**Compact binary formation: dynamics**

Now we'll think about the second major channel of compact binary formation, which involves dynamics. We'll start by thinking about dense stellar systems, for example globular clusters.

The main difference between the bulk of our Galaxy and dense stellar clusters such as globular clusters or nuclear star clusters is stellar number density. In the Solar vicinity there are roughly 0.15 stars per cubic parsec, but in the center of the densest globulars the density can be $10^6$ per cubic parsec. This still isn't enough to have stars collide directly with each other very often, but it does mean that binary systems, which act as if their collision cross sections are the sizes of the orbits, can have tens of collisionless binary-single and binary-binary encounters.

Let's demonstrate this quantitatively. The rate of encounters given a relative speed at large distance $v_\infty$, number density $n$, and effective cross section $\Sigma$ is $\tau^{-1} = n\Sigma v_\infty$. When gravitational focusing is taken into account, the effective cross section to get to a separation $a$ or less, when the total mass is $M$, is

$$\Sigma = \pi a^2 \left[\frac{2GM}{av_\infty^2} + 1\right] \; . \tag{1}$$

When $v_\infty$ is much larger than the escape speed at $a$, $v_{\rm esc} = \sqrt{2GM/a}$, this is just the geometric cross section $\pi a^2$. But in the opposite limit that $v_\infty \ll v_{\rm esc}$, gravitational focusing increases the effective cross section substantially, and in fact the cross section scales with $a$ rather than $a^2$. In a moderately rich globular cluster the velocity dispersion is $\sim 10$ km s$^{-1}$. If we think about a binary with a total mass of $M = 1\ M_\odot$ and an orbital semimajor axis of $a =$ 1 au, then the escape speed is about 40 km s$^{-1}$, so this is in the gravitational focusing limit. Putting in the numbers gives $\Sigma \approx 10^{28}$ cm$^2$. At a number density of $10^5$ pc$^{-3} \approx 3\times10^{-51}$ cm$^{-3}$ and a speed of 10 km s$^{-1} = 10^6$ cm s$^{-1}$, this gives a rate of $\tau^{-1} = n\Sigma v_\infty \approx 1/(7\times10^8$ yr). If the binary has a black hole and thus the total mass is $M \sim 10\ M_\odot$, then the rate is increased by a factor of 10. Given the $\sim 10^{10}$ yr age of globular clusters, these calculations support the idea that tens of interactions are possible.

The interactions are chaotic, but computer simulations show that when a binary and single interact, the binary that emerges from the interaction tends to contain the two most massive of the three original objects. Thus neutron stars and black holes, which are more massive than the average star in a globular, can swap into binaries and eventually find compact objects as companions. Moreover, the same simulations show that "hard binaries harden and soft binaries soften". That is, to a reasonable approximation, binaries tight enough that the total energy of the binary-single interaction, including the negative gravitational energy, is negative tend to tighten after a three-body interaction, whereas binaries wide enough that the total energy is positive tend to widen after a three-body interaction, more and more, until the binary is finally separated (also called "ionized"). Therefore, per stellar mass, globulars are expected to have far more double compact object systems than the low number density bulk of their host galaxies. For the same reason, there is a high rate per mass in globulars of low-mass X-ray binaries and millisecond pulsars (which are thought to have been spun up by accretion from a companion).

Another process that favors interactions of heavy objects is called "mass segregation": due to dynamical interactions, heavy things sink in the potential of a cluster. This is a process that might initially seem obvious (of course heavy things sink!) but then becomes inobvious (sink in what? This isn't a fluid). Basically, the idea is that in a two-body gravitational encounter between two objects, the speeds of the objects are determined by the gravitational potential of the cluster and are therefore likely to be close to each other. If one object is more massive than the other, it has more energy, so the drive toward equipartition of energy will tend to cause the heavier object to give energy to the lighter object. Lower energy means that the heavier object sinks in the gravitational potential. This has been shown in simulations to lead to a concentration of black holes in the central parts of clusters, where they can interact with each other.

