[09] Special Relativity (2/22/18)

Upcoming Items

1. Read Ch. S3 for next class and do the self-study quizzes
2. Homework #2 due on Tuesday
3. Advance warning: please read blackhole.pdf (in Files ->derivations) by next Thursday's class on BH

APOD 2/17/17: NGC 660

LEARNING GOALS

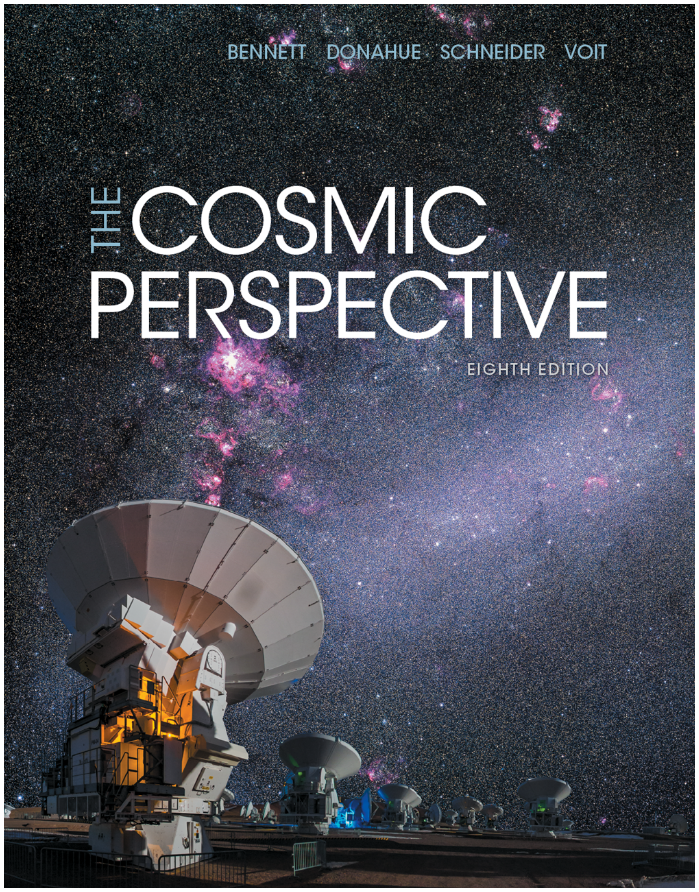
Ch. S2

For this class, you should be able to…

… *state the two postulates of special relativity and give an example of how they challenge our everyday intuition;*

… *explain what is meant by an inertial frame of reference and give an example;*

… *calculate how perception of time and measures of length and mass change depending on the relative motion of observers in inertial reference frames.*



Any astro questions?

Why do we care?

- In this class we'll talk about special relativity (which deals with reference frames that move at close to the speed of light relative to each other) and in the next one, general relativity (which is our best current theory of gravity)
- I hope that, like me, you'll find these to be fascinating and worthy of study in their own right
- But this is an astronomy class; why should we care?
- Because in cosmology and other applications, things appear to move very fast with respect to us
- And because white dwarfs, neutron stars, and black holes, as well as the universe as a whole, need to be described using general relativity

Special Relativity

- Postulates of special relativity:
  1. The laws of physics are the same in all inertial frames.
  2. The speed of light in vacuum is the same in all inertial frames.
- A frame that is at rest or at constant velocity is ***inertial***.
- Formulas of special relativity: $\gamma=1/(1-v^2/c^2)^{1/2}$; Lorentz factor
  - Time interval dilation: $t = \gamma(1\text{-}v/c)t_{\text{proper}}$ (toward); $\gamma(1+v/c)t_{\text{proper}}$ (away)
  - Length contraction: $l = l_{\text{proper}}/\gamma$.
  - Mass (momentum) increase: $m = \gamma m_{\text{proper}}$.
  - Velocity addition: $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$. For motion in a line
  - Doppler shift: $z = [(1 + v/c)/(1 - v/c)]^{-1/2} - 1$. For motion directly away
  - Mass-energy relation: $E = mc^2$.
- Relativity has been tested, and it agrees with all expts.

Group Q: The Twins Paradox

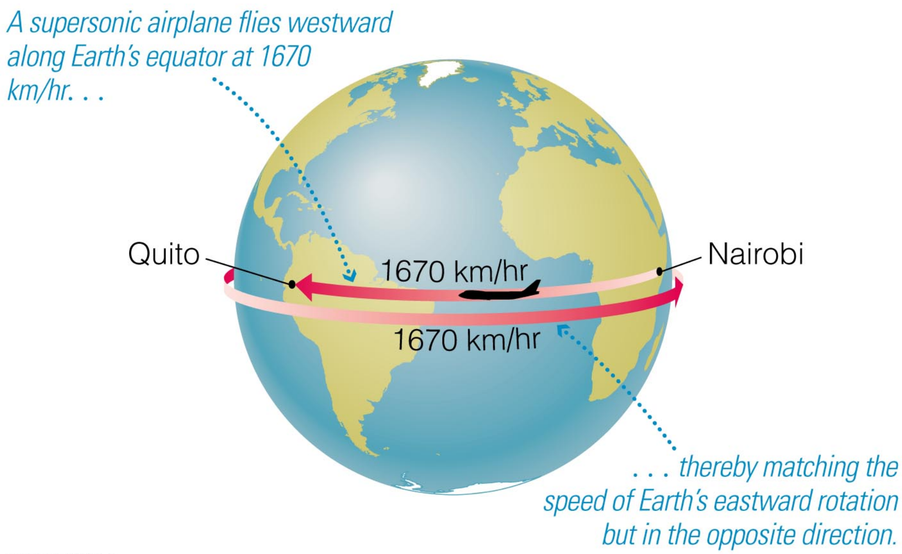
- Pat and Drew, identical twins, perform an experiment
- Pat stays put
- Drew starts in a spaceship going 0.9c directly away from Pat, travels for 10 light years, turns around and immediately goes at a speed of 0.9c directly toward Pat.
- When they meet again, and compare their ages, who has aged more?
- You might say: Pat sees Drew moving and thus sees Drew's clock moving slowly; expects Drew to be younger
- But we might say: Drew sees Pat moving and thus sees Pat's clock moving slowly; expects Pat to be younger
- Who is right? Hint: it has nothing to do with acceleration!

