Constraints on the Equation of State of Neutron Star Matter From Observations of Kilohertz QPOs

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The frequencies of the highest-frequency kilohertz QPOs recently discovered in some sixteen neutron stars in low-mass X-ray binary systems are most likely the orbital frequencies of gas in Keplerian orbit around these neutron stars. If so, these QPOs provide tight upper bounds on the masses and radii of these neutron stars and important new constraints on the equation of state of neutron star matter. If the frequency of a kilohertz QPO can be established as the orbital frequency of gas at the innermost stable circular orbit, this would confirm one of the key predictions of general relativity in the strong-gravity regime. If the spin frequency of the neutron star can also be determined, the frequency of the QPO would fix the mass of the neutron star for each assumed equation of state. Here we show how to derive mass and radius bounds, using the kilohertz QPOs, for nonrotating and slowly rotating stars, and discuss how these bounds are affected by rapid stellar rotation and radial radiation forces. We also describe observational results that would be strong evidence for the presence of an innermost stable circular orbit. No such strong evidence is present in current data, but future prospects are excellent.

1. INTRODUCTION

Determination of the equation of state of neutron stars has been an important goal of nuclear physics for more than two decades. Progress toward this goal can be made by establishing astrophysical constraints as well as by improving our understanding of nuclear forces.

Many ways of deriving constraints from astrophysics have been explored. One of the best known is pulse timing of pulsars in binary systems. Although binary pulsar timing has made possible stringent tests of general relativity (see, e.g., [16]), the $\approx 1.4\,M_\odot$ masses derived from timing (see [17]) are allowed by all equations of state based on realistic nuclear physics, and hence these observations have not eliminated any of the equations of state currently being considered. The highest known neutron star spin frequency, the 643 Hz frequency of PSR 1937+21 [1], is also allowed by all equations of state currently under consideration. Radius estimates based on the energy spectra of type I X-ray bursts and on observations of thermal emission from the surfaces of neutron stars are more restrictive in principle but currently have large systematic uncertainties (see [6,11]).

The discovery of high-frequency brightness oscillations from some sixteen neutron stars in low-mass X-ray binaries with the *Rossi* X-ray Timing Explorer (RXTE) holds great promise for providing important new constraints. Oscillations are observed both in the persistent X-ray emission and during type I X-ray bursts. The kilohertz quasi-periodic oscillations (QPOs) observed in the persistent emission have high amplitudes and relatively high coherences (see, e.g., [18]). A pair of kilohertz QPOs is commonly observed in a given source. Although the frequencies of these QPOs vary by up to a factor of $\sim$2, the frequency separation $\Delta\nu$ between a pair of kilohertz QPOs appears to be constant in almost all cases. In both the sonic-point [10] and magnetospheric [15] beat-frequency interpretations, the higher frequency in a pair is the orbital frequency at the inner edge of the Keplerian flow, whereas the lower frequency is the beat of the stellar spin frequency with this frequency. Such high orbital frequencies yield interesting bounds on the masses and radii of these neutron stars and interesting constraints on the equation of state of neutron star matter. In § 2 we describe how mass and radius bounds can be derived from the properties of the kilohertz QPOs, and discuss how these bounds are affected by rotation and by radial radiation forces.

The bounds on the mass and radius and on the equation of state would become particularly restrictive if the frequency of a kilohertz QPO can be securely established as the orbital frequency of gas at the innermost stable circular orbit. Indeed, if the QPO frequencies currently observed in some sources are the orbital frequency of gas at the innermost stable orbit, several currently viable equations of state are ruled out. Moreover, detection of the effects of the innermost stable orbit would, by itself, be a confirmation of one of the key predictions of general relativity in the strong-gravity regime. Several authors have recently suggested that these effects have already been observed and have argued that the neutron stars in some kilohertz QPO sources therefore have masses close to $2.0\,M_\odot$. In § 3 we discuss these suggestions, and describe observational results that would be strong evidence for the presence of an innermost stable orbit. We argue that the evidence cited to support detection of the innermost stable circular orbit was not compelling and that more recent observational results do not support these claims.

