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EFFECT OF RADIATION FORCES ON DISK ACCRETION BY WEAKLY MAGNETIC NEUTRON STARS

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ABSTRACT

Radiation forces are shown to be more important than general relativistic corrections to Newtonian gravitational forces in determining the motion of particles accreting onto a slowly rotating neutron star if the luminosity of the star is greater than $\sim 1\%$ of the Eddington critical luminosity L_E^∞. This is so even if the radius of the star is less than the radius of the innermost stable orbit. In particular, radiation drag causes matter accreting from a disk to lose angular momentum and spiral inward. At luminosities greater than $\sim 0.2L_E^\infty$, a substantial fraction of the accreting matter can transfer most of its angular momentum and gravitational binding energy to the radiation field before reaching the stellar surface. These results have important implications for the X-ray spectra, time variability, and spin evolution of neutron stars with very weak magnetic fields and the prospects for detecting general relativistic effects near such stars.

Subject headings: accretion, accretion disks — relativity — stars: neutron

1. INTRODUCTION

Some accreting neutron stars in low-mass X-ray binary systems (LMXBs) may have magnetic fields so weak that they do not significantly affect the motion of accreting matter. Prime candidates for such "nonmagnetic" stars include the so-called atoll sources and other type 1 burst sources (see Hoshi 1984; van Paradijs 1991; van der Klis 1992). It has generally been assumed that nongravitational stresses are dynamically unimportant near such stars, and hence that the inner edge of the Keplerian flow around them is determined either by interaction of the flow with the stellar surface, if the equatorial radius R_{eq} of the neutron star is greater than the radius R_{ms} of the innermost stable circular orbit, or by general relativistic effects, if $R_{eq} < R_{ms}$ (see, e.g., Kluźniak & Wagoner 1985; Czerny, Czerny, & Grindlay 1986; Sunyaev & Shakura 1986). The assumption that nongravitational stresses are dynamically unimportant has been used to calculate the X-ray spectrum, spin-up rate, and other properties of such stars (see, e.g., Kluźniak & Wagoner 1985; Sunyaev & Shakura 1986; Hanawa 1989; Fu & Taam 1990; Czerny, Czerny, & Grindlay 1986; Ebisawa, Mitsuda, & Hanawa 1991; Kluźniak & Wilson 1991; Biehle & Blandford 1993). It has also been suggested that general relativistic effects may be detectable via their dominant effect on the accretion flow near nonmagnetic neutron stars, if $R_{eq} < R_{ms}$ (see, e.g., Kluźniak & Wagoner 1985; Kluźniak, Michelson, & Wagoner 1990).

Of course, matter accreting onto a neutron star experiences a variety of nongravitational forces. In particular, radiation forces are unavoidable near such a star. Previous work (Lamb 1989, 1993; Fortner, Lamb, & Miller 1989, 1993) has shown that radiation drag (sometimes called Poynting-Robertson drag) can have an important effect on the motion of matter far from the star when the luminosity of the star is very close to the Eddington critical luminosity L_E^∞. This effect plays an important role in creating a radial component of the accretion flow in the unified model of the X-ray properties of the Z sources (Lamb 1989, 1993). However, at luminosities less than $\sim 0.9L_E^\infty$, radiation drag has relatively little effect on matter orbiting far from the star. The possible importance of radiation forces in changing the accretion flow near a neutron star undergoing an X-ray burst was pointed out by Walker & Mészáros (1989). Aspects of this problem has been considered further by Walker (1992).

