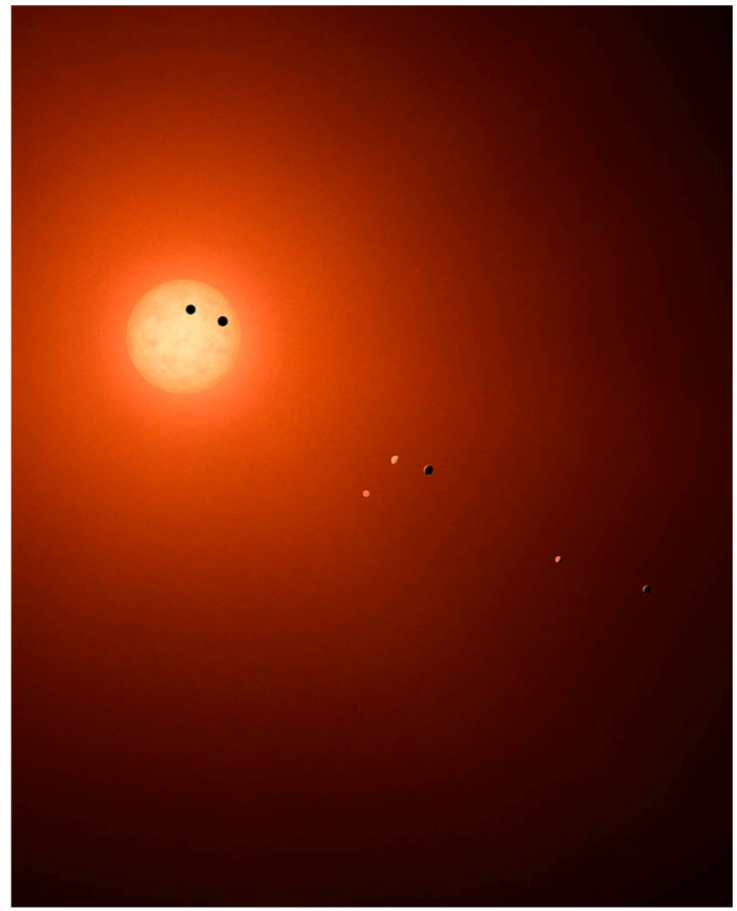
[10] General Relativity (2/27/18)

Upcoming Items

1. Read Ch. 18.3–18.4 for next class and do the self-study quizzes.
2. Read blackhole.pdf on Files->derivations
3. Homework #2 due today!
4. Midterm #1 in one week (Ch. S2, S3, 15–18). Cover sheet and formulae available: Files->exams
5. Exam reviews: Friday discussion and Monday 6-8 PM in this classroom Driven 100% by your Qs!

