Black Holes

We're now ready to start tackling the properties of black holes. For astronomers interested in compact objects, what is most important about black holes is the properties of their spacetime. Mathematically, a black hole can have only three properties: mass, angular momentum, and electric charge. However, like all other big things in the universe (here we'll take "big" to mean "of planetary or greater mass"), black holes are expected to have negligible net electric charge. That means we have only to worry about mass and angular momentum. We'll begin with nonrotating (or Schwarzschild) black holes and then talk about the effects of rotation.

Nonrotating black holes

(1) There is a unique spacetime outside spherically symmetric objects.

(2) Black holes have event horizons. For nonrotating black holes, the circumferential radius (circumference divided by 2π) at the horizon is $R_H = 2GM/c^2 \approx 3$ km $(M/M_\odot)$ for a black hole of mass M. Coincidentally, this is the radius at which the Newtonian escape velocity is c.

(3) There are unstable circular orbits close enough to a black hole or neutron star. Far away, a slight perturbation of an orbit just makes it a little elliptical. Close enough, however, a slight perturbation causes it to spiral in quickly. For a nonrotating object, the so-called innermost stable circular orbit, or ISCO, is at a circumferential radius $R_{\rm ISCO} = 6GM/c^2 \approx 9$ km $(M/M_\odot)$.

(4) Photons can be deflected dramatically close to a black hole. Indeed, a photon could undergo a circular orbit at a circumferential radius of $R_{\rm ph} = 3GM/c^2 \approx 4.5$ km $(M/M_\odot)$ around a nonrotating black hole, but because this is inside $R_{\rm ISCO}$ the orbit is unstable. The dramatic deflection of photon orbits means that there can be photon "rings", and thus if there is a sharp burst of radiation this can produce echoes as photons undergo different numbers of orbits. These are, however, successively fainter than the previous echo.

In Newtonian physics, the gravity around a spherically symmetric object is as if all the mass is concentrated at a point in the center. If the object explodes or contracts, but in a spherically symmetric way, it is still the case that the gravity outside that region is the same as before.

In general relativity, one gets a similar result: if the spacetime is spherically symmetric, then it is unique. Birkhoff's theorem says that the only vacuum, spherically symmetric gravitational field is static. One convenient metric for this spacetime is the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega \tag{1}$$

where $d\Omega = d\theta^2 + \sin^2\theta\, d\phi^2$. Here we follow convention in setting $G = c = 1$. Note here an important distinction: a *metric* is a description of a spacetime based on a particular choice of coordinates. The spacetime itself is more general. In Schwarzschild coordinates, t is the time between events measured by someone at infinity, θ and ϕ are their usual spherical coordinate equivalents, and r is $1/2\pi$ times the circumference of a circle drawn at Schwarzschild coordinate at r. *However*, you have to be careful: the proper radial distance from r_1 to r_2 is *not* $r_2 - r_1$. It is $\int_{r_1}^{r_2} (g_{rr})^{1/2}\, dr$, which is larger in general.

Because of differences in the way in which local or distant observers measure things, in GR one must always be precise when specifying the frame in which a quantity is measured. If necessary, one can then use transformation rules to figure out what that quantity would be in other frames. Confusion between frames is a common sticking-point for many crackpots!

The region $r = 2M$ (or $2GM/c^2$ when we put G and c back in) is the *event horizon*, which defines the "surface" of the black hole and defines the Schwarzschild radius. This is also called the *static limit*: static (non-moving) observers can't exist inside this radius. Now, this also means that if we at infinity watch something fall in to the black hole, it never appears to cross the horizon because of time dilation. It seems to "freeze" there, although it dims rapidly so we shouldn't expect to see a collection of frozen surprised aliens at the horizons of black holes! A person falling through $r = 2M$ does so in finite proper time (one can verify this by integration), and does not feel infinite tidal forces (those happen at $r = 0$, which is a genuine singularity). Note that as $r \longrightarrow 2M$ the redshift approaches infinity. A misunderstanding about what really happens at $r = 2M$ led many people, including Einstein, to disbelieve in black holes because they thought it meant a star somehow could hover just outside the horizon.