In this *Letter* we show that radiation forces are more important than general relativistic corrections to Newtonian gravitational forces in determining the motion of matter accreting onto a slowly rotating neutron star if the luminosity of the star is greater than $\sim 0.01L_E^\infty$.This is so even if the radius of the star is less than the radius of the innermost stable orbit. In particular, radiation drag causes matter accreting from a disk to lose angular momentum and spiral inward. The motion of accreting matter near the star therefore resembles the motion expected when R_{eq} is substantially less than R_{ms}, even if $R_{eq} > R_{ms}$. At luminosities greater than $\sim 0.2L_E^\infty$, a substantial fraction of the accreting matter can transfer most of its angular momentum and gravitational binding energy to the radiation field before reaching the stellar surface. For sufficiently high luminosities, radiation drag limits the radial velocity of the matter flowing toward the star. The persistent luminosities of most X-ray burst sources are $\gtrsim 0.01L_E^\infty$ while those of the most luminous LMXBs are $\sim L_E^\infty$; peak luminosities during X-ray bursts are also $\sim L_E^\infty$ (van Paradijs 1991; van der Klis 1992). Interaction between the radiation field and the accretion flow near these stars therefore has an important effect on their X-ray spectra, time variability, and spin evolution.

In § 2 we describe numerical calculations of the motion of test particles in the Schwarzschild geometry outside a nonrotating, nonmagnetic but radiating spherical star and summarize our results. In § 3 we discuss briefly some of the implications of our results for the X-ray spectra and spin-up rates of neutron stars in LMXBs and the prospects for detecting relativistic effects near such stars.

2. CALCULATIONS AND RESULTS

In the calculations reported here we assume that the only source of radiation is uniform, isotropic emission from the star's surface. The radiation stress produced by emission concentrated more toward the stellar equator would be greater; on the other hand the stress produced by radiation coming from the inner part of the disk may partially offset that produced by the radiation coming from the star.

Our numerical code can be used to compute photon trajectories, the radiation energy tensor, and the orbits of particles accelerated by gravitational and radiation forces in an arbitrary spacetime. We have used this code to calculate a wide variety of test particle trajectories by first computing the energy tensor of the radiation field outside the star and the luminosity at infinity by launching beams of radiation from the surface of the star and tracing their trajectories along null geodesics to large radii. The motion of each test particle was then determined by integrating the geodesic equation of motion with radiation forces included (see, e.g., Abramowicz, Ellis, & Lanza 1990, eqs. [3.12]–[3.14]).

For the Schwarzschild geometry considered here, our numerical results for the rate of increase of binding energy and loss of angular momentum by a test particle in a circular orbit in the radiation field produced by a star of radius $R \geq 3M$ agree with the exact analytical expressions (here and below we set $c = G = 1$ except where noted)

$$\frac{1}{\tilde{E}_b}\frac{d\tilde{E}_b}{d\tau} = +f(\alpha)\left(\frac{1-\tilde{E}_b}{\tilde{E}_b}\right)\frac{M\tilde{l}^2}{3R^2r^2}\frac{(1-2M/R)}{(1-2M/r)^2}\left(\frac{L^\infty}{L_{\rm E}^\infty}\right) \quad (1)$$

and

$$\frac{1}{\tilde{l}}\frac{d\tilde{l}}{d\tau} = -f(\alpha)(1-\tilde{E}_b)^2\frac{M}{3R^2}\frac{(1-2M/R)}{(1-2M/r)^3}\left(\frac{L^\infty}{L_{\rm E}^\infty}\right). \quad (2)$$

Here $\tilde{E}_b$ and $\tilde{l}$ are the specific binding energy and angular momentum of the test particle, τ is the proper time, L^∞ is the luminosity at infinity, $L_{\rm E}^\infty$ is the Eddington luminosity at infinity for the mass and cross section (assumed independent of energy and angle) of the test particles being considered, r and R are, respectively, the orbital radius of the particle and the radius of the star in Schwarzschild coordinates, and $f(\alpha) \equiv (8\text{–}9\cos\alpha + \cos^3\alpha)$, where α is half the apparent angle subtended by the star at the test particle and is given implicitly by

$$\sin\alpha = \left(\frac{R}{r}\right)\left(\frac{1-2M/r}{1-2M/R}\right)^{1/2}. \quad (3)$$

Equations (1)–(3) show clearly the effects of general relativity and the finite angular size of the star on the energy and angular momentum loss rates. For example, the rates of change of the binding energy and angular momentum in the Schwarzschild geometry are, respectively, 50% and 125% greater than in the Newtonian approximation at the surface of a star of radius $6M$, and 100% and 300% greater at the surface of a star of radius $4M$, for the same specific energy $\tilde{e}$. Equation (3) shows that gravitational lensing increases the apparent angular diameter of the star as seen by the test particle by a modest amount; at $6M$, for example, the apparent angular diameter of a star of radius $4M$ is 20% larger than it would be if the spacetime were flat. Equations (1)–(3) show that in general relativity, the finite angular size of the star increases the rates of energy and angular momentum transfer at the surface of the star by a factor of 8/3 over the rates for a point source with the same luminosity, just as in the Newtonian approximation (Guess 1962).

