 2 we show the total time lag assuming that the phase lag from the disk is a constant 0.05 radians, which is a rough approximation of the behavior seen in Cyg X-1, GX 339−4, and GS 2023+338 (Miyamoto et al. 1992) and GRO J0422+32 (Grove 1994). For closely spaced energies, as in Figure 2*a*, the shelf is barely detectable, but for energies with a large ratio, as in Figure 2*b*, the shelf is at several milliseconds, potentially within the reach of satellites such as *ASCA* or OSSE. Observation of a shelf, combined with estimates of τ and kT from the energy spectrum, will yield values or upper limits for the radius R of the corona, but only if a wide range of photon energies is seen and timed simultaneously.

4. TIME-DEPENDENT INPUT TO A TIME-DEPENDENT CORONA

If both the disk emission and the properties of the Comptonizing corona vary, what can be said about the emergent phase lags as a function of frequency? As in § 3, the corona itself cannot introduce time lags greater than the transit time across the corona, so for time lags from the disk greater than this time the only way that the corona could change the lag-frequency relation is by redistributing the photon energies in a time-dependent way.

Assume that photons emitted from the disk may be treated independently of each other, so that only photon-electron scattering need be considered. Other processes, such as induced Compton scattering and photon-photon scattering, are negligible (see, e.g., Itzykson & Zuber 1980, p. 358 for a treatment of photon-photon scattering). Assume also that the redistribution function is a function of time that repeats after a time T; thus, $R(E, E', t) = R(E, E', t + T)$ for any time t. Then, from § 2,

$$a(E, \omega') = \frac{1}{\pi} \int_{-\infty}^{\infty} I(E, t) \cos \omega' t \, dt = \frac{1}{\pi} \int_0^T \left[\sum_{n=-\infty}^{\infty} I(E, t_0 + nT) \cos \omega'(t_0 + nT) \right] dt_0 , \quad (18)$$

where

$$I(E, t_0 + nT) = \int_0^{\infty} \int_0^{\infty} dE' \, d\omega A(E', \omega) \times \sin \{\omega(t_0 + nT) + \phi[E', \omega]\} R(E, E', t_0) . \quad (19)$$

If we assume as before that $\phi(E, \omega) \ll 1$, and bring the summation sign inside the integrals, then the summation becomes

$$\sum_{n=-\infty}^{\infty} \sin [\omega(t_0 + nT) + \phi] \cos \omega'(t_0 + nT) = \sum_{n=-\infty}^{\infty} [\sin \omega(t_0 + nT) \cos \omega'(t_0 + nT) + \phi \cos \omega(t_0 + nT) \cos \omega'(t_0 + nT)] . \quad (20)$$

The only significant contribution comes from the second term when $\omega = \omega'$, so

$$a(E, \omega') = \frac{1}{\pi} \int_0^T dt_0 \int_0^{\infty} dE' A(E', \omega') \phi(E', \omega') R(E, E', t_0) \times \sum_{n=-\infty}^{\infty} \cos^2 \omega'(t_0 + nT) . \quad (21)$$

If we assume as before that $\phi(E', \omega') = e(E') f(\omega')$ and $A(E', \omega') = g(E') h(\omega')$, we then find that, as before,

$$\frac{a(E, \omega')}{b(E, \omega')} = f(\omega') G(E) \quad (22)$$

for some function $G(E)$. Therefore, even if the corona oscillates, the dependence of phase lag on frequency is unchanged by passage through the corona if the time lag is greater than the transit time.

5. CONCLUSIONS

The message of this paper reduces to three basic ideas. First, the phase lags in the emission from the disk emerge from Comptonization unchanged except for a scale factor (which may in principle be negative), even if the properties of the corona vary with time. Second, any frequency dependence of the coronal lag can come only from large-amplitude variation of the corona. Third, a generic plot of observed time lag versus Fourier frequency includes a "shelf" of constant lag equal to the coronal lag, with this constant being proportional to $\ln(E_2/E_1)$ for energies E_2 and E_1.

In more detail, the dependence of phase lags on Fourier frequency in the disk emission for time lags longer than ~ 1 ms is maintained through Comptonization if (*a*) the dependence of phase and amplitude on frequency does not change significantly in the energy range sampled by Comptonization and either (*b*) the phase lags are small enough that $\sin\phi \approx \phi$ and $\cos\phi \approx 1$ or (*c*) the phase does not change significantly over the Comptonization peak. If the coronal properties oscillate, then in addition the corona parameters must repeat with a period much less than the time of observation, or the oscillation amplitude must be small.

The time lag produced by transit through a Comptonizing corona comes for the difference in propagation times between photons of different energies. If the corona parameters are time-independent, the time lag produced by the corona does not depend on the Fourier frequency of analysis. If the corona parameters vary, the lag depends on frequency only as the square of the amplitude of the variation at the given frequency. For a corona whose parameters are independent of location, the time lag between energies E_1 and E_2 is proportional to $\ln(E_2/E_1)$. Lags longer than this are caused by the disk. In general, then, a graph of time lag versus Fourier frequency will consist of a model-dependent disk contribution superposed on a constant coronal lag proportional to $\ln(E_2/E_1)$.

Observations of Cyg X-1, GX 339−4, and GS 2023+338 (Miyamoto et al. 1992) and GRO J0422+32 (Grove 1994) all reveal time lags that are proportional to the reciprocal of the frequency and which have values as large as 0.1 s for energies differing by a factor of 2. These lags must come from the disk emission and are not the result of processing in a Comptonizing corona. The uniformity of this behavior, among four different black hole candidates, can be used to constrain models of the inner disk. Moreover, because none of the observations show a shelf above ~ 3 ms, the corona radius is constrained to be $R < 10^8\tau$ cm. Simultaneous gamma-ray and X-ray timing observations of a black hole candidate could improve this limit by a factor of ~ 4.

In conclusion, though optically thick coronae obscure the energy spectra of the inner parts of accretion disks, the temporal variations of the disk emerge largely unscathed. The observation of phase lags over a wide range of energies thus provides both a window onto the disk and an independent way of estimating properties of the corona.

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