[05] Life as a Low-mass Star (2/8/18)

Upcoming Items

1. Read Ch. 17.3–17.4 for next class and do the self-study quizzes.
2. Homework #1 due next class.
3. How many of you read virial.pdf since the last class?

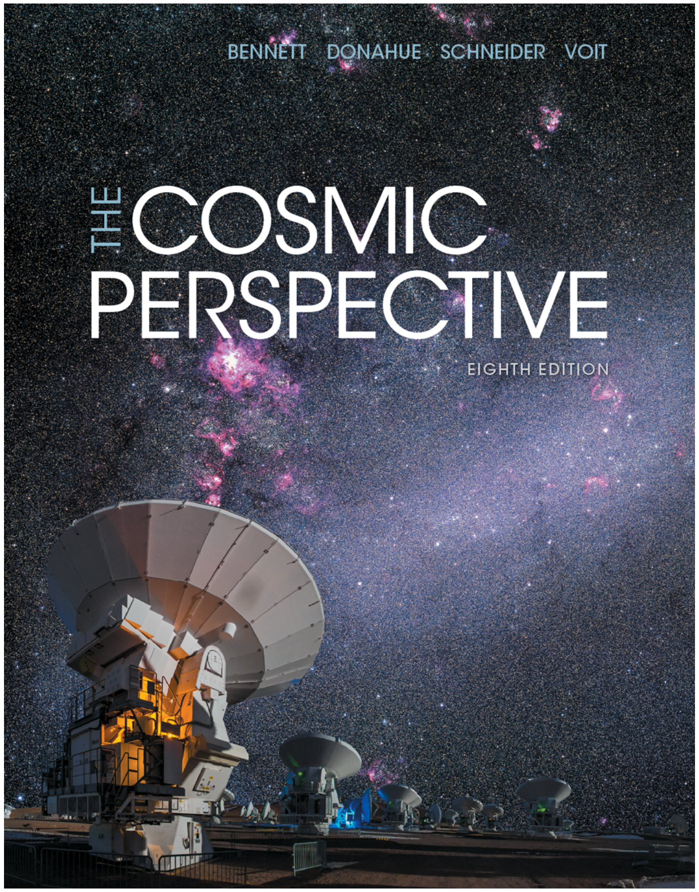
APOD 2/8/17: Butterfly Nebula

LEARNING GOALS

Ch. 17.1–17.2

For this class, you should be able to…

- … *estimate stellar lifetime given available fuel and burn efficiency, or using the stellar mass-luminosity relation;*
- … *predict the life track of a low-mass star on an H-R diagram given the processes that define its evolutionary stages, and explain why these stages occur;*



Any astro questions?

In-Class Quiz

1. What happens when a star can no longer fuse hydrogen to helium in its core?

A. The core cools off.
B. The core shrinks and heats up.
C. The core expands and heats up.
D. Helium fusion begins immediately.

2. Roughly how much shorter lived is a 16 $M_\odot$ star compared to a 1 $M_\odot$ star?

A. 10 times.
B. 100 times.
C. 1,000 times.
D. 10,000 times.

Hint: $\tau \propto M^{-2.5}$.