Figure 1*a* shows how the local azimuthal and radial components $v^{\hat\phi}$ and $v^{\hat r}$ of the particle velocity in the static frame vary with r as a test particle spirals inward from an initial radius $R_i = 6M$ toward the surface of a star of radius $4M$. Figure 1*b* shows how these same velocity components vary with r as a test particle spirals inward from $R_i = 9M$ toward the surface of a star of radius $6M$. In all cases the initial azimuthal velocity component $v_i^{\hat\phi}$ was set equal to the azimuthal velocity of a circular orbit at R_i. Curves are shown for four luminosities. All are less than the critical luminosity $0.71L_{\rm E}^\infty$ at which the outward radiation force on a static particle at the surface of a star of radius $4M$ exactly balances the inward force of gravity and all are therefore subcritical everywhere for all the stars considered here.

Except very near R_i, both $v^{\hat\phi}$ and $v^{\hat r}$ are insensitive to the initial radial velocity $v_i^{\hat r}$, as long as it is inward. For example, the radial velocity profiles for $v_i^{\hat r} = -0.001v_i^{\hat\phi}$ and $v_i^{\hat r} = -0.1\,v_i^{\hat\phi}$ differ by less than 15% at $r < 5.5M$ for particles spiraling inward from $6M$ to $4M$ and by less than 15% at $r < 8.6M$ for particles spiraling inward from $9M$ to $6M$. The azimuthal velocity profiles match even more closely. Because of their insensitivity to the particle's initial radial velocity, in this report we present velocity, binding energy, and angular momentum curves only for $v_i^{\hat r} = -0.01v_i^{\hat\phi}$.

Figures 1*a* and 1*b* show that radiation forces significantly affect the motion of particles near the star when the luminosity is greater than $\sim 0.01L_{\rm E}^\infty$. These forces are of two types: a radially outward force, which acts on a particle even if its velocity is zero, and a drag force, which opposes the radial and azimuthal motion of the particle and increases with increasing particle velocity. The sharp decrease in $v_i^{\hat\phi}$ and $v_i^{\hat r}$ with increasing luminosity reflects the decrease in the frequency of circular orbits at R_i caused by the increase in the outward radiation force. As Figure 1*a* shows, particles injected at the radius $6M$ of the marginally stable orbit spiral inward even in the absence of radiation drag, due to the strong gravitational field there. However, radiation drag reduces their azimuthal velocity at $5M$ by a factor of ~ 2 for $L^\infty = 0.2L_{\rm E}^\infty$ and by a factor of ~ 4 for $L^\infty = 0.5L_{\rm E}^\infty$. As a result, the radial velocity at $5M$ is more than 3 times greater if $L^\infty = 0.05L_{\rm E}^\infty$ than if $L^\infty = 0$. For $L^\infty = 0.5L_{\rm E}^\infty$, the radiation drag is so large that the radial velocity reaches a maximum and then decreases toward the star.

In the absence of drag, particles with initial radii $R_i > 6M$ do not spiral inward, because circular orbits at $r > 6M$ are stable. However, as Figure 1*b* shows, even particles with initial radii as large as $9M$ spiral rapidly inward toward a star of radius $6M$ if its luminosity is greater than $\sim 0.01L_{\rm E}^\infty$, due to radiation drag. In fact, for $L^\infty \gtrsim 0.2L_{\rm E}^\infty$, the radial velocities of such particles at $6M$ are comparable to the radial velocities of particles that have spiraled inward from $R_i = 6M$ to the surface of a star of radius $4M$ in the absence of radiation (see Fig. 1*a*).