Equivalence Principle, Part 1

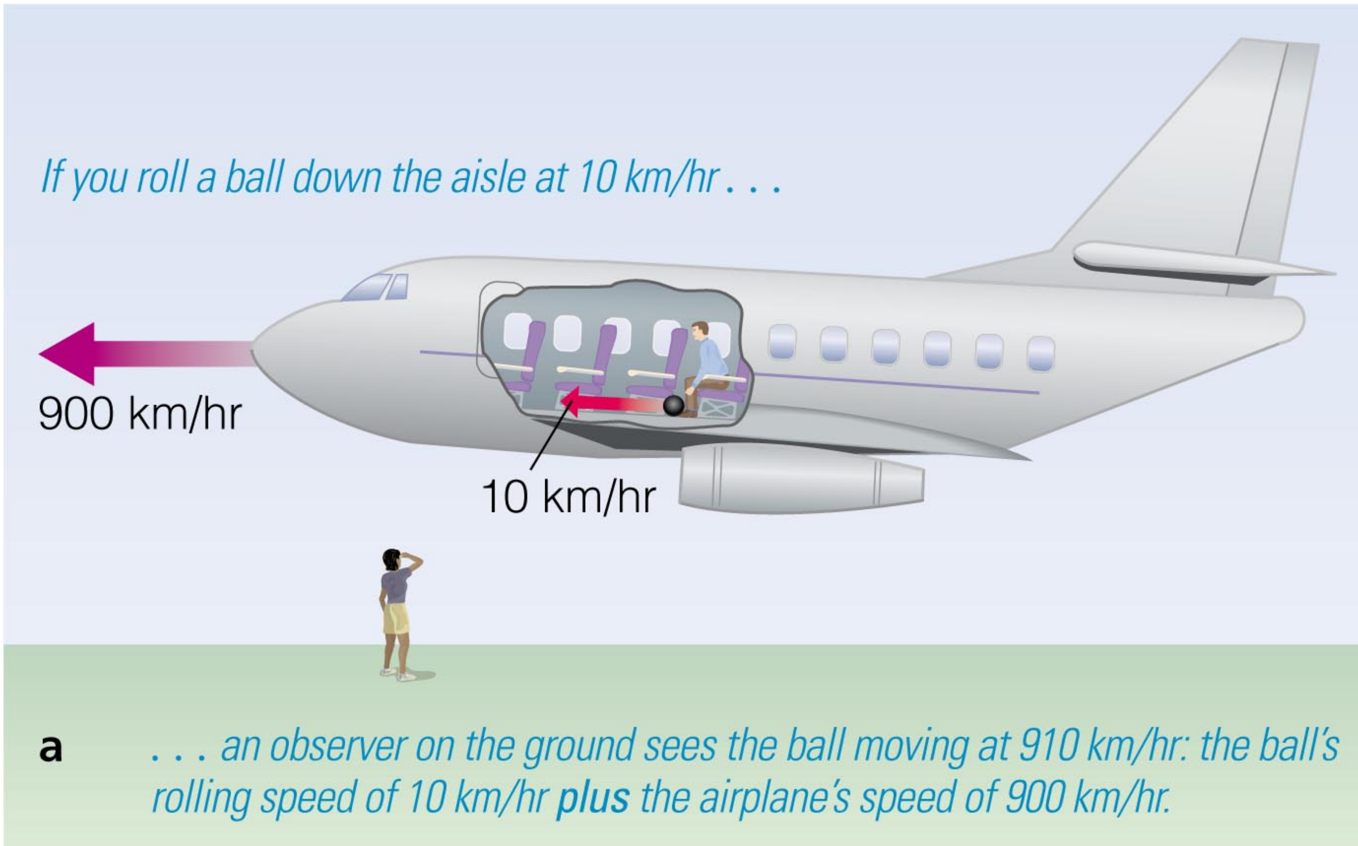
- We'll return to this in a more general form next class
- But let's say that you are in a small container without any view of the outside world
- In this container, you feel *no* acceleration, not even gravity
- Then the result of any experiment you perform will be the same as the result of the same experiment as performed by any other observer in their own small container who doesn't feel any acceleration
- This is true even if the two of you have relative motion
  Even if that motion is close to the speed of light
- There is no "absolute" reference frame, or "true rest"
  Observer in plane, boat, ground; all equivalent
  Motion is relative

Relativity of Motion

- Motion is not absolute—we must measure the speed of one object *relative* to another.
- Example: A plane moving along the equator at 1670 km/hr from east to west would appear from space to be standing still.