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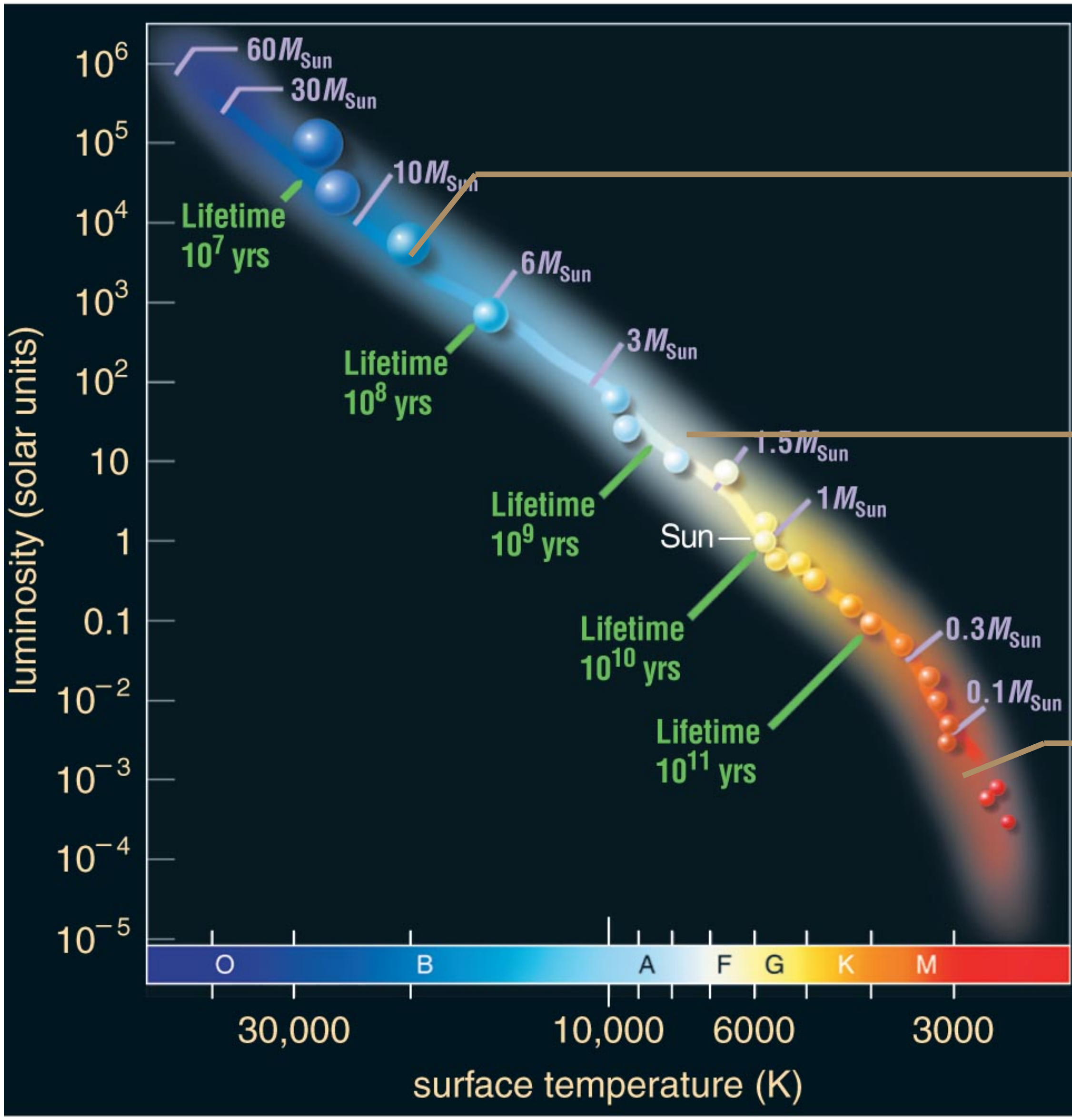
The destiny of a star

- Seen from a distance, stars are fairly simple objects
  Just balls of gas!
- Thus it might not be surprising that the destiny of a star is tied to its initial mass
- There's another correlated property, and for that we'll bring in a guest speaker...

Stellar Mass and Fusion

- The mass of a main-sequence star determines its core pressure and temperature.
- Stars of **higher** mass have higher core temperature and more rapid fusion, making those stars both more luminous and shorter-lived.
- Stars of **lower** mass have cooler cores and slower fusion rates, giving them smaller luminosities and longer lifetimes.