Comparison of Figures 1*a* and 1*b* shows that *the motion of particles near a star with $R \geq R_{\rm ms}$ is qualitatively the same as that near a star with R substantially less than $R_{\rm ms}$, if the luminosity of the star exceeds a few percent of $L_{\rm E}^\infty$*. Indeed, the motion of particles near a *radiating* star with $R > R_{\rm ms}$ is qualitative similar to that near a *nonradiating* star with R substantially less than $R_{\rm ms}$.

Figure 1*c* illustrates the effect of radiation forces on the binding energy of particles arriving at the stellar surface. Plotted is the specific energy $\tilde{e}$ as a function of L^∞, for particles spiraling from $R_i = 9M$ toward stars of radius $6M$, $5M$, and $4M$. If $L^\infty = 0.2L_{\rm E}^\infty$, the particles spiraling toward a star of radius $6M$ release 68% of their final binding energy *before* reaching the stellar surface, while if $L^\infty = 0.5L_{\rm E}^\infty$, they release

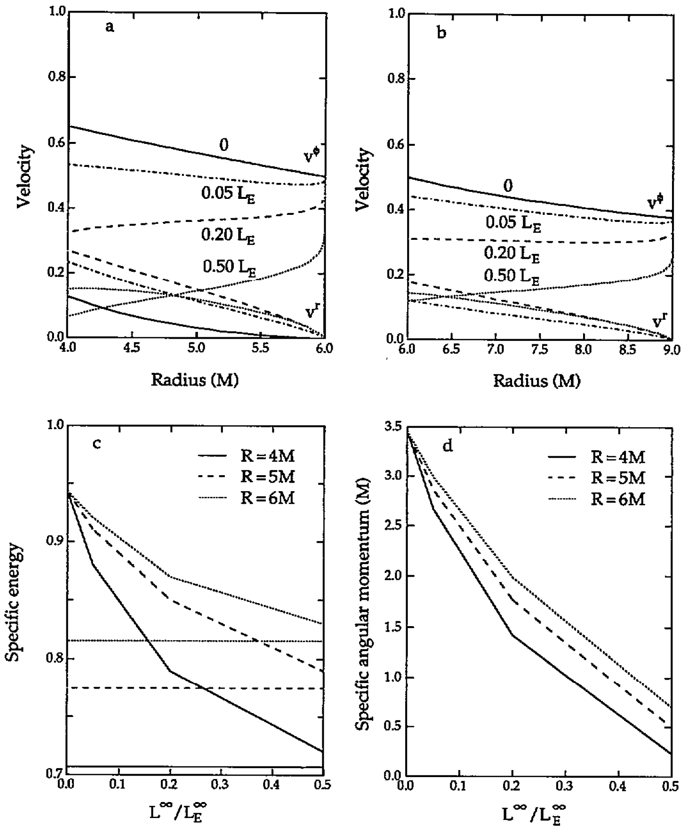

FIG. 1.—Particle trajectories with and without radiation forces. (*a*) Locally measured radial and azimuthal velocity components v^ϕ and v^r (in units of c) as functions of radius, illustrating the effect of radiation forces on the spiral of particles inward from $R_{ms} = 6M$ to a stellar surface at $4M$. Curves are labeled by the luminosity at infinity. (*b*) Same as in (*a*), but for an initial radius of $9M$ and a stellar radius of $6M$. Note the qualitative similarity to (*a*), even though $r \geq R_{ms}$ everywhere. (*c*) Specific energy at impact (in units of c^2) as a function of L^∞ for particles spiraling from $9M$ to the surfaces of stars of radius $6M$, $5M$, and $4M$. The horizontal lines show the specific energy of a particle at rest on the surface of a star of the given radius. (*d*) Specific angular momentum at impact (in units of the gravitational mass M of the star) as a function of L^∞ for the same particle spirals as in (*c*).

