

## Problem set 7

1. Suppose you were to observe the spin frequency of a pulsar with high precision over a long time. If you can measure the “braking index”  $\Omega\ddot{\Omega}/\dot{\Omega}^2$ , then you can in principle discriminate between spindown due to gravitational radiation and spindown due to magnetic torques. Recall that gravitational radiation gives  $\dot{\Omega} \propto \Omega^5$ , and magnetic dipole torques give  $\dot{\Omega} \propto \Omega^3$ . Use these to compute the braking indices for pure gravitational radiation and for pure magnetic dipole torques.

Suppose that a neutron star with zero magnetic field accretes matter from a companion at a rate  $\dot{M}$ . The rate at which angular momentum is accreted is  $\sqrt{12}\dot{M}GM/c$ . In the following, assume that the mass  $M$  and moment of inertia  $I$  of the star do not change significantly in the period of interest. In addition, we are working under the assumption that a lump, rather than a wave, produces gravitational radiation.

2. As we said in the notes, magnetic torques are likely to set the equilibrium spin frequencies of accreting neutron stars. However, just for the sake of argument, suppose that torques from gravitational radiation balance the accretion torques at a frequency  $\Omega_{\text{equil}}$ . As a function of  $\dot{M}$ ,  $M$ ,  $I$ , and  $\Omega_{\text{equil}}$ , calculate the required ellipticity  $\epsilon$ . Approximately what is the value of  $\epsilon$  if  $\dot{M} = 10^{17} \text{ g s}^{-1}$ ,  $M = 3 \times 10^{33} \text{ g}$ ,  $I = 10^{45} \text{ g cm}^2$ , and  $\Omega_{\text{equil}} = 2\pi \times 300 \text{ Hz}$ ?

3. Suppose that an accreting nonmagnetic neutron star has an observed electromagnetic flux  $F$ , which we assume is produced by accretion onto the surface: the luminosity is  $L = 0.2\dot{M}c^2$ , where the 0.2 is the efficiency of energy release. In the same spirit as the previous problem, suppose that the torque from the accreting matter is balanced exactly by gravitational radiation. Given this assumption, show that the flux in gravitational radiation has a simple relation to the electromagnetic flux, and find that relation. Assume isotropic emission of both.

4. One burst source some people have proposed is pulsar glitches. In a glitch, the spin frequency of the pulsar changes suddenly, due (we think) to a sudden coupling between the normal matter and the superfluid matter in the crust. The energy release is  $I\Omega\Delta\Omega$ , but  $I$  is the moment of inertia of the crust, which is perhaps  $10^{43} \text{ g cm}^2$ , or 1% of the moment of the inertia of the star (because the crust exists only at low densities). In a really big glitch,  $\Delta\Omega \sim 10^{-6}\Omega$ . Let’s say that such a glitch happens to a star with  $\Omega = 100 \text{ rad s}^{-1}$ , and that *all* the energy comes out in gravitational waves with frequency 2000 Hz (comparable to double the sound crossing frequency), in a period of only 1 second. If this is a very close source, at 1 kpc (or about  $3 \times 10^{21} \text{ cm}$ ), could this be seen with Advanced LIGO (sensitivity  $\sim 2 \times 10^{-23} \text{ Hz}^{-1/2}$  at 2000 Hz)?

5. Dr. Sane has heard about the possibility of seeing continuous wave emission from lumpy neutron stars using high-frequency ground-based detectors. He suggests that lumpy white dwarfs might be good sources for low-frequency space-based detectors.

To evaluate this suggestion, do the following. Recall that  $h \sim (1/r)(\partial^2 I/\partial t^2)$ . For a rotating source only the nonaxisymmetric part contributes, so let's say that  $I \sim \epsilon MR^2$ , where  $\epsilon \ll 1$ . Also, let  $\partial^2/\partial t^2 \sim \Omega^2$ , where  $\Omega$  is the rotation frequency. Supposing that the ellipticity  $\epsilon$  is the same for neutron stars and white dwarfs, how does the maximum possible  $h$  at a given  $r$  for white dwarfs compare with the maximum possible  $h$  for neutron stars? **Hint:** how fast can they rotate? Combine this with our calculation that gravitational waves can just barely (if at all!) be detected from rotating neutron stars, and the fact that high-frequency detectors have a better  $h$  sensitivity than low-frequency detectors, to evaluate Dr. Sane's idea.

6. Darth Sidious has taken an interest in the generation of gravitational waves. He plans to use the Death Star to destroy the planet Alderaan in such a way as to make the resulting gravitational waves visible throughout the universe to detectors that (coincidentally?) have roughly the sensitivity of Advanced LIGO. He has brought you in to consult on this plan. Can he do it? **Hint:** look at the notes about core-collapse supernovae, and remember that a planet like Earth has  $\sim \text{few} \times 10^{-6}$  times the mass of the core of a massive star. Be careful how you deliver your answer: the Emperor does not like bad news.