Continuous and Burst Sources, and Stochastic Backgrounds

We'll end these lectures next time by discussing the future of ground-based gravitational wave astronomy. In this lecture we will instead discuss the other categories of sources. All non-binary sources are of unknown strength, which is another way of saying that if they are detected, we can learn a lot of astrophysics.

The first of these uncertain classes of sources that we will treat is continuous sources. A binary increases its frequency as it loses energy, and the lifetime of such sources is short (minutes at most) in the frequencies accessible to ground-based detectors. In contrast, a spinning source can in principle emit gravitational waves at a single frequency for a long time, so the signal builds up in a narrow frequency bin. As a result, particularly for high frequencies observable with ground-based detectors, continuous-wave sources are interesting because they can in principle be seen even at relatively low amplitudes.

What amplitude can we expect? From the first lecture we know that if the moment of inertia is I, then the amplitude is

$$h \sim (G/c^4)(1/r)(\partial^2 I/\partial t^2) \ . \qquad (1)$$

For binaries we argued that $I \sim MR^2$, and also we also imposed a relation between $\Omega^2 \sim \partial^2/\partial t^2$ and M and R. However, for a spinning source these relations do not have to hold. For a gravitationally bound source (e.g., neutron stars and not strange stars, which if they exist are self-bound and can therefore in principle rotate faster), Ω cannot be greater than the Keplerian angular velocity, but it can certainly be less. In addition, unlike for binaries, only a small fraction of the moment of inertia is involved in gravitational wave generation (indeed, if the spinning source is axisymmetric, no gravitational radiation is emitted). Let us say that some fraction ϵ of the moment of inertia is nonaxisymmetric. Generically this could be, e.g., a lump or a wave. Therefore, $h \sim (G/c^4)(1/r)\Omega^2\epsilon I$.

The luminosity is then

$$\begin{aligned} L &\sim r^2 h^2 f^2 \\ &= (32/5)(G/c^5)\epsilon^2 I_3^2 \Omega^6 \ , \end{aligned} \qquad (2)$$

where we have put in the correct factors for rotation around the minor axis of an ellipsoid (here I_3 is the moment of inertia around that axis), and we are now defining ϵ to be the ellipticity in the equatorial plane: $\epsilon = (a-b)/(ab)^{1/2}$, where the lengths of the principal axes of the ellipsoid are $a \geq b \geq c$.

Note the extremely strong dependence on Ω. When the correct factors are put in, we find that the strain amplitude from a pulsar of period P seconds at a distance r is

$$h_c \approx 4 \times 10^{-24} \epsilon P^{-2} (1 \ \mathrm{kpc}/r) \ . \qquad (3)$$

Pulsars spin down with time, and a benchmark is whether it would be possible to detect gravitational waves from a pulsar if *all* of the spindown were due to gravitational wave torques (this is called the spindown limit). The answer is that gravitational waves at the spindown limit would be detectable for a growing number of pulsars (led by the Crab pulsar), but nothing has yet been seen. We can also use models of neutron stars to judge how large ϵ could be. The answer is that, optimistically, ϵ could be as large as few $\times 10^{-6}$. For some millisecond pulsars, the limit based on nondetection of gravitational waves is $\epsilon <$ few $\times 10^{-9}$, which means that we're placing good limits. Now, the maximum *possible* (if you were to set up the neutron star carefully) isn't the same thing as the maximum we might actually expect, but it's still interesting that we're able to use nondetections to probe aspects of neutron stars that can't be analyzed in other ways!

Another possibility is that actively accreting neutron stars might balance the accretion torque by gravitational radiation losses of angular momentum. There is no *need* for this, given that magnetic torques can balance accretion torques, but if accretion produces a lump on the star (e.g., a "mountain" supported by locally strong magnetic fields) or if there is an ongoing wave (such as an "r-mode", which would have $\sim 2/3$ of the stellar rotation frequency), gravitational waves could be substantial. The extra challenge compared with searches for continuous waves from nonaccreting pulsars is that accreting neutron stars seldom present us with ultraregular pulses, and the accretion process itself changes the spin frequency. Thus it is not trivial to stack many cycles on top of each other. In the general case one must take into account the possibility of frequency wander, which requires enormous computational power. In any case, no continuous signals have yet been seen in gravitational wave data.

Burst Sources

The next category of gravitational wave sources is burst sources. These refer to events of very limited duration that do not have to have any special periodicity. Data analysis for these will be very challenging indeed, but since they are by definition associated with violent events, we could potentially learn a great deal from detection of gravitational radiation.

The most commonly discussed burst sources are core-collapse supernovae, and here we also include those core-collapse supernovae that are thought to produce long gamma-ray bursts (GRBs). When the core of a massive star collapses, it will not do so in a perfectly symmetric fashion. For example, convection will introduce asymmetries. What fraction of the mass-energy will therefore be released as gravitational radiation? This is a question that has to be answered numerically, but it is an extraordinarily challenging problem because it has to be done in three dimensions, with general relativistic magnetohydrodynamics and excellent radiation and neutrino transport, and a wide variety of scales are important!