93%. The transfer of energy to the radiation is even more pronounced for the particles spiraling toward the star of radius $4M$. If $L^\infty = 0.2L_E^\infty$, they release 71% of their final binding energy before reaching the stellar surface, while if $L^\infty = 0.5L_E^\infty$, they release 96%.

Figure 1*d* shows the effect of radiation forces on the specific angular momentum of particles arriving at the stellar surface. Plotted is the specific angular momentum $\tilde{l}$ as a function of L^∞, for the same particle spirals as in Figure 1*c*. If $L^\infty = 0.2L_E^\infty$, the particles spiraling toward the star of radius $6M$ reach its surface with only $\sim$60% of the angular momentum they would have there in the absence of radiation, while if $L^\infty = 0.5L_E^\infty$, they reach the surface with only $\sim$20% of this angular momentum. Again, the transfer of angular momentum to the radiation is even more pronounced for the particles spiraling toward the star of radius $4M$. If $L^\infty = 0.2L_E^\infty$, these particles arrive at the stellar surface with only $\sim$40% of the angular momentum they would have had in the absence of radiation, while if $L^\infty = 0.5L_E^\infty$, they reach the surface with only 7% of this angular momentum.

3. DISCUSSION

The results presented in the previous section show that radiation forces can dramatically affect the motion of test particles near a nonrotating, nonmagnetic neutron star, if the luminosity of the star is greater than a few percent of the Eddington luminosity. The effect of radiation forces on the flow near a neutron star accreting from a disk depends on the spin of the star, the strength of its magnetic field, the accretion rate and luminosity, and the structure of the flow.

Suppose the persistent luminosity of the star is supplied entirely by accretion, i.e., $L = \epsilon \dot{M} c^2$ for some efficiency ϵ. Then the mass accretion rate $\dot{M}$ is directly related to the luminosity and is greater than a few percent of $\dot{M}_E \equiv L_E/\epsilon c^2$ for luminosities greater than a few percent of L_E. For mass accretion rates this large, the magnetic field of the star is dynamically unimportant if the field strength at the stellar surface is $\ll 10^7$ Gauss (Lamb & Miller 1993). The spin of the star has little effect on the exterior metric and the properties of radiation emitted from the stellar surface if $\Omega \ll c/R \sim 3 \times 10^4$ rad s^{-1}.

To estimate the fraction of the angular momentum of the accreting matter that can be removed by the radiation field in the Newtonian approximation, consider scattering of a radially propagating photon by matter orbiting with velocity $v \ll c$. This interaction will change the photon momentum by an amount $\Delta p_\gamma \approx (E_\gamma/c^2)v$, where E_γ is the photon energy. As a result, the photon will carry away angular momentum $r \times \Delta p_\gamma \approx (E_\gamma/c^2)l$, where l is the specific angular momentum of the matter in the flow. Assuming that a fraction f of the radiation interacts with the matter, the maximum rate at which the radiation can remove angular momentum is $\sim f(L/c^2)l = \epsilon f \dot{M} l$, which is a fraction $\sim f\epsilon$ of the rate of accretion of angular momentum. Near the star, the finite angular size of the radiation source (see § 2) coupled with special and general relativistic effects may increase this fraction to $\sim$2–3$f\epsilon$. These estimates show that the radiation forces produced by the accretion luminosity may be important even if the accretion flow near the star is optically thin (cf. Kluźniak & Wilson 1991) and that neglect of radiation drag (see, e.g., Biehle & Blandford 1993) may lead to inaccurate results. The radiation forces produced by the luminosity of a nuclear flash can be much larger.

Consider now how the effects of radiation forces vary with accretion rate. For $\dot{M} \ll 0.01\dot{M}_E$, the luminosity is low and radiation stresses are weak. Moreover, the accretion flow near the star is likely to be optically thin, in which case f is small and the radiation field is almost unaffected by its interaction with the flow. For $\dot{M} \sim 0.01\dot{M}_E$, it happens that if $\epsilon \sim 0.25$, as expected, the vertical optical depth of the flow near a star of radius $5M$ is of order unity whether or not radiation forces on the flow are included, although for mass fluxes of this size radiation forces are significant, as shown in § 2. Because the optical depth is of order unity, f can be ~ 1, in which case the radiation field is substantially changed by its interaction with the flow. The above estimate of the fraction of the angular momentum of the accreting matter that can be removed by the radiation shows that radiation can carry off much of the angular momentum of the flow near the star.