High-mass stars
$> 8\ M_\odot$

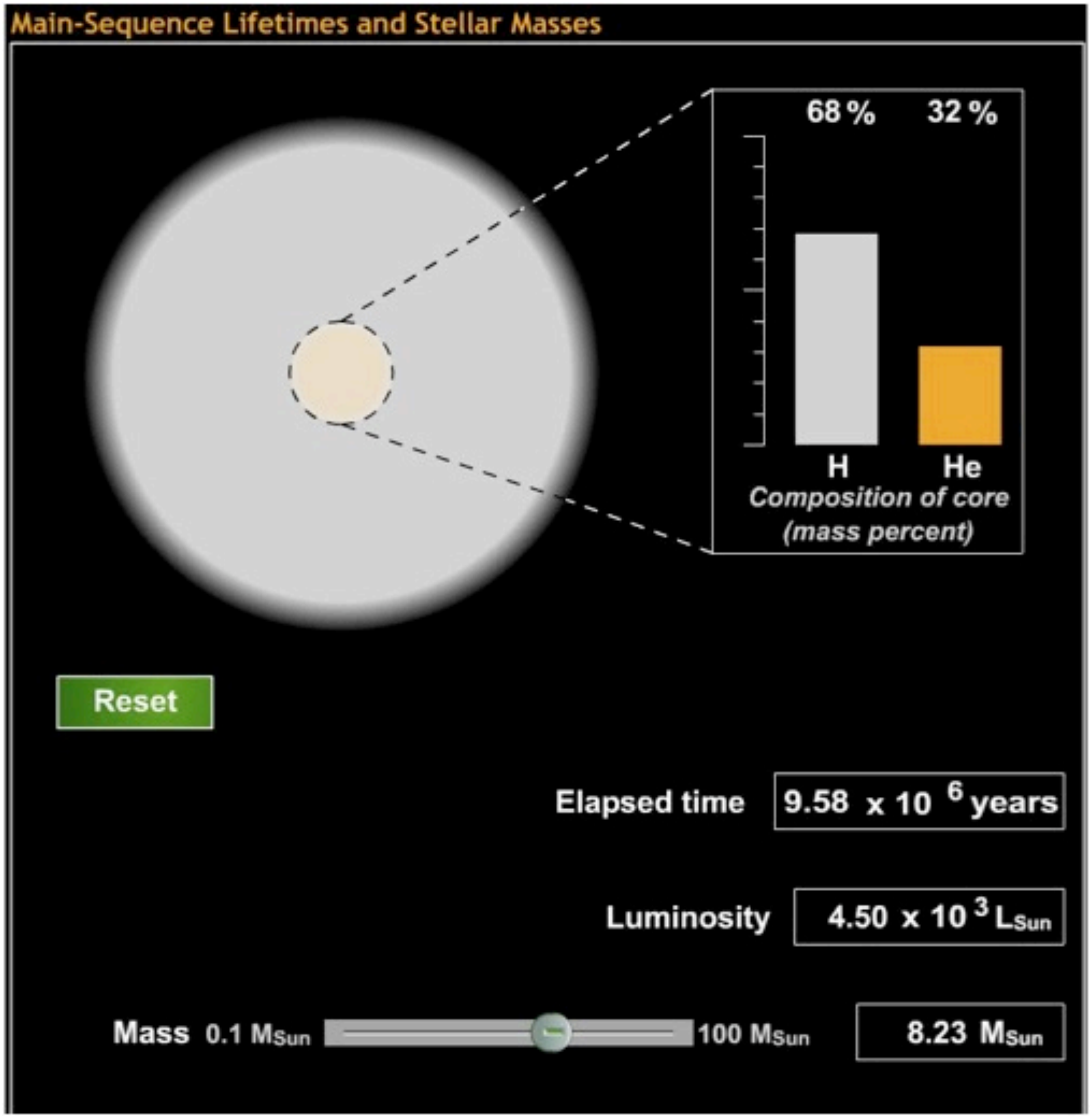
Intermediate-mass stars

Low-mass stars
$< 2\ M_\odot$

Brown Dwarfs

Star Clusters and Stellar Lives

- Our knowledge of the life stories of stars comes from comparing mathematical models of stars with observations.
- Star clusters are particularly useful because they contain stars of different mass that were born about the same time.



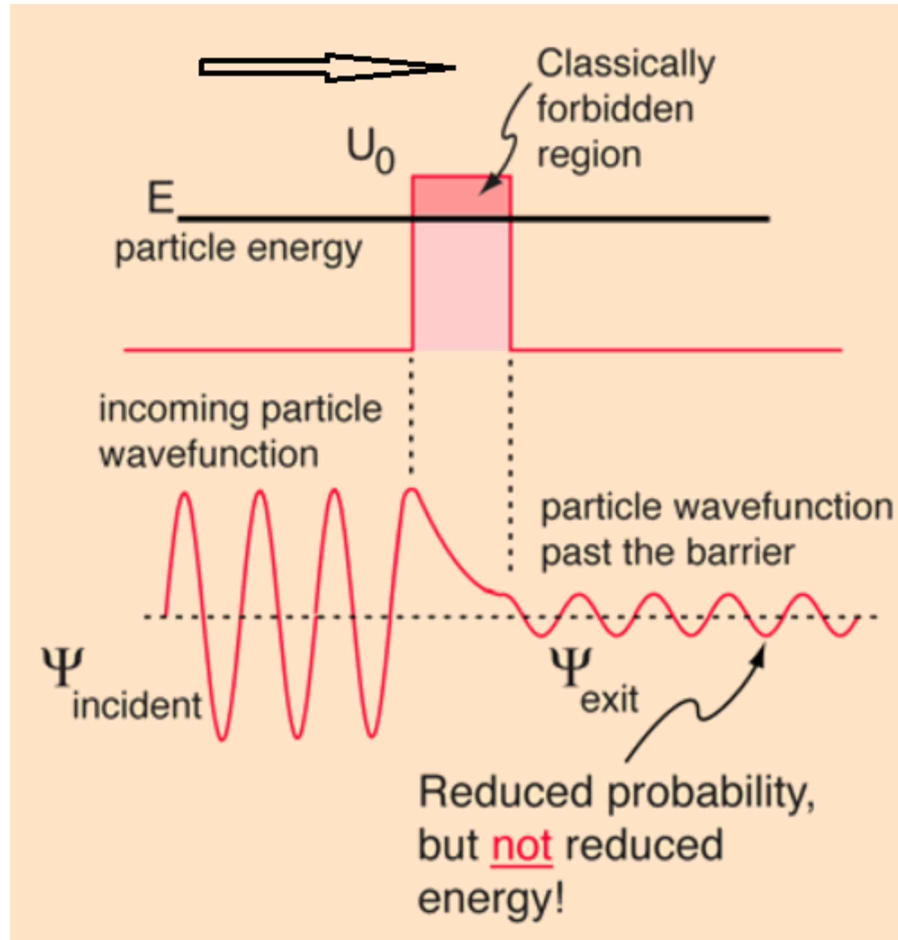
A star remains on the main sequence as long as it can fuse hydrogen into helium in its core.