Another popular process that can happen with multiple stars is the Kozai resonance (or Kozai-Lidov, or even more recently von Zeipel - Kozai - Lidov; it can be challenging to track down the origin of an idea). Kozai and Lidov discovered independently in the early 1960s (and von Zeipel discovered in the early 1900s) that if a binary is orbited by a third object in a hierarchical triple (such that the system has long term stability), and the binary orbital axis is strongly tilted with respect to the orbit of the tertiary, then over many orbits of both the binary and the tertiary, the relative inclination of the binary to the tertiary cycles between low and high values, while conserving the semimajor axis. Most importantly for merger possibilities, when the inclination goes down the eccentricity goes up and vice versa. Thus in the right range of orientations the binary could be driven to such a high eccentricity and thus such a low pericenter distance that gravitational radiation grinds it down to merger. Careful observations of massive binaries in our Galaxy suggest that 10% or more of them could actually be triples, so in principle such systems could evolve naturally to high-eccentricity states. As a caveat, if the system is susceptible to such evolution it is likely that it would be driven to collisions on the main sequence or giant branch rather than when the objects become compact. In dense stellar systems such as globular clusters, hierarchical Kozai-susceptible triples can be created *after* evolution to compact objects, for example as an outcome of binary-binary interactions.

Yet another possibility, although one with much smaller probability, is that two initially unbound compact objects could pass close enough to each other that the gravitational radiation they emit during their closest passage carries away enough energy to bind the objects together. They would then coalesce quickly. The reason that this is a very low-probability event is that the objects would have to come very close to each other to radiate the required energy, and thus the cross section for the process is tiny. If this happens, it seems most likely to happen during the many chaotic interactions that occur during a binary-single interaction rather than during a random encounter between two single objects. In any case, high stellar densities are obviously important for this mechanism to have any chance.

If such a coalescence does happen in a dense stellar system, then in principle it could have another coalescence later on. We could imagine the following sequence: (1) black hole is born in a globular cluster, (2) after wandering for a while, the hole encounters a binary and swaps in for one of the components, (3) later, another black hole swaps in for the remaining normal star, (4) a number of encounters later, the black hole binary has hardened enough and/or become eccentric enough that it mergers, leaving a single black hole, (5) that single black hole wanders and swaps into a new binary, and so on. This might even seem fairly probable, given that from the equation above we see that more massive objects have larger cross sections of interaction. In this way, it would be possible to have multiple generations of mergers and thus possibly have some kinds of mergers that aren't easy in other ways (for example, it would be straightforward to populate the putative pair instability mass gap if the black holes were actually the result of previous mergers).

But there's a kicker (pun intended). When a binary-single encounter tightens (or "hardens") the binary, then both the binary and the single star get a kick compared with their original motion. We can understand this because when a binary tightens its total energy (kinetic plus potential) becomes more negative. Conservation of energy then means that the kinetic energies have to become larger than they were, and thus both components get a kick. Moreover, the tighter the binary, the stronger the kick after a binary-single interaction. For systems such as globular clusters, with relatively low escape speeds of just a few times their velocity dispersion (thus a few tens of km $s^{-1}$), this means that it is very likely that prior to merger, a binary-single interaction will kick the binary out of the cluster. Furthermore, when two black holes merge, asymmetry in the emitted gravitational radiation can produce an additional kick, which might typically be tens to hundreds of km $s^{-1}$ but could reach thousands of km $s^{-1}$ for an optimal combination of mass ratio and black hole spins. A merger can then happen, but not a second merger. You could do better with a more concentrated star cluster, such as a nuclear star cluster in the center of a galaxy (which could have an escape speed of hundreds of km $s^{-1}$). The problem in that case, though, is that when the velocity dispersion is higher, more binaries are soft in the sense described above, so they are destroyed and thus the binary fraction is lower.

But another channel might be able to ride to the rescue: compact binary coalescences in the accretion disks of active galactic nuclei (AGN)! To understand how that might happen, we can start with the picture that the center of a galaxy usually has a supermassive black hole and lots of stars. If there is also an accretion disk going into the black hole, then this is an active galaxy. Among those stars are stellar-mass black holes. You might hope that as those black holes punch repeatedly through the accretion disk they might slow down and be captured, but that turns out to be improbable. However, it is believed that massive stars can *form* in AGN in some circumstances. If that happens and they end up forming black holes, then they can be dragged by the accreting gas toward the supermassive black hole. Because the rate at which they are dragged will in general depend on the mass of the stellar-mass black hole, two could meet, form a binary in the disk, and merge. Or, it has been suggested, there can be "migration traps" in the disk in which the net torque on an object is zero and thus a black hole could stall in its inspiral. In that case, multiple mergers might occur before a black hole can break out. A potentially exciting aspect of this scenario is that because the orbital speed at the putative migration traps is tens of thousands of km $s^{-1}$, kicks of hundreds or even thousands of km $s^{-1}$ can't eject a merged black hole, which makes a sequence of mergers easier than it is in other scenarios.

Now that we've talked about a few different ways to get compact objects together, how can we judge between them on the basis of observations? That turns out to be a lot harder than people had hoped. We'll talk about it in the next lecture, where we describe the observations so far.