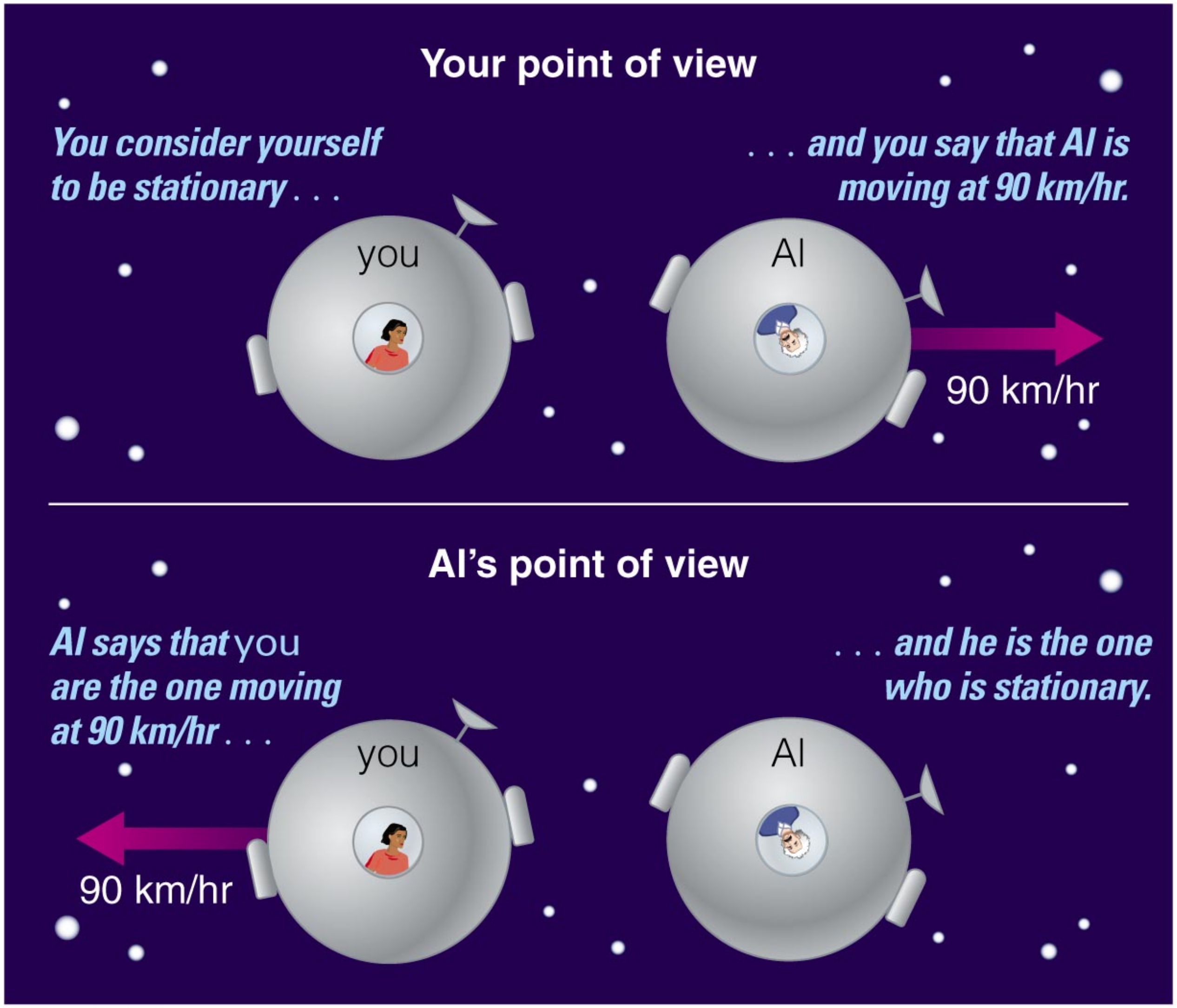


Reference Frames

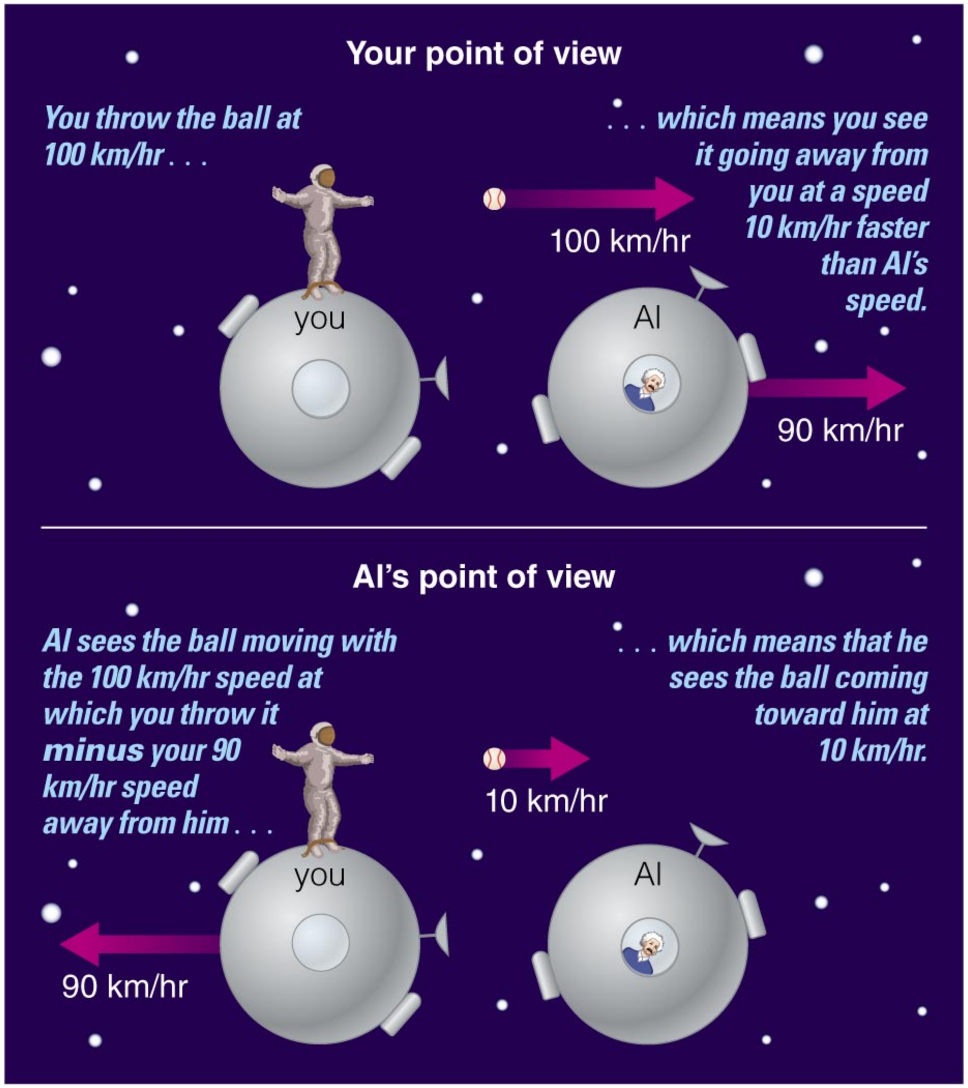


- Motion can only be defined with respect to a particular frame of reference.

Relativity of Motion at Low Speeds



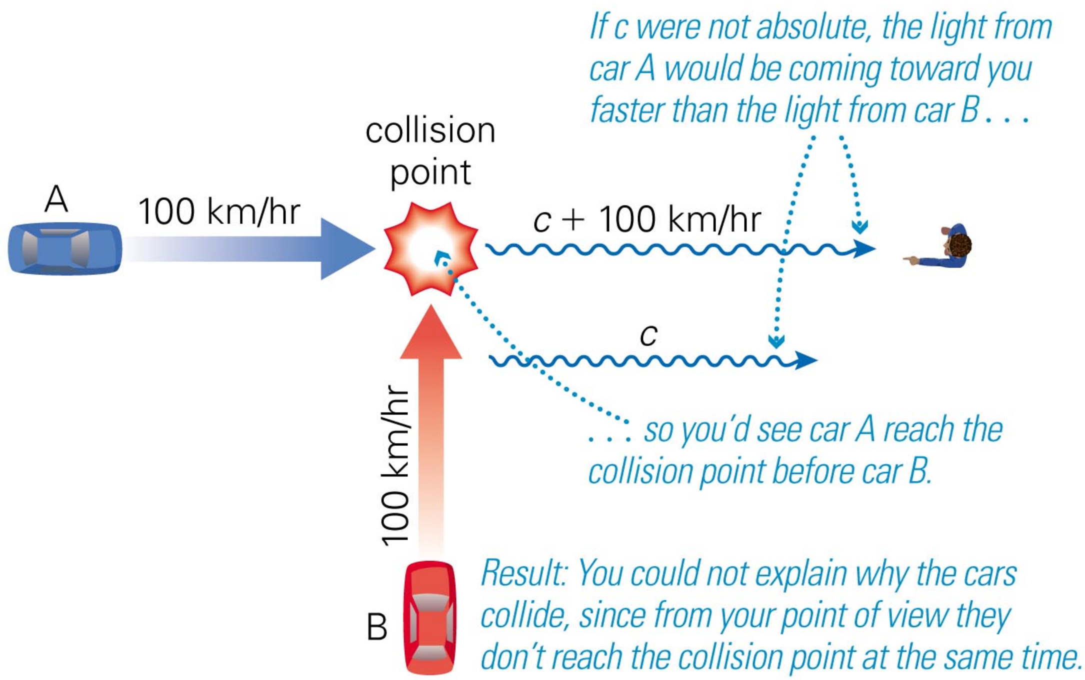
Relativity of Motion at Low Speeds



What about speeds near that of light?