Group Question: T for Fusion

- For protons to fuse together, they must approach within a distance $r \sim 10^{-15}$ meters
- The electrostatic energy at a distance r is
  $E_{elec} = e^2/r \sim (2.3\times10^{-28}\ \mathrm{J\ m})/r$
- You can equate this to the temperature you would need:
  $kT = E_{elec}$, and $k = 1.38\times10^{-23}\ \mathrm{J\ K^{-1}}$
- Your questions for discussion:
  What temperature do you need to get to 10^{-15} meters?
  What is the actual central temperature of the Sun?
  What does this imply?
  What is the resolution?

Quantum Tunneling!

- In quantum mechanics, particles are represented by wavefunctions
- Related to the probability of a particle being in a region
- A “classically forbidden” region (where the particle’s energy is too low) means an exponential decrease in the probability (but not to zero!)
- Result: “tunneling” can happen, which lowers the necessary T for fusion a lot!



https://i.stack.imgur.com/RkbTd.png

Context: How *Rapid* is Stellar Fusion?

- Fusion produces a lot of energy. But how long does it take the typical proton in the core of the Sun to fuse?
  Seconds?
  Years?
  Millions of years?
  Other?

Context: How *Rapid* is Stellar Fusion?

- Fusion produces a lot of energy. But how long does it take the typical proton in the core of the Sun to fuse?
  Seconds?
  Years?
  Millions of years?
  Other?
- The answer is *billions* of years. That's how long the Sun has lived, and will live.
- Thus a fusion event is very rare per proton; it's just that there are a whole lot of protons in the Sun!
- Exponential sensitivity of fusion to temperature means that main sequence stars all have *roughly* the same core temperature (why does that follow?)

Density, Mass, and the Virial Theorem

- This brings us back to our friend George McFly...
- In virial equilibrium (what does that mean?)
  Kinetic energy = -1/2 x gravitational potential energy
- The gravitational potential energy for a particle of mass m in a star of mass M and radius R is ~ -GMm/R
- The kinetic energy for that particle, at temperature T, is roughly kT
  Thus T~M/R for fixed particle mass
- We just argued that T is roughly the same for all main sequence stars (varies only by factor of 3 overall)
- Thus M/R is nearly constant, so R~M (or pretty close)
- Density ~ $M/R^3 \sim M/M^3 \sim 1/M^2$
- Lower-mass stars are denser on the main sequence
  Density really is destiny!

Review: Hydrogen Fusion

- Discussed this in the context of the Sun last semester.
- Basic process: 4H $\rightarrow$ He.
  - 0.7% of mass is converted to energy…
  - I.e., the efficiency of this process is $\eta \approx 0.007$:

$$\text{efficiency} = \frac{\text{energy released}}{(\text{total mass processed})c^2}.$$

  - About 10^6 times more efficient that chemical burning.
- Actual process by which hydrogen is fused depends upon the mass of the star.
- Question: what are the nuclear particles (not electrons) that make up four hydrogen nuclei? One helium nucleus? Are they the same? If not, what does that imply?

Hydrogen Fusion

- I don’t need you to know the details on the next two slides, but I do want you to know that there are multiple ways for hydrogen fusion to happen, and that the primary way changes as the mass increases
- We need to change four hydrogen nuclei (four protons) into one helium nucleus (two protons and two neutrons)
- Can this be done by having all four protons hit each other at the same time, with two of them converting into neutrons at that moment?

Proton-proton Chain

- For mass $M < 1.3\ \mathrm{M}_{\odot}\ldots$
  - Reactions proceed via **proton-proton chain**.
  - Reaction rate is exponential with temperature generally, but is proportional to $\sim T^4$ in the most relevant temperature range

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu$$

$$^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H}$$

CNO Cycle

- For mass $M > 1.3\ \mathrm{M}_{\odot}\ldots$
  - Reactions proceed via the **CNO cycle**.
  - Essentially, carbon acts as a "catalyst."
  - Reaction rate proportional to T^{20} in some T range
    Exponential generally

Incredibly steep temperature dependence!