2. CALCULATIONS

Suppose that, as in the sonic-point model, the frequency $\nu_{\rm QPO2}$ of the higher-frequency QPO in a kilohertz pair is the orbital frequency of gas in a nearly circular Keplerian orbit around the neutron star and that the highest observed value of $\nu_{\rm QPO2}$ from a given star is $\nu^*_{\rm QPO2}$.

2.1. Nonrotating stars

Assume first that the star is not rotating and is spherically symmetric. Then the exterior spacetime is the Schwarzschild spacetime and the orbital frequency (measured at infinity) of gas in a circular orbit at Boyer-Lindquist radius r around a star of mass M is

$$\nu^0_{\rm K}(M,r) = (1/2\pi)(GM/r^3)^{1/2}\ . \tag{1}$$

Here and below the superscript 0 indicates that the relation is that for a nonrotating star. Equation (1) may be solved for the mass of the star as a function of the radius of the orbit with frequency $\nu^*_{\rm QPO2}$, with the result

$$M^0(R_{\rm orb},\nu^*_{\rm QPO2}) = (4\pi^2/G)R^3_{\rm orb}(\nu^*_{\rm QPO2})^2\ . \tag{2}$$

The mass of the star is related to the radius of the innermost stable orbit by the expression

$$M^0(R_{\rm ms}) = (c^2/6G)R_{\rm ms}\ . \tag{3}$$

The radius of the star must be smaller than the radius $R_{\rm orb}$ of the gas with orbital frequency $\nu^*_{\rm QPO2}$, so the representative point of the star in the R,M plane must lie to the left of the curve $M^0(R_{\rm orb},\nu^*_{\rm QPO2})$. In addition, in order to produce a wave train with tens of oscillations, the gas producing the QPO must be outside the radius $R_{\rm ms}$ of the innermost stable circular orbit, so the representative point must also lie below the intersection of $M^0(R_{\rm orb},\nu^*_{\rm QPO2})$ with the curve $M^0(R_{\rm ms})$. Figure 1a shows the allowed region of the R,M plane for $\nu^*_{\rm QPO2} = 1220$ Hz. The maximum allowed mass and radius are [10]

$$M^0_{\rm max} = 2.2\,(1000\ {\rm Hz}\,/\nu^*_{\rm QPO2})\ M_\odot \tag{4}$$

$$R^0_{\rm max} = 19.5\,(1000\ {\rm Hz}\,/\nu^*_{\rm QPO2})\ {\rm km}. \tag{5}$$

For example, the 1220 Hz QPO observed in the atoll source 4U 1636−536 would constrain the mass of this neutron star to be less than $1.8\,M_\odot$ and the radius to be less than 16.0 km, if it were not rotating. Figure 1b compares the mass-radius relations for nonrotating stars given by five equations of state with the regions of the radius-mass plane allowed for three values of $\nu^*_{\rm QPO2}$.