The ISCO is important astrophysically because matter that gets into a black hole typically can't fall in directly; everything has some angular momentum, and black holes are small for their mass, so gas spirals in through an accretion disk. If there are no other forces involved, then when the gas reaches the ISCO it spirals in to the black hole without further loss of energy or angular momentum. In reality there *are* other forces (e.g., gas pressure gradients or magnetic fields), but this isn't usually a terrible approximation. For a nonrotating black hole, a particle in a circular orbit at the ISCO radius $R_{\rm ISCO} = 6M$ has a specific energy (where "specific" means "per mass") of $\sqrt{8/9}c^2$, which means that if it started far away and moving slowly compared with the speed of light, it had to release $1 - \sqrt{8/9} \approx 0.057c^2$ of energy per mass. That's a lot; nuclear fusion of hydrogen to helium only releases $0.007c^2$ per mass.

Rotating black holes

A dimensionless parameter that characterizes the angular momentum of a rotating object is $\hat{a} \equiv cJ/(GM^2)$, where J is the angular momentum and M is the mass of the black hole.

Black holes can have $0 \leq |\hat{a}| \leq 1$. This isn't an absolute limit on the angular momentum of things in the universe (for example, for the Earth, $\hat{a} \approx 700$), but nothing with $\hat{a} > 1$ can have a horizon, and as far as we know we can't spin a black hole up to have $\hat{a} > 1$. People often use a (without the hat) to represent $\hat{a}M$.

Whereas the spacetime outside a spherically symmetric object is always the Schwarzschild spacetime, the spacetime outside a rotating object depends on the properties of the object. For a black hole, in the most commonly-used coordinates in astrophysics (Boyer-Lindquist coordinates), the metric line element is

$$ds^2 = -(\Delta/\rho^2)[dt - a\sin^2\theta\, d\phi]^2 + (\sin^2\theta/\rho^2)[(r^2+a^2)d\phi - a\, dt]^2 + (\rho^2/\Delta)dr^2 + \rho^2\, d\theta^2 \ , \quad (2)$$

where $\Delta \equiv r^2 - 2Mr + a^2$ and $\rho^2 \equiv r^2 + a^2\cos^2\theta$. This is the Kerr line element in these coordinates, and it reduces to the Schwarzschild line element when $a = 0$. Comparing this carefully with Schwarzschild, we note that whereas Schwarzschild is diagonal (the only nonzero elements are dt^2, dr^2, $d\theta^2$, and $d\phi^2$), for Kerr we have $dtd\phi$ and (symmetrically) $d\phi dt$ terms. This introduces an important new feature of spacetime around rotating objects: frame-dragging. That is, spacetime itself is dragged in the direction of rotation of the object. This changes the horizon radius in Boyer-Lindquist coordinates to

$$r_+ = M + (M^2 - a^2)^{1/2} \ , \quad (3)$$

which reduces to the Schwarzschild expression $r_+ = 2M$ in the nonrotating limit $a = 0$. When the hole rotates, we have a new, important radius: the "ergosphere". This is at

$$r_0 = M + (M^2 - a^2\cos^2\theta)^{1/2} \ , \quad (4)$$

where θ is the colatitude ($\theta = 0$ at one pole and $\theta = \pi$ at the other, and $\theta = \pi/2$ at the rotational equator). Note that there is room between r_0 and the horizon radius r_+ for $a < 1$ (and even for $a = 1$ it turns out that the horizon and ergosphere are distinct). Inside this radius, test particles (massive or massless) must rotate with the hole, even if they are outside the horizon. The ergosphere is therefore the static limit; things inside can't remain stationary with respect to infinity. This is an extreme example of frame-dragging. Even outside the ergosphere, spacetime is dragged in the direction of rotation of the hole. The natural frequency for this frame-dragging (i.e., the frequency of a particle with no angular momentum) is $\omega \approx \frac{2Ma}{r^3}$. The ergosphere is named thus ("ergo" being Greek for energy) because in principle something sent in there could split in two and emerge with more than the original energy of the particle, thereby tapping the spin energy of the hole. More astrophysically plausible is that threaded magnetic fields down the hole might extract its spin energy (the "Blandford-Znajek process").