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \gamma + \nu$$

$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma$$

$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma$$

$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu$$

$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{16}\mathrm{O} + \gamma$$

$$^{16}\mathrm{O} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} + \gamma$$

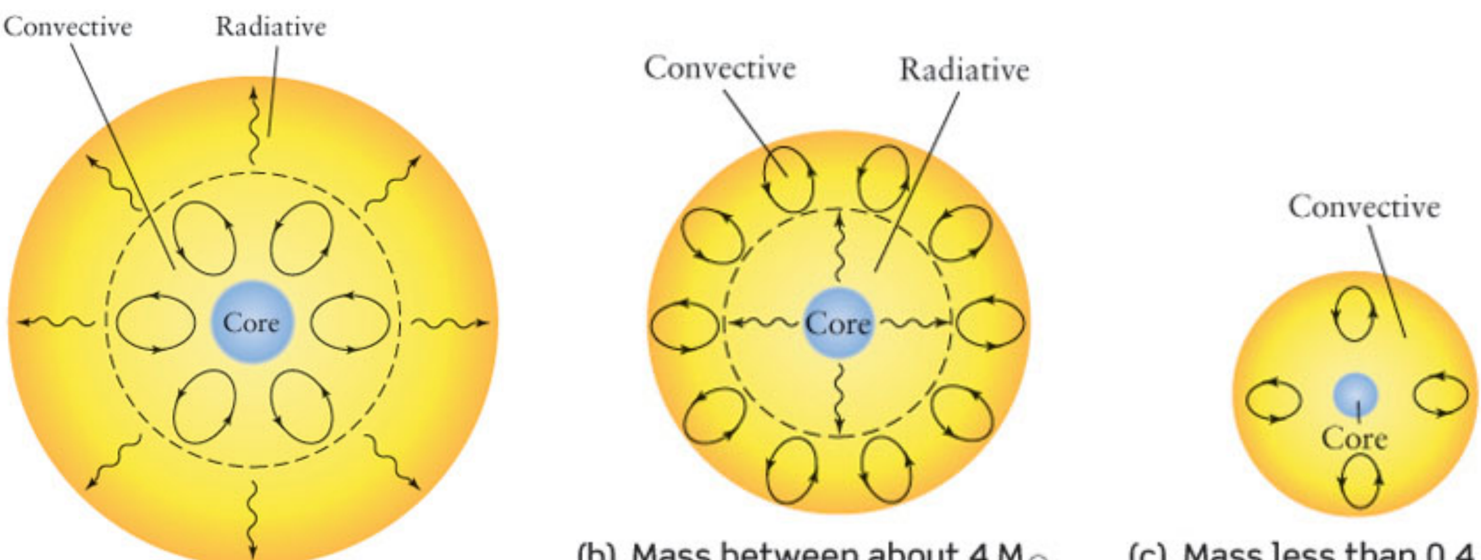
Main Sequence Lifetime

- A star leaves the main sequence once it exhausts its supply of hydrogen in the core.
- This lifetime depends on…
  - The luminosity L.
  - The efficiency of the fusion η.
  - The mass of the star M.
  - The fraction of M that can participate in the fusion reactions f.
- Total energy available to the star is: $E = \eta fMc^2$.
- So (assuming constant luminosity), the lifetime is:

$$\tau = \frac{E}{L} = \frac{\eta fMc^2}{L}.$$

Mixing

- What determines f ?
  - This is controlled by how effectively gas within the star is mixed.
  - Convective zones mix well; radiative zones mix poorly.
  - Stars with high rotation could mix almost completely
    Thus rather than inert He sitting in center, H is mixed in to fuse



(a) Mass more than about 4 $M_\odot$: Energy flows by convection in the inner regions and by radiation in the outer regions.

(b) Mass between about 4 $M_\odot$ and 0.4 $M_\odot$: Energy flows by radiation in the inner regions and by convection in the outer regions.

(c) Mass less than 0.4 $M_\odot$: Energy flows by convection throughout the star's interior.