APOD 2/23/17: TRAPPIST-1



LEARNING GOALS

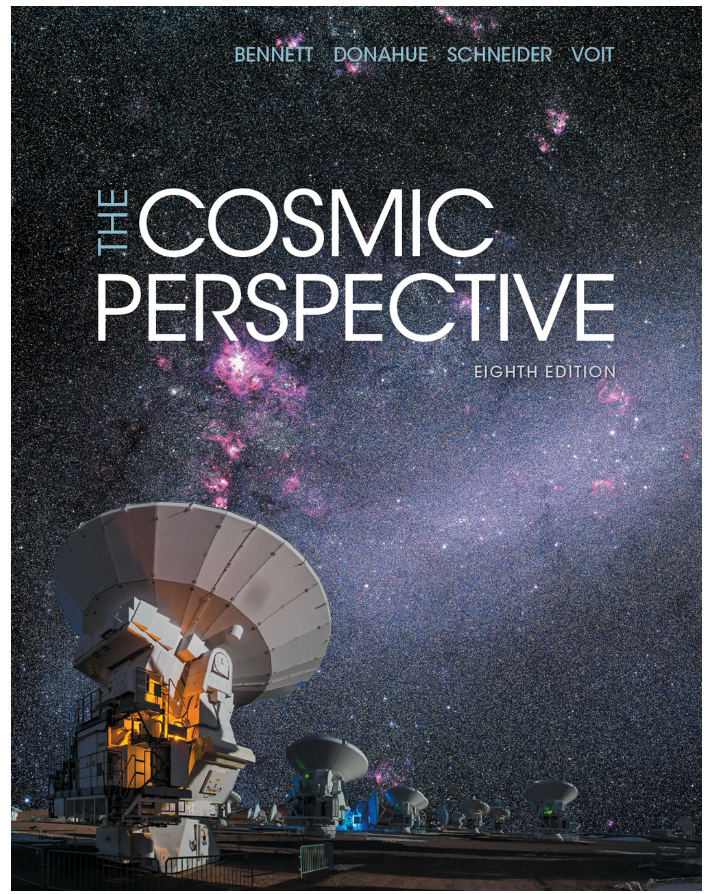
Ch. S3

For this class, you should be able to…

… *show that the Einstein tower implies that the presence of mass causes light to be redshifted and makes clocks run slow;*

… *describe how the equivalence principle leads to the conclusion that a light path must appear to bend near mass;*

… *describe a successful test of general relativity in an astronomical context.*



Any astro questions?

In-Class Quiz

1. According to or by invoking the equivalence principle, which one of the following statements is **false**?

A. If you feel weightless in a closed box, you cannot tell if the box is in empty space or in free-fall over a planet.
B. If you feel your own weight in a closed box, you cannot tell if the box is accelerating in empty space or resting on the surface of a planet.
C. Free-fall is an inertial frame.
D. In the presence of mass, light moves in a straight line.

2. Which of the following is ***not*** a test of general relativity?

A. The extra precession of the pericenter of Mercury
B. The extended lifetime of muons in particle accelerators
C. Gravitational lensing
D. Gravitational redshifts
E. Gravitational waves