- We might think that such “velocity addition” would work at any speed, including the speed of light in a vacuum
- But let’s consider a situation that should motivate us to give that a second thought

A Paradox of Non-relativistic Thinking

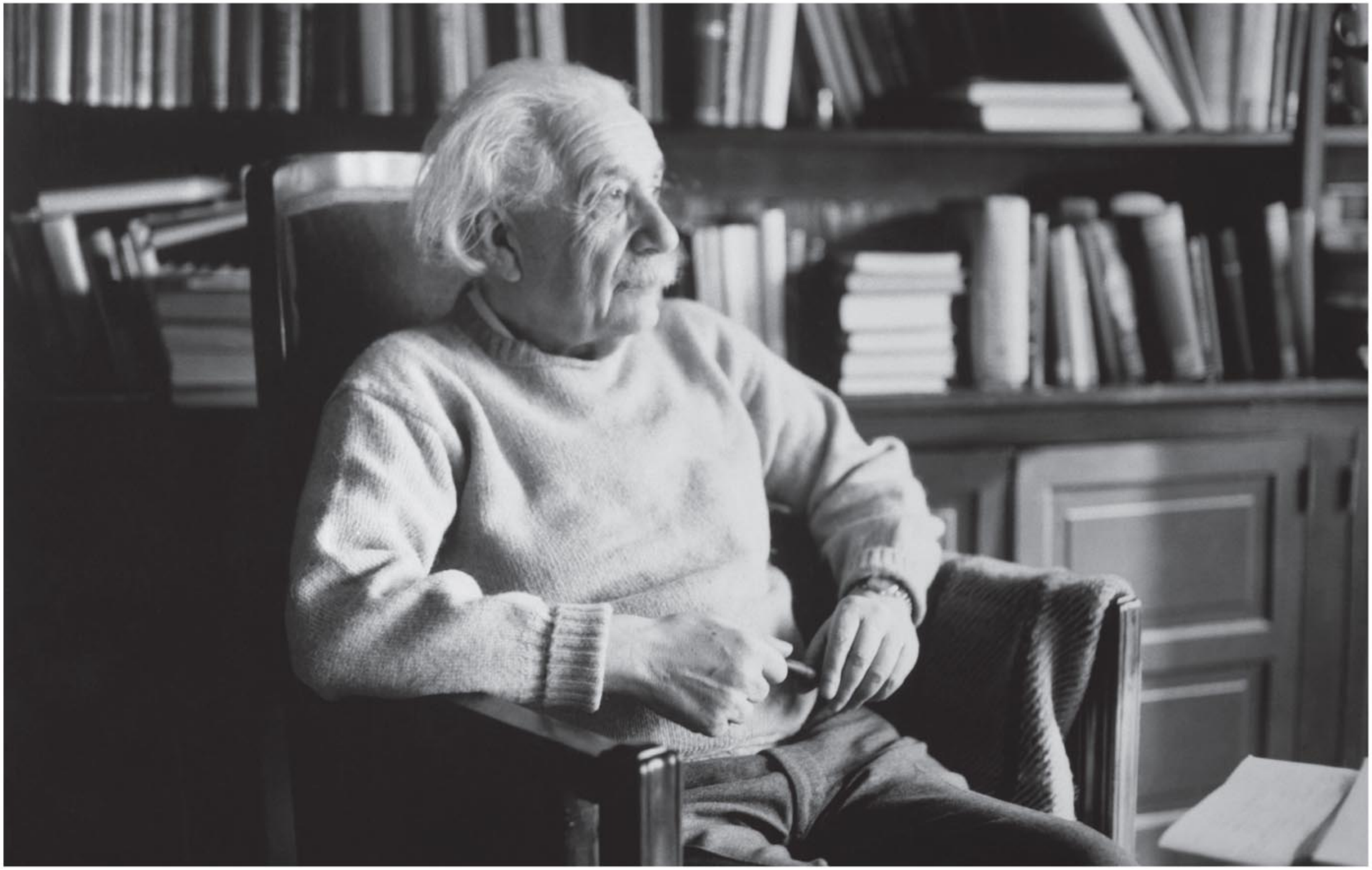


- If the speed of light were not absolute, you would see the car coming toward you reach the collision point before the car it struck.
- There is no paradox if light speed is same for everyone.

The lesson, and why we accept SR

- The lesson of the previous paradox is ***not*** that we could arrive at special relativity by pure thought
  In fact, if you think deeply about the "paradox", there are some loopholes
- Instead, the idea was to get you to at least consider that something funny might happen at high speeds
- But the reason that we accept special relativity, or general relativity, or quantum mechanics, or any other scientific theory is that its predictions match with what we observe
  That's science!
- For SR, GR, or QM, the realm is very different from our everyday lives; so the results can contradict our intuition!

Einstein's Theories of Relativity



Einstein's Theories of Relativity

- Special Theory of Relativity (1905)
  - Usual notions of space and time must be revised for relative speeds approaching light speed (c).
  - $E = mc^2$ (equivalence of mass and energy).
- General Theory of Relativity (1916)
  - Expands the ideas of SR to include a surprising new view of gravity.