Main Sequence Lifetime

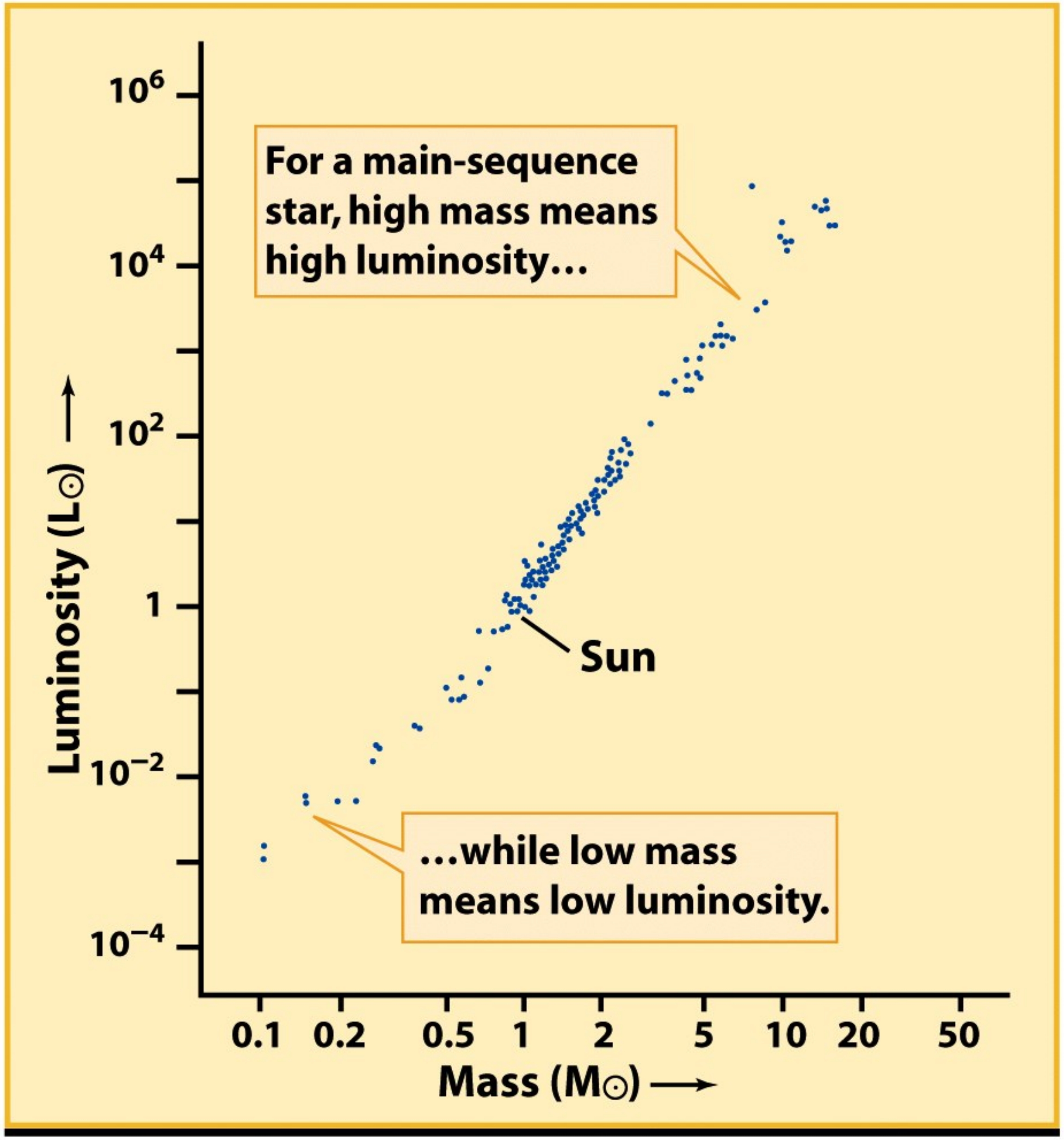
- For Sun, $f \sim 0.1$, so lifetime $\approx 1.0 \times 10^{10}$ yr (recall, $\eta \sim 0.007$).
- What about other stars?
  - Observations show that, for all but the most massive stars, $L \propto M^{3.5}$.
  - So, using formula for lifetime…

