Neutron Star Structure

We now enter the study of neutron stars. Like black holes, neutron stars are one of the three possible endpoints of stellar evolution (the remaining one being white dwarfs). Also like black holes, neutron stars are very compact objects, so GR is important in their description. Unlike black holes, they have surfaces instead of horizons, so they are a lot more complicated than black holes. We'll start with an overall description of neutron stars, then discuss some of the physics of degenerate objects. We will conclude by talking a bit about white dwarfs.

Summary of Neutron Stars

A typical neutron star has a mass of $1.2 - 2\,M_\odot$ and a radius of $\sim 11 - 13$ km, so their bulk average densities can be a few times larger than "nuclear saturation density" (the density at the centers of large nuclei on Earth) $\rho_{\rm sat} \approx 2.6 \times 10^{14}$ g cm^{-3}. Known neutron stars have spin frequencies up to 716 Hz and their inferred surface magnetic field strengths are $\sim 10^8 - 10^{16}$ G. The surface gravity is few $\times\, 10^{14}$ cm s^{-2}, so mountains of even perfect crystals can't be higher than < 1 mm, meaning that these are the smoothest surfaces in the universe. They have many types of behavior, including pulsing (in radio, IR, opt, UV, X-ray, and gamma-rays, but this is rarely all seen from a single object), glitching, accreting, and possibly gravitational wave emission. They are the best clocks in the natural universe. Their cores are at several times nuclear density, and may be composed of exotic matter such as quark-gluon plasmas, strange matter, kaon condensates, or other weird stuff. In their interiors they are superconducting and superfluid, with transition temperatures around $10^8 - 10^9$ K. All these extremes mean that neutron stars are attractive to study for people who want to push the envelope of fundamental theories about gravity, magnetic fields, and high-density matter. A few thousand neutron stars are known, mostly from their radio pulsations, and a few gravitational wave events have involved neutron stars (two events that were probably NS-NS, and a few more that might have been BH-NS).

High densities

Let's talk about the quantum physics that enters at high densities. An essential new concept that is introduced at high densities is *Fermi energy*. The easiest way to think about this is in terms of the uncertainty principle,

$$\Delta p \Delta x \geq \hbar/2\ . \tag{1}$$

If something is localized to a region of size Δx, then its momentum must be at least $\sim \hbar/\Delta x$ (where you see that we have dropped the factor of 2; we're looking for general insight rather than precise values). That means that in a dense environment, there is a momentum, and hence an energy, associated with the confinement. Therefore, squeezing something increases its total energy, and this Fermi energy acts as a pressure (sometimes called degeneracy pressure). The existence of this energy has a profound role in the structure of white dwarfs, and especially neutron stars. In fact, if degeneracy pressure dominates, then unlike normal stars, which get larger as they get more massive, degenerate stars are *smaller* at higher masses. In particular, an approximate relation is that $R \sim M^{-1/3}$ for a degenerate star, although for neutron stars the poorly known details of nuclear physics beyond the density of an atomic nucleus means that things aren't as clear.

Now let's get some basic numbers. If the energy and momentum are low, then the Fermi energy E_F is related to the Fermi momentum $p_F \sim \hbar/\Delta x$ by $E_F \approx p_F^2/2m$, where m is the rest mass of the particle (this is the standard nonrelativistic expression for energy as a function of momentum). Since $\Delta x \sim n^{-1/3}$, where n is the number density of the particle, in this nonrelativistic regime $E_F \sim n^{2/3}$. At some point, however, $E_F > mc^2$. Then in the extreme relativistic limit $E_F \sim p_F c$, so $E_F \sim n^{1/3}$. For electrons, the crossover to relativistic Fermi energy happens at a density $\rho \sim 10^6$ g cm^{-3}, assuming a fully ionized plasma with two nucleons per electron. For protons and neutrons the crossover density is about 6×10^{15} g cm^{-3} (it scales as the particle's mass cubed). The maximum density in neutron stars is no more than 10^{15} g cm^{-3}, so for most of the mass electrons are highly relativistic but neutrons and protons are at best mildly relativistic.