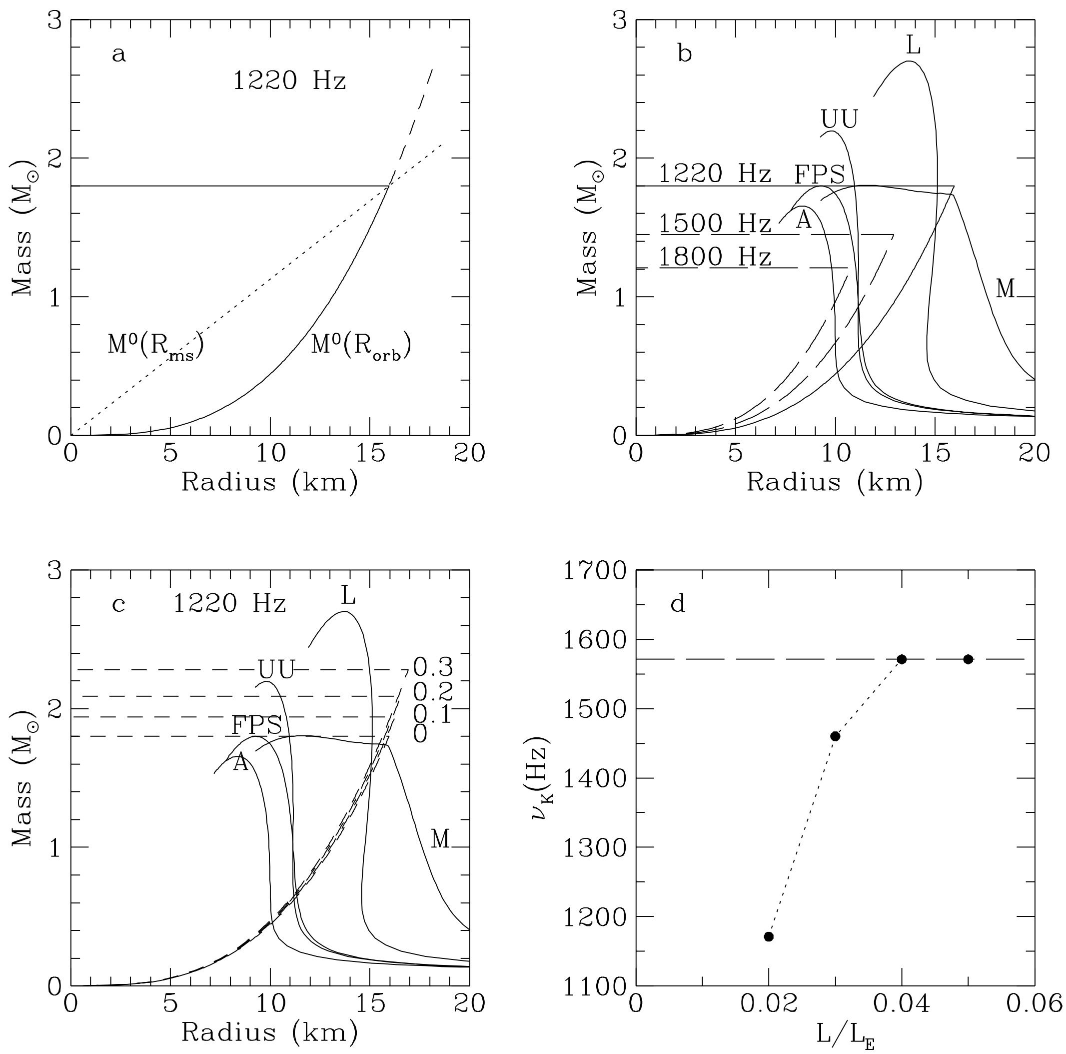

Figure 1. (a) Radius-mass plane, showing the region allowed for a nonrotating neutron star with $\nu^*_{\rm QPO2} = 1220$ Hz. (b) Comparison of the mass-radius relations for nonrotating neutron stars given by five representative equations of state with the regions of the mass-radius plane allowed for nonrotating stars and three different Keplerian orbital frequencies. The light solid curves show the mass-radius relations given by equations of state A [12], FPS [7], UU [20], L [14], and M [13]. (c) Regions allowed for rotating neutron stars with various values of j and $\nu^*_{\rm QPO2} = 1220$ Hz, when first-order effects of the stellar spin are included. (d) Illustrative Keplerian QPO frequency given by fully general relativistic calculations of the gas dynamics and radiation transport in the sonic-point model [10].

2.2. Slowly rotating stars

Rotation affects the structure of the star and the spacetime, altering the mass-radius relation, the frequency of an orbit of given radius, and the radius $R_{\rm ms}$ of the innermost stable orbit.

The parameter that characterizes the importance of rotational effects is the dimensionless quantity $j \equiv cJ/GM^2$, where J and M are the angular momentum and gravitational mass of the star. The value of j that corresponds to a given observed spin frequency depends on the neutron star mass and equation of state, and is typically higher for lower masses and stiffer equations of state. For the spin frequencies ~ 300 Hz inferred in the kilohertz QPO sources, $j \sim 0.1$–0.3. For such small values of j, a first-order treatment is adequate. To this order, analytical expressions are available for the relevant quantities and one can prove the existence of upper bounds on the mass and radius [10]. However, the bounds must be computed numerically.