$$\tau \propto \frac{M}{L} \propto M^{-2.5}.$$

  - Scaling from the Sun we get,

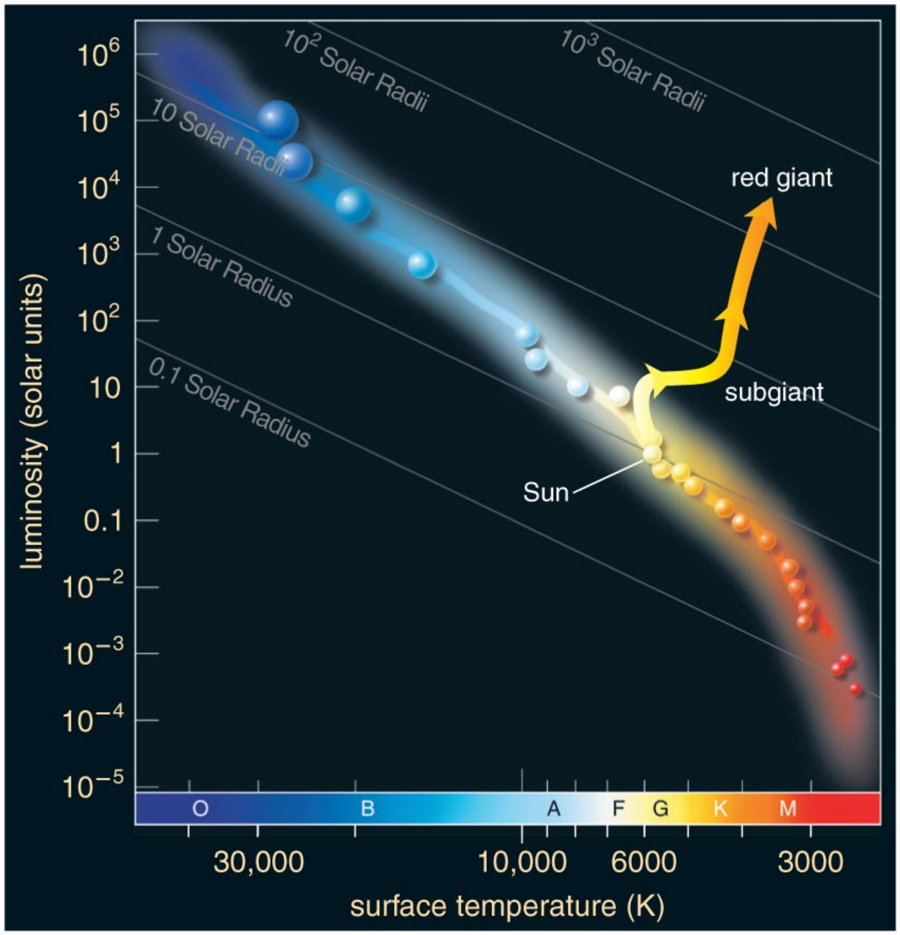
$$\tau \approx 1.0 \times 10^{10} \left(\frac{M}{M_{\mathrm{Sun}}} \right)^{-2.5} \text{ yr.}$$

  - Very important result!
    - Low-mass stars live for a long time.
    - High-mass stars have short lives.
  - Eddington luminosity ~M, so highest-mass stars have L~M and T~L/M=constant! About 2.5 million years for typical f.



For a main-sequence star, high mass means high luminosity…
…while low mass means low luminosity.
Sun
Luminosity (L⊙)
Mass (M⊙)
10^6
10^4
10^2
1
10^−2
10^−4
0.1
0.2
0.5
1
2
5
10
20
50

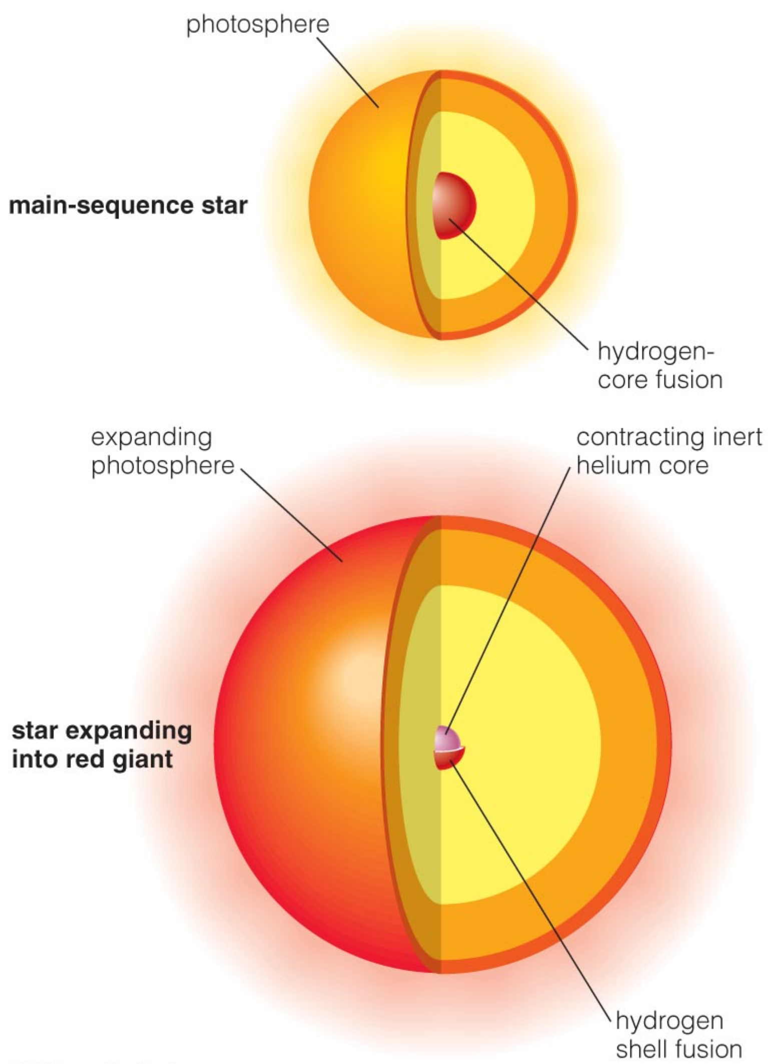
Life Track after Main Sequence



- Observations of star clusters show that a star becomes larger, redder, and more luminous after its time on the main sequence is over.