Suppose that we have matter in which electrons, protons, and neutrons all have the same number density. For a low density, which has the highest Fermi energy? The electrons, since at low densities the Fermi energy goes like the inverse of the particle mass. Given what we said before, what is the approximate value of the electron Fermi energy when $\rho = 10^6$ g cm^{-3}? That's the relativistic transition, so $E_F \approx m_e c^2 \approx 0.5$ MeV. Then at 10^7 g cm^{-3} the Fermi energy is about 1 MeV, and each factor of 10 doubles the Fermi energy since $E_F \sim n^{1/3}$ in the relativistic regime. What that means is that the energetic "cost" of adding another electron to the system is not just $m_e c^2$, as it would be normally, but is $m_e c^2 + E_{F,e}$, where the "e" subscript means "electron". It therefore becomes less and less favorable to have electrons around as the density increases.

Now, in free space neutrons are unstable. This is because the sum of the masses of an electron and a proton is about 1.5 MeV short of the mass of a neutron, so it is energetically favorable to decay (for the pedantic: what we *really* mean is that decay is favorable because it increases the total number of states accessible to the system and thus increases the system's entropy). **Ask class:** what happens, though, at high density? If $m_p + m_e + E_{F,e} > m_n$, then it is energetically favorable to combine a proton and an electron into a neutron. Therefore, at higher densities matter becomes more and more neutron-rich. First, atoms get more neutrons, so you get nuclei such as ^{120}Rb, with 40 protons and 80 neutrons. Then, at about 4×10^{11} g cm^{-3} it becomes favorable to have free neutrons floating around, along with some nuclei (this is called “neutron drip” because the effect is that neutrons drip out of the nuclei). At even higher densities, the matter is essentially a smooth distribution of neutrons plus a $\sim 5 - 10\%$ smattering of protons and electrons. At higher densities yet (here we’re talking about nearly 10^{15} g cm^{-3}), the neutron Fermi energy could become high enough that it is favorable to have other particles appear.

It is currently unknown whether such particles will appear, and this is a focus of much present-day research. If they do, it means that the energetic “cost” of going to higher density is less than it would be otherwise, since energy is released by the appearance of other, exotic particles instead of more neutrons. In turn, this means that it is easier to compress the star: squashing it a bit doesn’t raise the energy as much as you would have thought. Another way of saying this is that when a density-induced phase transition occurs (here, a transition to other types of particles), the equation of state (the pressure as a function of energy density) is “soft”. Maybe the new phase of matter is hard, but at the transition itself the matter is soft.

Soft matter can’t support as much mass as hard matter. That’s because as more mass is added, the star compresses more and more, so its gravitational compression increases. If pressure doesn’t increase to compensate, in it goes and forms a black hole. What all this means is that by measuring the mass and radius of a neutron star, or by establishing the maximum mass of a neutron star, or (highly relevant to gravitational waves) by determining how deformed a star is under the influence of an external gravitational tidal field, one gets valuable information about the equation of state (EOS: this is the pressure as a function of energy density), and hence about nuclear physics at very high density. This is just one of many ways in which study of neutron stars has direct implications for microphysics. We’ll talk about more when we discuss what we’ve already learned from gravitational wave detections.

White Dwarfs

The overwhelming majority of stars will end their lives as white dwarfs rather than neutron stars or black holes; only if the initial mass is $> 8 - 9\ M_\odot$ will the star become something other than a white dwarf. A white dwarf will typically have a mass of $\sim 0.6\ M_\odot$ and a radius roughly that of Earth. This makes its average density something like 10^6 times that of water, and means that the maximum frequency it can reach (whether by rotation, or in a binary, or because of a sound wave that involves most of the star) is typically a few tenths of a Hertz. This is well below what we’d expect to access using current or future ground-based gravitational-wave detectors, but is comfortably above what we will see using the space-based gravitational-wave detector LISA (Laser Interferometer Space Antenna). This means that LISA will be able to see detached (i.e., non-accreting) double white dwarf (DWD) binaries. Since these have low mass and (compared with LVK events) low frequency, their gravitational wave luminosity is small and thus we only expect to see them if they are in the Milky Way galaxy or possibly in a nearby satellite galaxy. But they make up for their weakness with sheer numbers: we expect something like a hundred million DWD binaries in our Galaxy, which is so many that from few $\times 10^{-4}$ Hz to few $\times 10^{-3}$ Hz, it is the confusion of unresolved DWDs, rather than instrumental sensitivity, which will limit the detection of other sources using LISA.