To first order in j, the orbital frequency (measured at infinity) of gas in a prograde Keplerian orbit at a given Boyer-Lindquist radius r is

$$\nu_K(r,M,j) \approx [1 - j(GM/rc^2)^{3/2}]\,\nu_K^0(r,M) \qquad (6)$$

and the radius of the innermost stable orbit is

$$R_{\rm ms}(M,j) \approx [1 - j(2/3)^{3/2}]\,R_{\rm ms}^0(M)\;, \qquad (7)$$

where ν_K^0 and $R_{\rm ms}^0$ are the Keplerian frequency and radius of the innermost stable orbit for a nonrotating star. Hence, to first order in j, the frequency of the prograde orbit at $R_{\rm ms}$ around a star of given mass M and dimensionless angular momentum j is (see [4])

$$\nu_{\rm K,ms} \approx 2210\,(1+0.75j)(M_\odot/M)\,{\rm Hz}\;. \qquad (8)$$

As a result, for slowly rotating stars the upper bounds on the mass become

$$M_{\rm max} \approx [1+0.75j(\nu_{\rm spin})]M_{\rm max}^0 \qquad (9)$$

and

$$R_{\rm max} \approx [1+0.20j(\nu_{\rm spin})]R_{\rm max}^0\;, \qquad (10)$$

where $j(\nu_{\rm spin})$ is the value of j for the observed stellar spin rate at the maximum allowed mass for the equation of state being considered and $M_{\rm max}^0$ and $R_{\rm max}^0$ are the maximum allowed mass and radius for a nonrotating star (see above). Expressions (9) and (10) show that the bounds are always greater for a slowly rotating star than for a nonrotating star, regardless of the equation of state assumed.

Figure 1c illustrates the effects of stellar rotation on the region of the radius-mass plane allowed for spin rates $\sim$ 300 Hz, like those inferred for the kilohertz QPO sources, and $\nu^*_{\rm QPO2} = 1220$ Hz, the frequency of the highest-frequency QPO so far observed in 4U 1636−536, which is also the highest-frequency QPO so far observed in any source. Our calculations show that *the mass of the neutron star in 4U 1636−536 must be less than* $\sim 2.2\,M_\odot$ *and its radius must be less than* $\sim$ *17 km*. As just explained, the precise upper bounds depend on the equation of state assumed. For further details, see [10].

2.3. Rapidly rotating stars

If the stellar spin is ~ 500 Hz or higher, spin affects the structure of the star as well as the exterior spacetime. The exterior spacetime of such a rapidly rotating star differs substantially from the Kerr spacetime and must be computed numerically for each assumed equation of state. Derivation of bounds on the mass and radius of a given star for an assumed equation of state requires construction of a sequence of stellar models and spacetimes for different masses using the assumed equation of state, with $\nu_{\rm spin}$ as measured at infinity held fixed. The maximum and minimum possible masses and radii allowed by the observed QPO frequency can then be determined.

Such computations have been carried out by Miller, Lamb, & Cook [9]. They find that if the neutron star is spinning rapidly, the constraints on the equation of state are tightened dramatically. For instance, if the spin frequency of the neutron star in 4U 1636−536 is ~ 580 Hz, the frequency of the single brightness oscillation observed during X-ray bursts from this source, then the tensor-interaction equation of state of Pandharipande and Smith [13] is ruled out by the kilohertz QPO frequencies already observed from this source. They also find that observation of a 1500 Hz orbital frequency would constrain the mass and radius of the neutron star to be less than $\sim 1.7\,M_\odot$ and ~ 13 km, ruling out several equations of state that are currently astrophysically viable, regardless of the star's spin rate.

2.4. Effects of the radial radiation force

The luminosities of the Z sources are typically ~ 0.5–$1\,L_E$, where L_E is the Eddington luminosity. Hence, in the Z sources the outward acceleration caused by the radial component of the radiation force can be a substantial fraction of the inward acceleration caused by gravity. The radially outward component of the radiation force reduces the orbital frequency at a given radius. For example, if the star is spherical and nonrotating and emits radiation uniformly and isotropically from its entire surface, the orbital frequency (measured at infinity) of a test particle at Boyer-Lindquist radius r is given by

$$\frac{\nu_{\rm K}(L)}{\nu_{\rm K}(0)} = \left[1 - \frac{(1-3GM/rc^2)^{1/2}}{(1-2GM/rc^2)}\frac{L}{L_E}\right]^{1/2} , \quad (11)$$

where L is the luminosity of the star measured at infinity and $\nu_{\rm K}(0)$ is the Keplerian frequency in the absence of radiation forces. Thus, the Boyer-Lindquist radius of a circular orbit with a given frequency is smaller in the presence of the radial radiation force and the constraints on the mass and radius of the star are therefore strengthened.