Key Ideas of Special Relativity

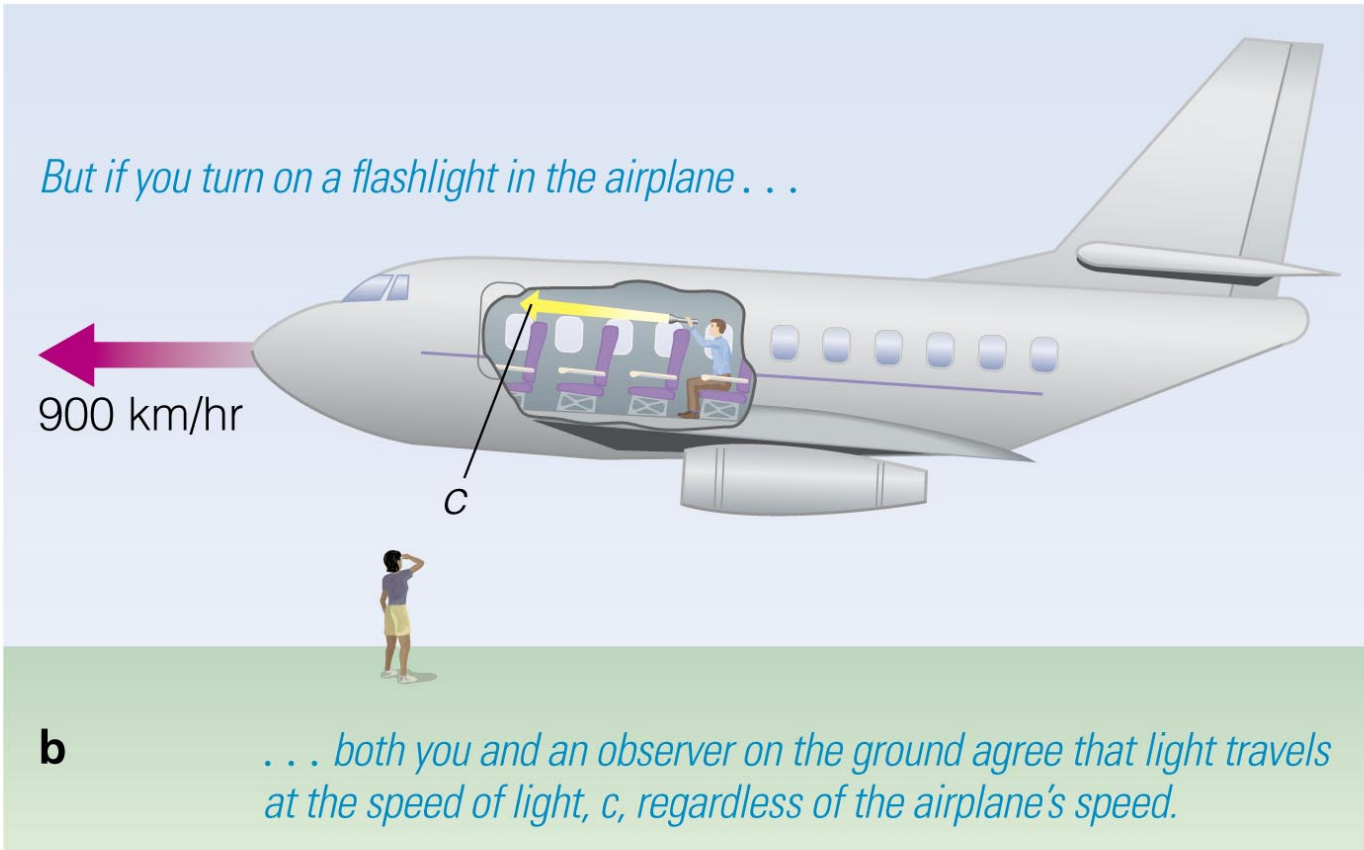
- Nothing can travel faster than the speed of light in a vacuum; nothing with nonzero rest mass can even travel ***at*** the speed of light in a vacuum
- If something moves past you near light speed:
  - Its time slows down according to you.
  - Its length contracts in direction of motion according to you.
  - Its mass increases according to you.
- Whether or not two events are simultaneous depends on your perspective.
  But causality is ***not*** relative! If I fire a gun and kill you, everyone will see it in that order

Postulates of Special Relativity

1. The laws of nature are the same for everyone.
2. The speed of light in a vacuum is the same for everyone.

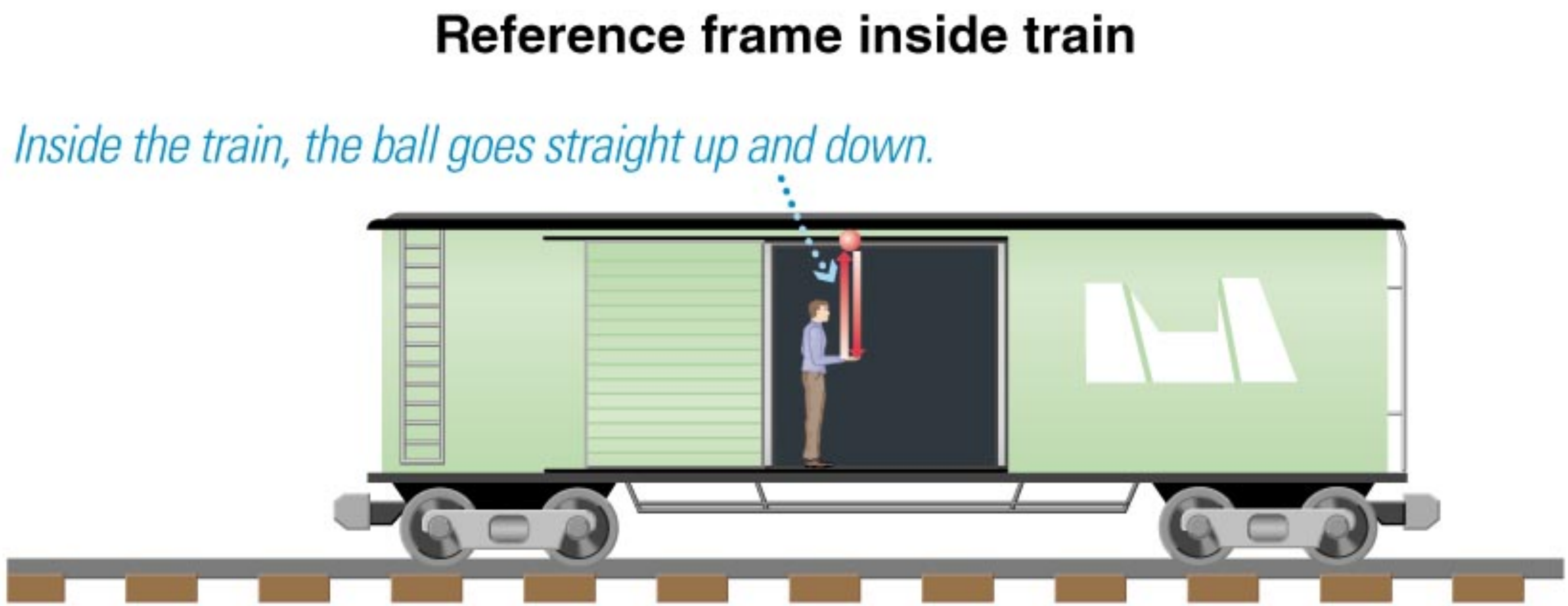
All of special relativity follows from these two ideas!