For accretion rates $\dot{M} \gg 0.01\dot{M}_E$, the flow near the star is likely to be optically thick. In this case radiation forces act primarily on accreting gas within one mean free path of the inner edge and surfaces of the disk. However, the increase in the radial velocity of the gas in these layers caused by rapid removal of angular momentum by radiation drag reduces the density, allowing radiation from the star and boundary layer to penetrate further into the disk. This is likely to produce a more widespread change in the velocity structure of the accretion flow, including the conditions under which supersonic inflow occurs near the star and, when it does, the location of the sonic point (compare Muchotrzeb-Czerny 1986; Abramowicz et al. 1988). Although a quantitative determination of the radiation field and the accretion flow in the optically thick regime requires a self-consistent computation of the radiation and flow, the results of § 2 and the arguments just given show that radiation forces are likely to be very important.

These results have significant implications for the detectability of general relativistic effects near accreting neutron stars. If $R_{eq} < R_{ms}$, general relativity predicts that matter orbiting at $r < R_{ms}$ will spiral inward, even if nongravitational forces are absent (see Misner, Thorne, & Wheeler 1973). This effect is absent in Newtonian gravity and underlies most spectral and temporal phenomena that have been suggested as tests of general relativity, based on test particle calculations that included only gravitational forces (Kluźniak & Wagoner 1985; Kluźniak et al. 1990; Kluźniak & Wilson 1991). Slowly rotating $\sim 1.4\ M_\odot$ neutron stars have $R_{eq} < R_{ms}$ if the equation of state of neutron star matter is relatively soft; depending on the equation of state, $R_{eq} < R_{ms}$ is possible even for rapidly rotating neutron stars with masses near the maximum stable mass (Kluźniak & Wagoner 1985; Friedman, Ipser, & Parker 1986; Kluźniak & Wilson 1991; Miller et al. 1993).

As noted in § 1, matter accreting onto a neutron star experiences a variety of nongravitational forces. If these stresses remove angular momentum from orbiting matter, it will spiral inward even at radii $r > R_{ms}$. Whether the accretion flow changes significantly at R_{ms} then depends on whether matter spirals inward much more rapidly inside R_{ms} than outside. To the extent that magnetic and fluid stresses within the flow can be neglected, streamlines will follow the test particle trajectories computed in § 2. If $L^\infty \gtrsim 0.05 L_E^\infty$, these trajectories are qualitatively the same for accretion by stars with $R > R_{ms}$ as for accretion by stars with $R < R_{ms}$, indicating that the effects of general relativity may be very difficult to detect.

Our results also have important implications for the X-ray spectra, X-ray variability, and spin-up rates of accreting neutron stars. As noted in § 1, it has been generally assumed that Keplerian flow around a neutron star is halted either by interaction with the stellar surface or by general relativistic effects. The results presented here show that radiation forces may be equally or even more important. By removing energy and angular momentum from the flow near the star, radiation forces may extend the region of non-Keplerian flow between the Keplerian disk and the stellar surface and change its structure. Transfer of energy from the accretion flow to the radiation field in such a region may play an important role in producing the X-ray spectra of LMXBs. Moreover, removal of energy from the upstream flow may greatly reduce the heating and X-ray emission caused by particle bombardment or viscous dissipation at the stellar surface (Sunyaev & Shakura 1986; Kluźniak & Wilson 1991). Removal of angular momentum by radiation stresses decreases the stellar spin-up rate produced by a given accretion rate, and may reduce the asymptotic spin rate that can be achieved by accretion.

In this report we have demonstrated that radiation forces can dramatically affect accretion flows near slowly rotating neutron stars. We also expect radiation forces to have important effects on the flow of accreting matter near black holes and rapidly rotating neutron stars. Numerical computations of the effects on such flows are in progress and will be reported elsewhere.

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