For the atoll sources, which have luminosities $\lesssim 0.1\,L_E$, the change in the Keplerian frequency is at most $\sim 5\%$. For the Z sources, on the other hand, which have luminosities $\approx L_E$, the change may be much larger, although the change in the sonic-point Keplerian frequency may be smaller than would be suggested by a naive application of equation (11), if a substantial fraction of the radiation produced near the star is scattered out of the disk plane before it reaches the sonic point.

3. INNERMOST STABLE CIRCULAR ORBIT

Establishing that an observed QPO frequency is the orbital frequency of the innermost stable circular orbit in an X-ray source would be an important step forward in our understanding of strong-field gravity and the properties of dense matter, because it would (1) confirm one of the key predictions of general relativity in the strong-field regime and (2) fix the mass of the neutron star in that source, for each assumed equation of state.

Given the fundamental significance of the innermost stable orbit, it is very important to establish what would constitute strong, rather than merely suggestive, evidence that the innermost stable orbit has been detected. Clear signatures of the innermost stable orbit include the following (see [10] for more details):

(1) Probably the most convincing signature would be a fairly coherent, kilohertz QPO with a frequency that reproducibly increases steeply with increasing accretion rate but then becomes constant and remains nearly constant as the accretion rate increases further. The constant frequency should always be the same in a given source. As shown in Figure 1d, this behavior emerges naturally from general relativistic calculations of the gas dynamics and radiation transport in the sonic-point model [5,10].

(2) A possible signature of the innermost stable orbit would be a simultaneous sharp drop in the amplitudes of both QPOs in a kilohertz QPO pair, or a sharp drop in the amplitude of the lower-frequency QPO, at a frequency that is always the same in a given source.

(3) A less likely but possible signature of the innermost stable orbit would be a steep and reproducible drop in the coherence of both QPOs in a pair (or in the coherence of the higher-frequency QPO, if the lower-frequency QPO is not visible) at a certain critical frequency (the frequency of the innermost stable orbit), as the frequencies of both QPOs increase steadily with increasing accretion rate. The critical frequency should always be the same in a given source.

Several authors have recently suggested that innermost stable orbits have already been observed. Zhang et al. [22] suggested that the similarity of the highest QPO frequencies seen so far indicates that innermost stable circular orbits are being detected. Kaaret, Ford, & Chen [3]3 suggested that the roughly constant frequencies of the 800–900 Hz QPOs seen in 4U 1608−52 [2] and 4U 1636−536 [21] were generated by the beat of the spin frequency against the frequency of the innermost stable circular orbits in these sources. This would imply that the neutron stars in all the kilohertz QPO sources have masses close to $2.0\,M_\odot$. However, *no strong signatures of the innermost stable circular orbit have so far been seen in any of these sources.*

Indeed, more recent observations of both 4U 1608−52 [8] and 4U 1636−536 [19] are inconsistent with the suggestion that the QPO frequencies seen initially are related to the frequencies of the innermost orbits in these sources. The 1171 Hz QPO frequency assumed by Kaaret et al. [3] to be the frequency of the innermost stable orbit in 4U 1636−536 and used by them to determine the mass of the star was later seen to increase to 1193 Hz and then to 1220 Hz ([19]; W. Zhang, personal communication). There is as yet no evidence for a maximum QPO frequency in 4U 1636−536 and hence there is no basis for the suggestion that an innermost stable orbit has been seen in this source.

A recent analysis of 4U 1608−52 data by Méndez et al. [8] shows that this source has twin kilohertz QPOs that vary with countrate just like the other atoll sources. There is as yet no evidence for a maximum QPO frequency in 4U 1608−52 and hence there is no basis for the suggestion that an innermost stable orbit has been seen in this source, either.

Even though no strong evidence for the effects of the innermost stable circular orbit has yet been seen in any kilohertz QPO data, there is reason for optimism, because the highest QPO frequencies observed so far are only 100–200 Hz below the expected frequencies of the innermost stable orbit around these neutron stars. Given the rapid pace of discoveries in this field, the prospects for obtaining clear evidence of an innermost stable circular orbit in the future appear excellent.

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