Absoluteness of Light Speed



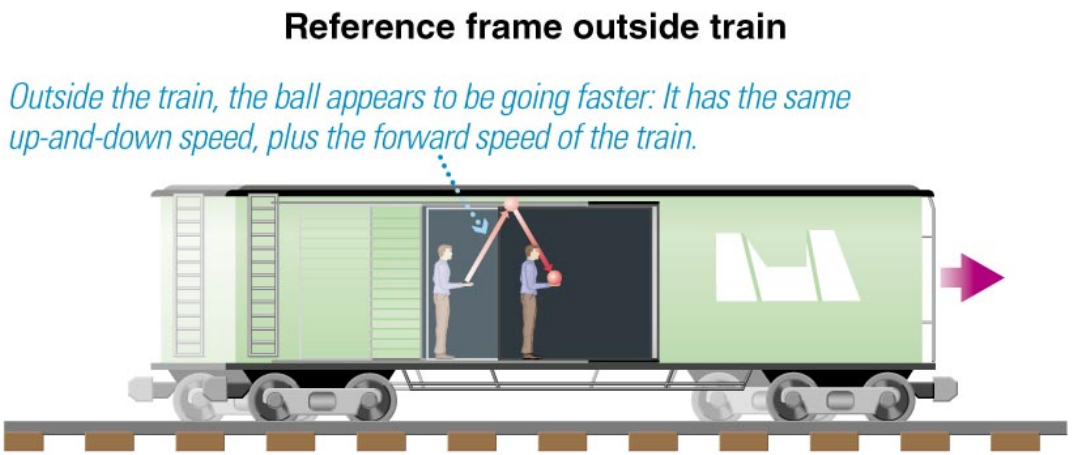
- Einstein claimed that light in a vacuum moves at exactly c in all reference frames (now experimentally verified).

Path of Ball in a Stationary Train



- Thinking about the motion of a ball on a train will prepare us for the next thought experiment.

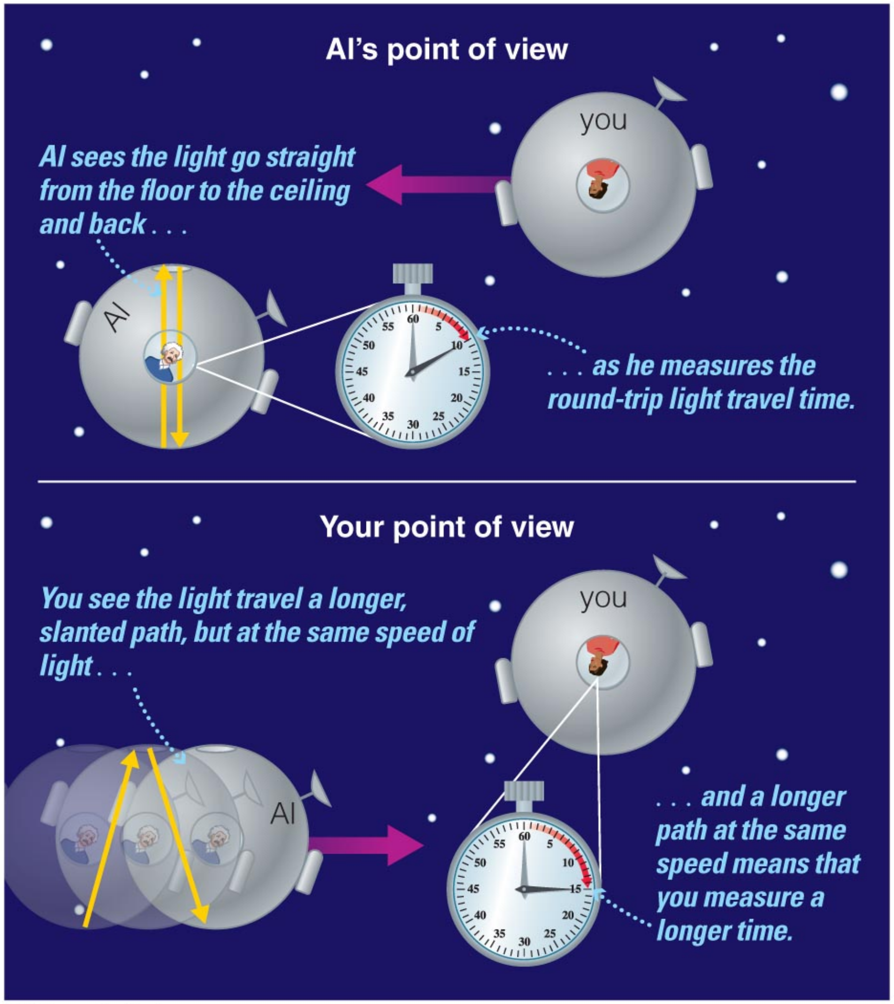
Path of Ball in a Moving Train



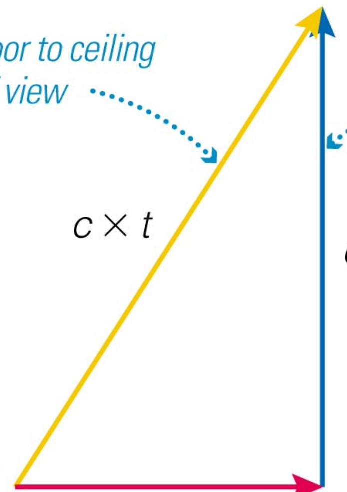
- Someone outside the train would see the ball travel a longer path in one up-down cycle.
- The faster the train is moving, the longer that path would be.

Time Dilation

- We can perform a thought experiment with a light beam replacing the ball.
- The light beam, moving at c, travels a longer path in a moving object.
- Time must be passing more slowly there.