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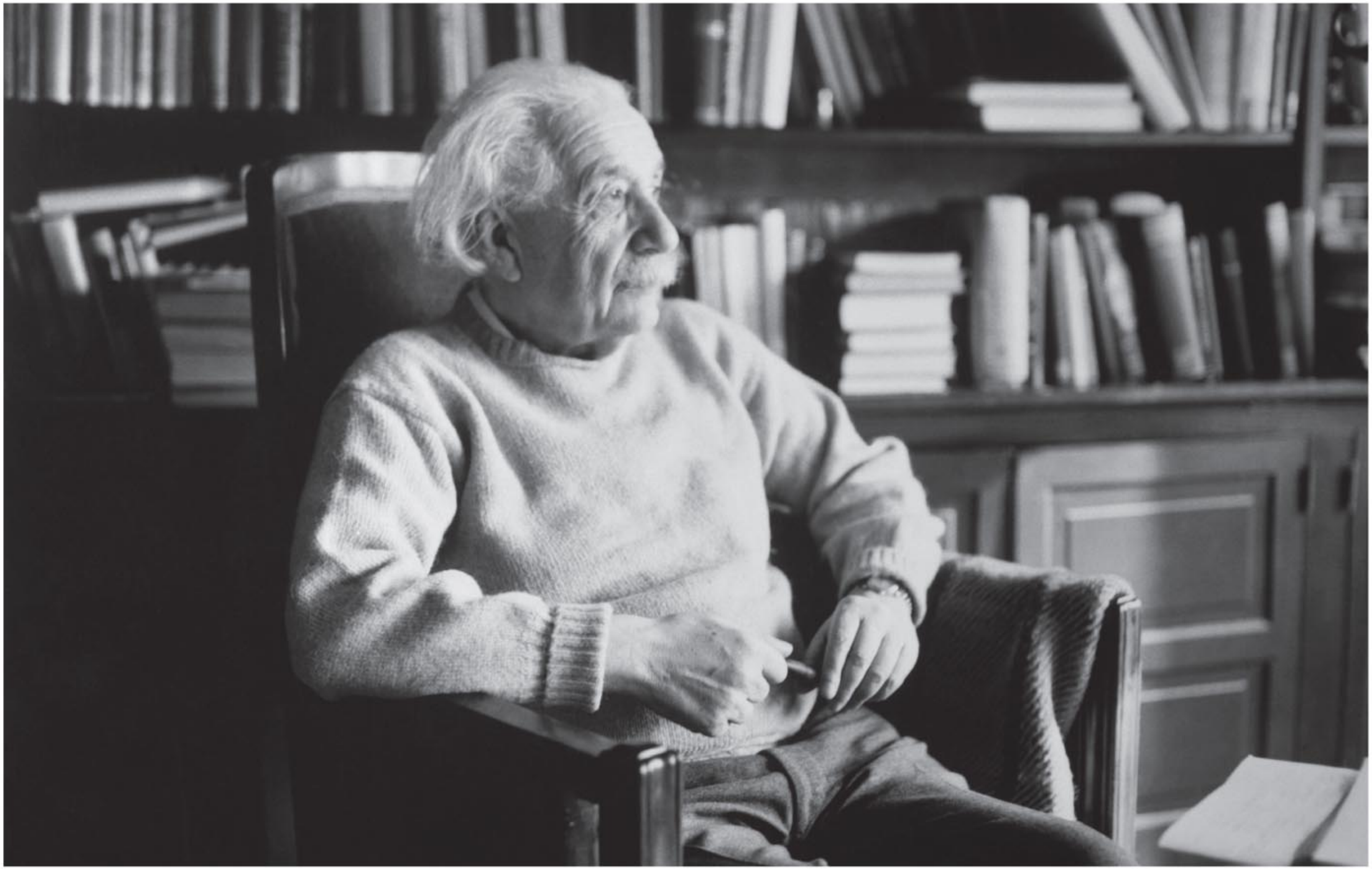
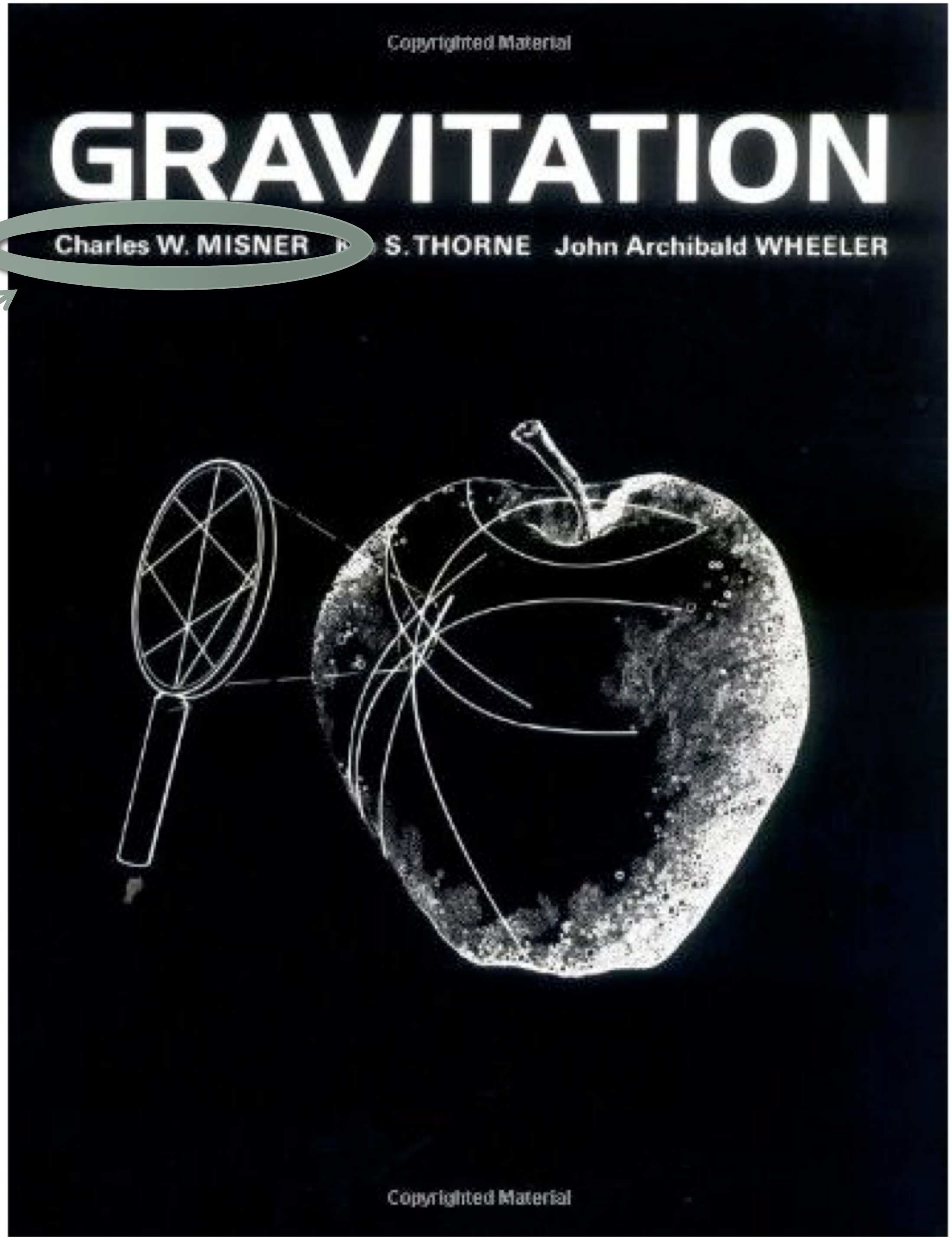
Einstein's Theories of Relativity