Rotation also changes the location of the ISCO and thus the total energy that is extracted from test particles or fluids if they spiral toward a black hole in a quasicircular accretion disk. The full expression for the ISCO radius is more complicated than is worth writing down (see Bardeen, Press, and Teukolsky 1972, ApJ, 178, 347), but some interesting limits are our previous $r_{\rm ISCO} = 6M$ for $\hat{a} = 0$ (and 5.7% of the energy is extracted), $r_{\rm ISCO} = 9M$ for $\hat{a} = 1$ but retrograde accretion (i.e., the disk spirals the opposite way from the black hole spin), and $r_{\rm ISCO} = M$ for $\hat{a} = 1$ and prograde accretion (the disk spirals the same way as the black hole spin). In the prograde $\hat{a} = 1$ case, about 42% of the mass-energy of a particle needs to be radiated to get to the ISCO. There are some complications when astrophysics is put back in. For example, Kip Thorne showed (1974, ApJ, 191, 507) that in a real accretion disk, the capture of photons with negative angular momentum would limit the spin parameter to $\hat{a} < 0.998$, which might seem pretty close to 1 but that tiny change limits the energy efficiency to $\sim 30\%$ instead of 40%. When even more effects are considered (e.g., magnetic fields in the accretion disk), the limit might be $\hat{a} = 0.95$ or even a bit less. Another nonideality is that when a real accretion disk is involved, pressure gradients in the disk allow the innermost part of the disk to move inside the ISCO by a bit before they spiral into the black hole.

Despite such complications, the spin-dependent properties of orbits around black holes are a key to understanding their properties. The spectrum of an accretion disk, particularly as it relates to spectral lines in X-rays, depends on the spin parameter. With that in mind, various groups have estimated the spin parameters of black holes electromagnetically, and have found a full range, from consistent with 0 to consistent with $\hat{a} > 0.9$. This is an interesting contrast with the spins inferred from gravitational wave observations, which are mostly consistent with zero and are definitely *not* consistent with values close to 1.

Astrophysical black holes

Identifying probable black holes with electromagnetic observations is tougher than you'd think, because their existence is something of a negative proposition: if nothing else fits, maybe it's a black hole. Also, black holes by themselves are essentially silent; for context, there are probably $\sim 10^8$ black holes in our galaxy (based on how stars evolve), but we only have decent confidence about a few tens of them, because they need to be accreting gas or at least causing a companion star to move for us to notice them. However, good evidence is to be had from a variety of quarters. Neutron stars can't have $M > 3\,M_\odot$, so if a compact X-ray source in a binary is seen that has a mass greater than this, it's probably a black hole. On a larger scale, dynamical measurements of stellar movements in the cores of several galaxies imply $M > 10^5\,M_\odot$ or so in small region ($\sim$pc), and one can at least say that if the central object isn't a black hole, it is something a lot more exotic! Gravitational wave detections are fully consistent with the expectations from GR and black holes. Recently, the Event Horizon Telescope collaboration has shown that there are "shadows" of the expected sizes from two supermassive black holes (in our Galactic center and in the center of the galaxy M87), although they have *not* demonstrated the existence of an event horizon. Overall, the electromagnetic evidence is consistent with black holes in the $\sim 3-20\ M_\odot$ range (stellar-mass black holes) and in the $\sim 10^5$ – few $\times\ 10^{10}\ M_\odot$ range (supermassive black holes in galactic centers); an ongoing question is whether black holes in the middle range of the masses, i.e., "intermediate-mass black holes" exist.

Can we be absolutely sure that the things that are currently identified as black holes are, in fact, the black holes described by general relativity? A reasonable perspective is that if it walks, acts, and quacks like a duck then maybe, well, it's a duck. There is no evidence that these things aren't GR black holes. But GR has singularities, which are uncomfortable, and most physicists think that quantum gravity (whatever its proper description) will eventually take over from GR, so it is reasonable to ask (1) can deviations from GR manifest themselves at the scale of astrophysical black holes, and (2) even with GR, are there ways to avoid an event horizon? The answer to both is that we don't know. If the things that look like black holes have surfaces rather than horizons, then for various reasons their surfaces have to be very close to the horizon. For example, if a nonrotating object has a surface at $r = 2.1M$, just outside what would have been the radius of the horizon, then infalling matter will hit that surface and radiate close to 100% of their mass-energy, which would lead to dramatically different predictions than what we observe. But, perhaps strangely, if the surface is much, much closer to the horizon (say, a few Planck lengths), then radiation has a much more difficult time escaping (it can escape only in an almost-radial direction), and it might be possible to evade detection.

One hope from the gravitational wave standpoint is that if there is a surface close to the horizon, then there will be "echoes" of the ringdown waveform that can be seen after the primary ringdown. This has even been claimed, but not at a level that has convinced the community.

What other signatures of a very compact surface would you expect?