White dwarfs are supported against gravity by gradients of electron degeneracy pressure, in contrast to the neutron degeneracy pressure that holds up neutron stars. Electrons are much less massive than neutrons (by around a factor of 2000), and this turns out to very roughly explain the difference in their radii: at an approximate level, white dwarfs are around 1000 times larger than neutron stars.

A remarkable fact about white dwarfs that was discovered by Chandrasekhar (and, as it turns out, a couple of years earlier by Edmund Stoner) is that they have a maximum mass. The basic idea is that whereas when the degenerate electrons are nonrelativistic the star can settle into a stable equilibrium, when the degenerate electrons are relativistic the star either reaches a minimum total energy by expanding into nonrelativistic degeneracy (if the mass is below a threshold) or it can't reach a minimum energy at all and thus collapses. The threshold mass, for an iron white dwarf, is about $M_{\rm Ch} = 1.35\ M_\odot$ (where the subscript is for Chandrasekhar). Fundamentally, this is what drives a core-collapse supernova, in which a massive star produces an iron core, which cannot fuse to produce energy; when the core reaches $M_{\rm Ch}$ (more or less, with some caveats), it collapses, at least temporarily produces a neutron star, and the energy blows apart the remaining portions of the star (with details that are still tricky after many decades of study).

If you apply the same logic to get the maximum mass of a neutron star, your initial estimate would be around $5.6\ M_\odot$. But it's actually much less than that (the best current estimate is $< 2.5\ M_\odot$), because of various subtleties involved in general relativity and nuclear physics. Nonetheless, that neutron stars *have* a maximum mass is crucial in making the argument that certain things are black holes; if it's non-luminous and heavier than the neutron star maximum, it's likely a black hole.

But back to white dwarfs. Their nuclei are basically ordinary, and the interactions of degenerate electrons are not a mystery. There are always details (e.g., how they cool is still being analyzed carefully), but why should we be interested in them and in particular gravitational waves from DWD binaries?

The most compelling answer is that, at least currently, DWD mergers are the best candidate to explain the so-called Type Ia supernovae (also known as SNe Ia), which are fantastic cosmological standard candles. For example, it was the study of SNe Ia which convinced people that the expansion of the universe is accelerating, and thus that dark energy makes up most of the mass-energy density in the universe.

Backing up a bit, after considerable study it was determined that there are two fundamentally different types of supernovae that have been detected. For historical and observational reasons, the two types are SNe Ia and everything else (including other SNe I designations). SNe Ia involve white dwarfs, and everything else is the core collapse of a massive star. Other types of supernovae have been postulated (e.g., pair-instability supernovae) but not yet clearly seen.

One observational clue about the distinction between the types is that because non-Ia types of SNe involve massive stars, which live short lifetimes (millions rather than billions of years), you expect to find them, and do, in galaxies with very active star formation. But SNe Ia are also found in galaxies without active star formation, which means that they have to be able to happen in older systems. For a long time the favorite idea was the "single-degenerate" origin, in which a white dwarf accretes mass from a companion star. If the white dwarf has enough mass, the idea went, then it could become unstable to thermonuclear fusion of (say) carbon into oxygen. Enough energy could be released in such fusion, if it happened throughout the star, that it would blow the star apart and leave no remnant.

But more recent observations have not found the signs of such systems. For example, given the rate of SNe Ia we would have expected to see a number of accreting systems where a high-mass white dwarf had matter actively falling on its surface, which we would see in ultraviolet. But not nearly enough systems have been seen. Thus a different idea has taken center stage: that SNe Ia are the result of the inspiral and merger of two white dwarfs that were in a binary. If this is the case then it means that the many DWD systems that LISA will see and identify individually are precursors to SNe Ia. Gravitational-wave detection of those systems will (1) confirm (or not!) that the rates are compatible, and if confirmation is achieved then (2) the characterization of those systems will tell us about their typical masses and mass ratios and (optimistically) might allow such systems to be better-modeled and thus allow them to be even more precise standard candles. Oh, also, this will give the people interested in white dwarfs on their own a lot more data!