Nonetheless, the current best guess is that only a very small fraction of the total mass-energy will come out in gravitational radiation, perhaps $\sim 10^{-10}$. If so, supernovae outside our galaxy will be undetectable. However, the rate of core-collapse supernovae in our Milky Way is estimated to be one every few decades, which means that there is a probability of tens of percent per decade that a supernova will occur within ~ 10 kpc. Current calculations suggest that the strain amplitude at 10 kpc could be $h \sim 10^{-20}$ for a millisecond or so, and maybe 10^{-21} for tens of milliseconds, which would be detectable with advanced ground-based instruments.

It could be that GRBs are the birth events for rapidly rotating black holes. If the rotation is rapid enough to produce triaxial ellipsoids then there could be much more substantial gravitational wave production. But even these would only be visible to a few tens of megaparsecs, and gamma-ray bursts are much farther than that, so the prospects aren't great.

Stochastic Backgrounds

For our last topic, we will focus on stochastic backgrounds, with an emphasis on primordial gravitational waves. Remember that "stochastic" means "the superposition of many individually unresolvable sources". Thus we need to think in terms of broad bands of frequency with many sources, rather than the signal produced by an individual source.

A background due to processes in the early universe (say, before the production of the cosmic microwave background) would be very exciting because it would contain information that is unavailable otherwise. In principle, one could see gravitational waves from very early in the universe, because gravitons have a very small interaction cross section. We need to state clearly that, even by the standards of gravitational wave astronomy, these processes are *highly* speculative. One consequence of this is that although it would be extremely exciting to detect a background of early-universe gravitational radiation, a nondetection would not be surprising.

Two primary mechanisms that have been explored for the production of an early-universe stochastic gravitational wave background are production during inflation, and production during a phase transition.

Various models of inflation have been discussed, but one that is considered relatively realistic is slow-roll inflation. In this model, the universe had a scalar field that, at the beginning of the inflationary period, was not at its minimum. The field value "rolls" towards the minimum and as it does so it drives rapid expansion of the universe. The rolling process means that the Hubble parameter (which relates the apparent recession speed to the distance: $v = Hd$) is not constant during inflation. Therefore, fluctuations that leave the Hubble volume during inflation and re-enter later have a tilt with respect to other fluctuations. The net result of calculations is that if standard inflation is correct then, unfortunately, there is no hope of detecting a gravitational wave background in the LVK frequency range, because the amplitude is orders of magnitude below what current or planned detectors could achieve. Variants of, or substitutes for, standard inflation have been proposed that might lead to detectable gravitational radiation, including bouncing-universe scenarios and braneworld ideas, but whether these encounter reality at any point is anyone's guess!

If phase transitions in the early universe (e.g., from a quark-gluon plasma to baryonic matter) are first-order, then by definition some thermodynamic variables are discontinuous at the transition. If the transition occurs in localized regions ("bubbles") in space, collisions between the bubbles could produce gravitational radiation. In addition, turbulent magnetic fields produced by the fluid motion could generate secondary gravitational radiation, but these are weaker. The most optimistic estimates suggest that this signal would peak in the millihertz range and would be detectable using LISA, but don't bet on it.

In 2014 there was an announcement from the BICEP2 team that they had detected the signature of gravitational waves from, likely, the inflationary epoch, in polarization patterns in the cosmic microwave background (CMB). The claim was based on the fact that, especially at degree scales and larger, the only source in the early universe that we think can produce polarization patterns with a nonzero curl (i.e., $\nabla\times$) is gravitational waves. See Wayne Hu's outstanding pedagogical pages at
http://background.uchicago.edu/∼whu/polar/webversion/node7.html for more details. The BICEP2 team did indeed see these so-called "B mode" polarization patterns, and they interpreted this as a confirmation of a prediction of inflationary theory (which has wide latitude in the amplitude of the predicted spectrum).

However, note that I said that gravitational waves are the only source in the *early* universe that we know can produce B modes. Scattering of light off of dust can perfectly well produce B modes in the current universe. The BICEP2 team thought they had accounted for this by looking at a dust map, but it turns out that this map was incomplete and it is now believed that their signal was dominated by dust effects. Nonetheless, there are many existing or planned experiments that will search for B modes in the CMB in the next few years. A claim that they have detected true primordial gravitational waves rather than dust will hinge on two important checks: (1) multifrequency observations will be needed to distinguish dust from gravitational wave signals, and (2) just as the standard CMB temperature power spectrum has characteristic peaks and dips, so does the predicted B mode power spectrum, so detection of those peaks at the right places, which will necessitate broad angular coverage, will be a critical test of the nature of the signal.

If any of these scenarios comes true and in fact there is a cosmological background of gravitational waves detected with planned instruments, this will obviously be fantastic news. However, what if it isn't seen? That would be disappointing, but there has been discussion about missions to go after weaker backgrounds. It is often thought that the $0.1-1$ Hz range is likely to be least "polluted" by foreground vermin (i.e., the rest of the universe!). This may be, but it is worth remembering that there are an enormous number of sources out there in even that frequency range, and that to see orders of magnitude below them will required *extremely* precise modeling of all those sources. Either way, whether we see a background or "merely" detect a large number of other sources, gravitational wave astronomy has wonderful prospects to enlarge our view of the cosmos.