The Time Dilation Formula



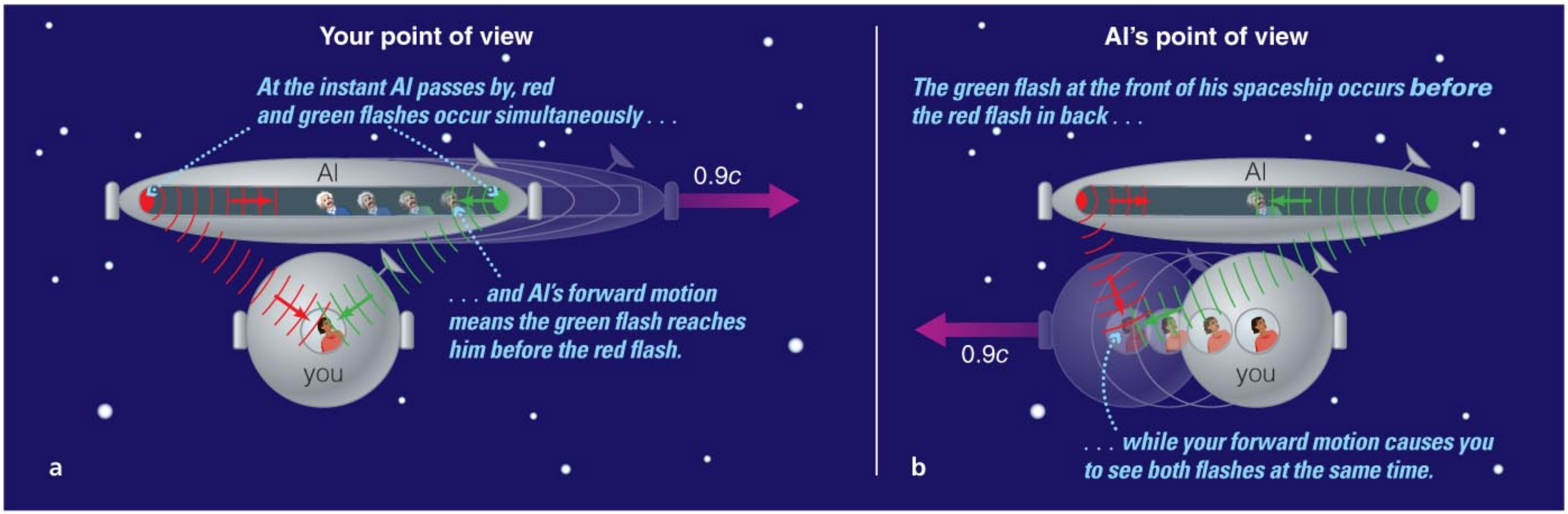
Clocks moving transverse to you run slow.

Here t is the time YOU measure;
t' is the time Al measures, i.e.,
the "proper" time in the rest frame
($t' = t_{\text{proper}}$ on formula sheet)

- Use Pythagoras:
  $$v^2t^2 + c^2t'^2 = c^2t^2$$
- Solve for t:
  $$t^2 = \frac{c^2}{c^2 - v^2} t'^2$$
  $$\boxed{t = \frac{1}{\sqrt{1 - v^2/c^2}} t'}$$
- The quantity $\frac{1}{\sqrt{1-v^2/c^2}}$ is called the *Lorentz factor*, gamma (γ).

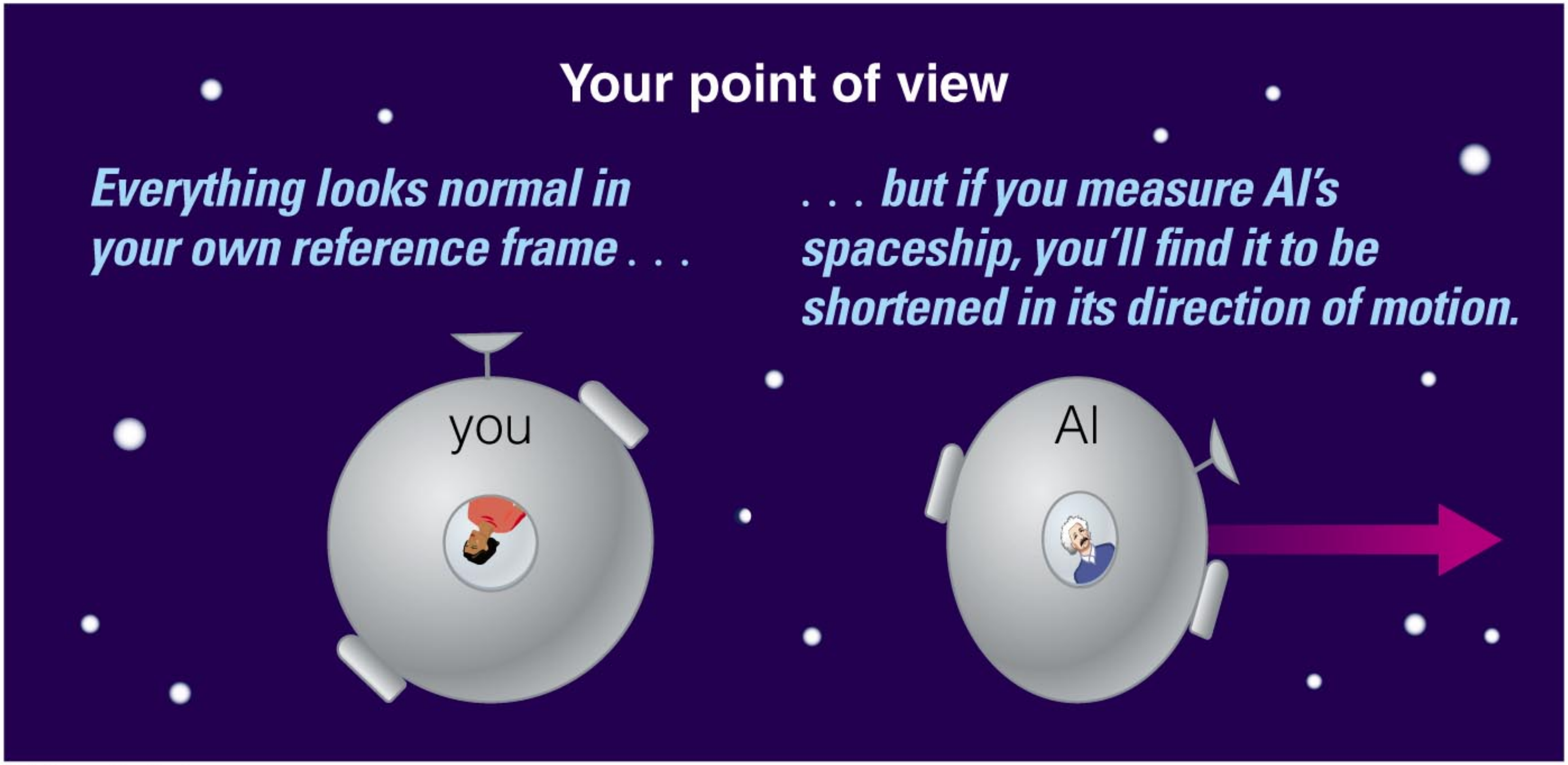
Note: Al sees *your* time dilated by exactly the same factor that you see *his* dilated!

Simultaneous Events?