Einstein's Theories of Relativity

- Special Theory of Relativity (1905)
  - Usual notions of space and time must be revised for speeds approaching light speed (c).
  - $E = mc^2$ (equivalence of mass and energy).
- General Theory of Relativity (1916)
  - Expands the ideas of special theory to include a surprising new view of gravity.

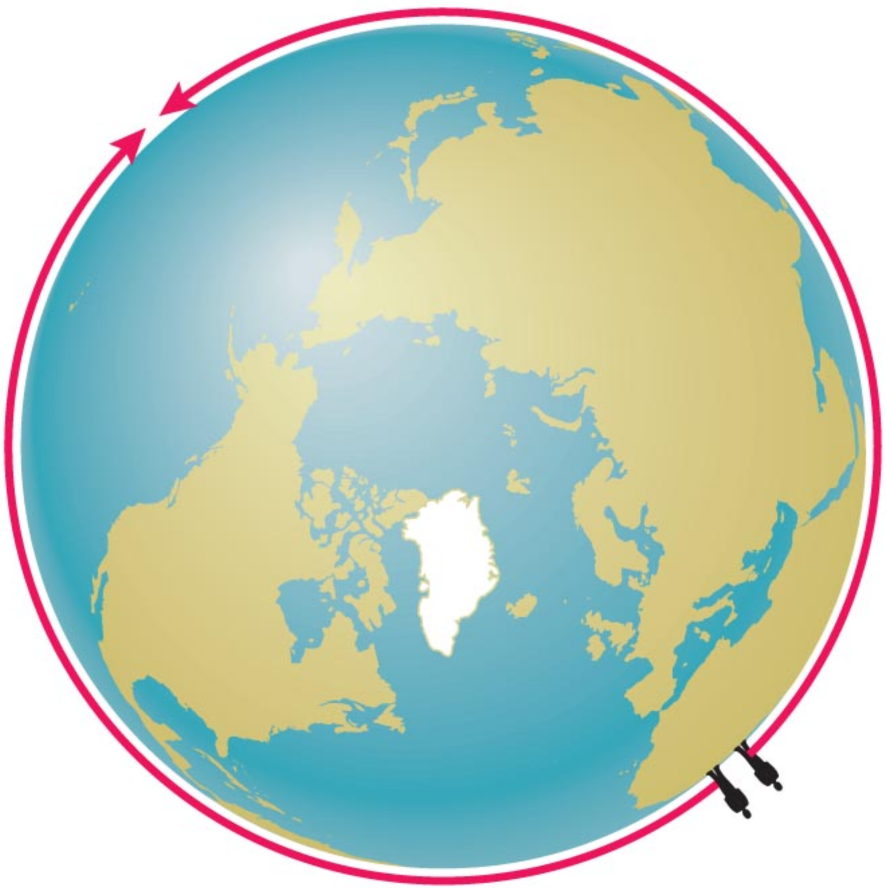
Professor Emeritus in Physics



Spacetime

- Special relativity showed that space and time are not absolute.
- Instead, they are inextricably linked in a four-dimensional combination called **spacetime**.