- Let's figure out why!

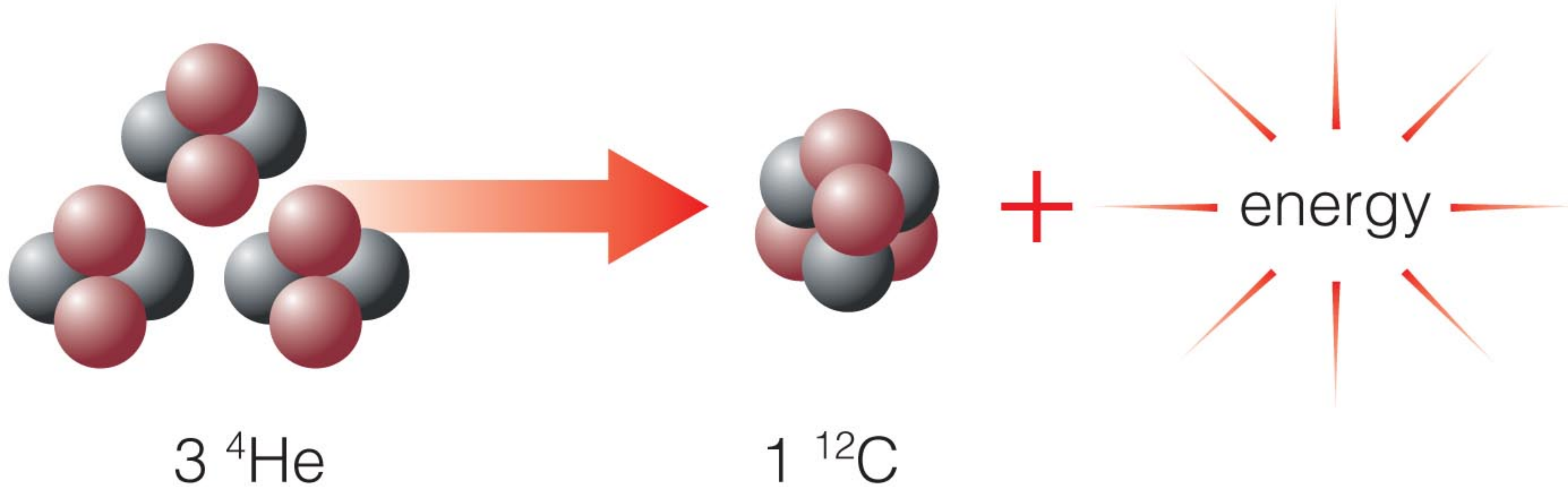
Red Giants: Broken Thermostat



- As helium core contracts, H will fuse to He in a *shell* around the core.
- Increasing fusion rate in shell does not stop the *core* from contracting.
- As core keeps shrinking, shell fusion gets more vigorous ➔ rest of star expands to form *red giant*.

The Core-Envelope Effect

- As core contracts and heats up, envelope expands and cools, despite greater luminosity.
  - The energy is spread over a greater surface area.
  - Expanding gases cool (ideal gas law).
- Once helium fusion begins, the core expands and cools, while the envelope contracts and heats up.
  - Envelope gravitational potential energy returned to thermal energy.
  - Luminosity higher than on main sequence because He burning more vigorous than H burning.
- Detailed solutions require running computer models.



- He-fusion doesn’t begin right away because it requires higher temperatures than H-fusion—larger charge leads to greater repulsion.
- Fusion of two He nuclei doesn’t work (^{8}Be is unstable), so He-fusion must combine *three* He nuclei to make carbon (C).

Triple-alpha Process

- At temperatures around 100 million K:

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \leftrightarrow {}^{8}\mathrm{Be} + \gamma$$

$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + e^{+} + e^{-}$$

- As a side-effect, also get a little of:

$$^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma$$

- Notice that unlike with hydrogen fusion, helium fusion does not require any protons to convert to neutrons
  Three heliums have six protons, six neutrons; 12C too!
- Thus no delay for weak decays, neutrinos
  As a result, in the right circumstances, can have burst!

Helium Flash

- In a low-mass red giant, the core *slows* its shrinking due to partial electron degeneracy pressure support, but continues to heat up from contraction.

- When the core is hot enough for helium fusion, initially the degeneracy pressure is higher than the thermal pressure, so the core *does not expand*—it just gets hotter!

- The core heats up very fast and the helium fusion rate skyrockets until thermal pressure takes over and expands the core again.