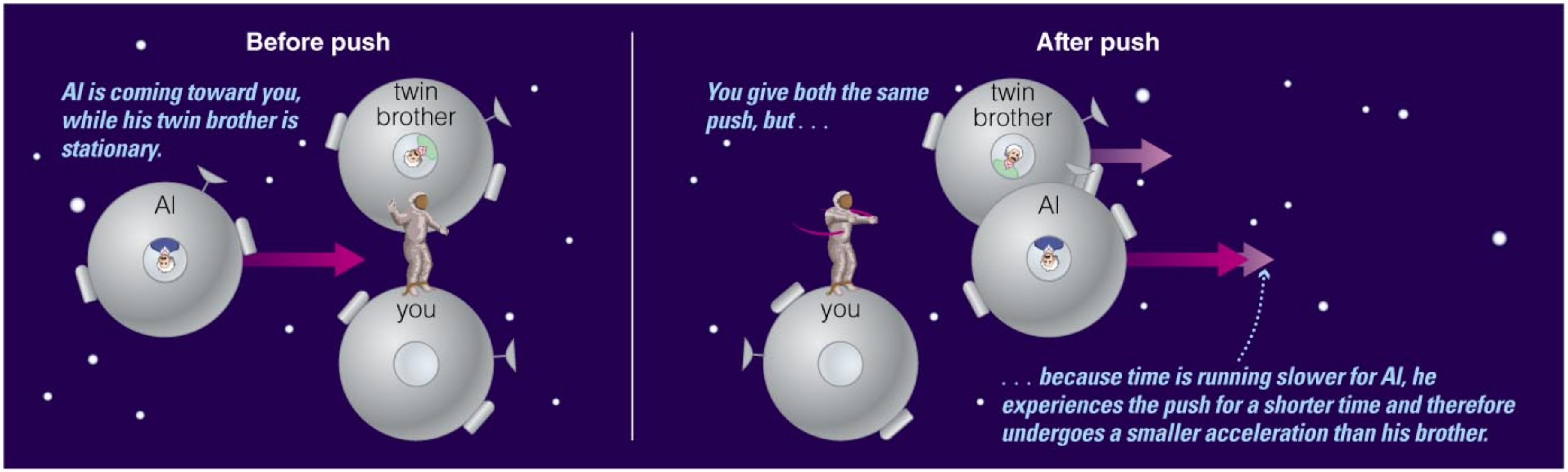
- In your reference frame, red and green lights on other spaceship appear to flash simultaneously.
- But someone on the other spaceship sees the green light flash first—simultaneity is relative! Causality is ***not*** relative

Length (Fitzgerald) Contraction



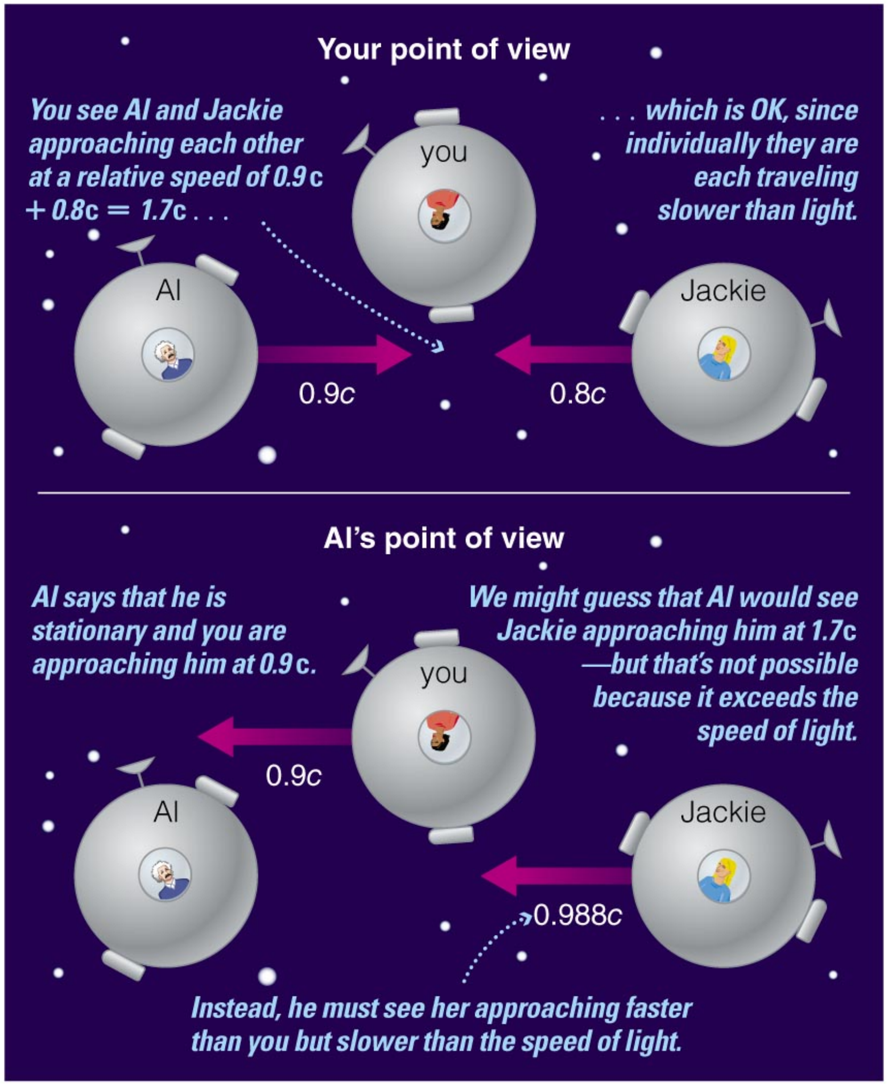
- Similar thought experiments tell us that an object's length becomes shorter in its direction of motion.

Mass (Momentum) Increase



- A force applied to a rapidly moving object produces less acceleration than if the object were motionless.
- This effect can be attributed to a mass increase in the moving object.

Velocity Addition (1-D)



- Suppose 2 spaceships approach each other (in your frame) at speeds v_1 and v_2.
- Then, from the point of view of someone in one of the ships, the other is approaching at speed

$$v = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2}.$$

- If either v_1 or $v_2 = c$, the result $= c$.

Tests of Relativity

- First evidence for absoluteness of speed of light came from the Michelson-Morley experiment performed in 1887.
- Time dilation happens routinely to subatomic particles that approach the speed of light in accelerators.
- Time dilation has also been verified through precision measurements in airplanes moving at much slower speeds.
- Prediction that $E = mc^2$ is verified daily in nuclear reactors and in the core of the Sun.

More info on Michelson-Morley and the luminiferous ether from Khan Academy: https://goo.gl/3OObM7