Curved Space

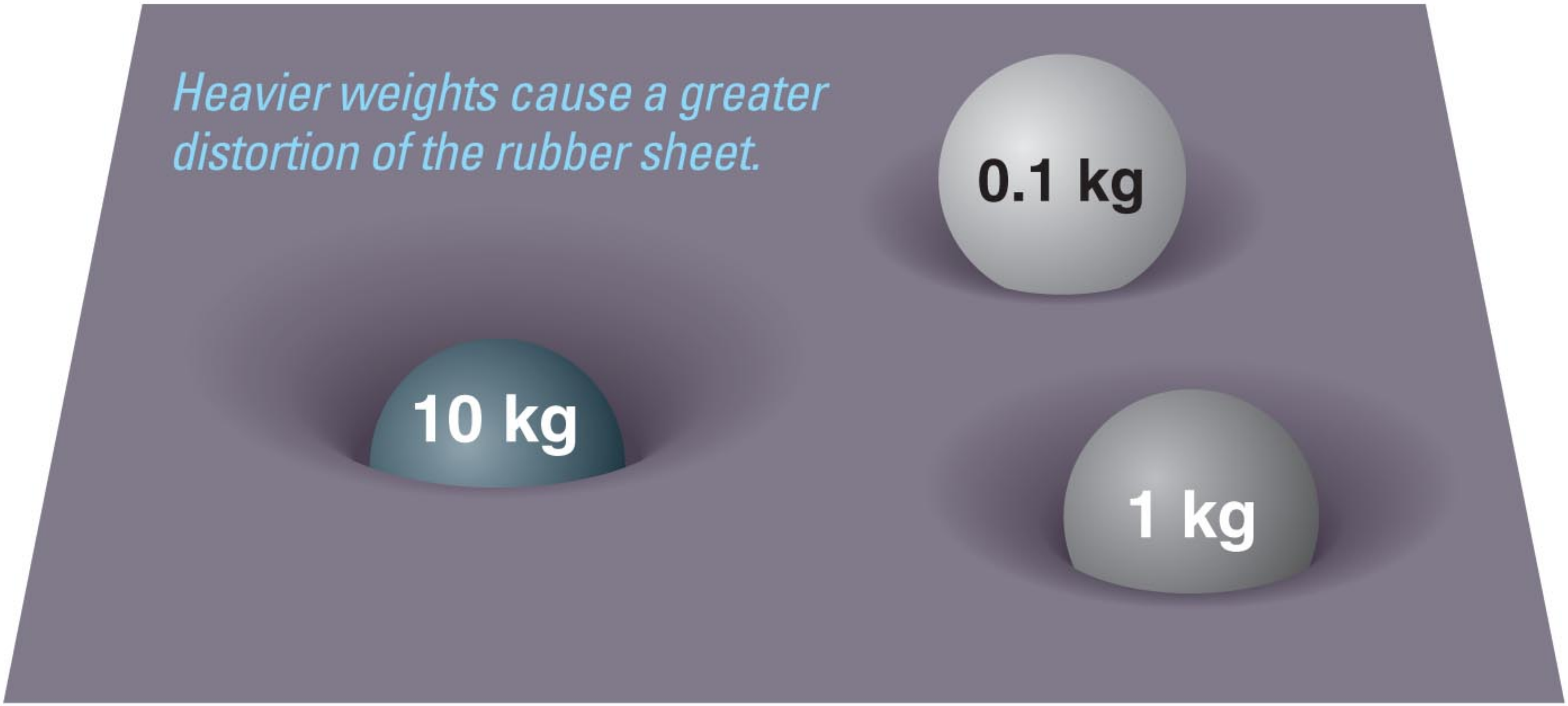


- Travelers going in opposite directions in straight lines will eventually meet.
- Because they meet, the travelers know Earth's surface cannot be flat—it must be curved.

Curved Spacetime

- Gravity can cause two space probes moving around Earth to meet.
- General relativity says this happens because spacetime is curved.

Rubber Sheet Analogy



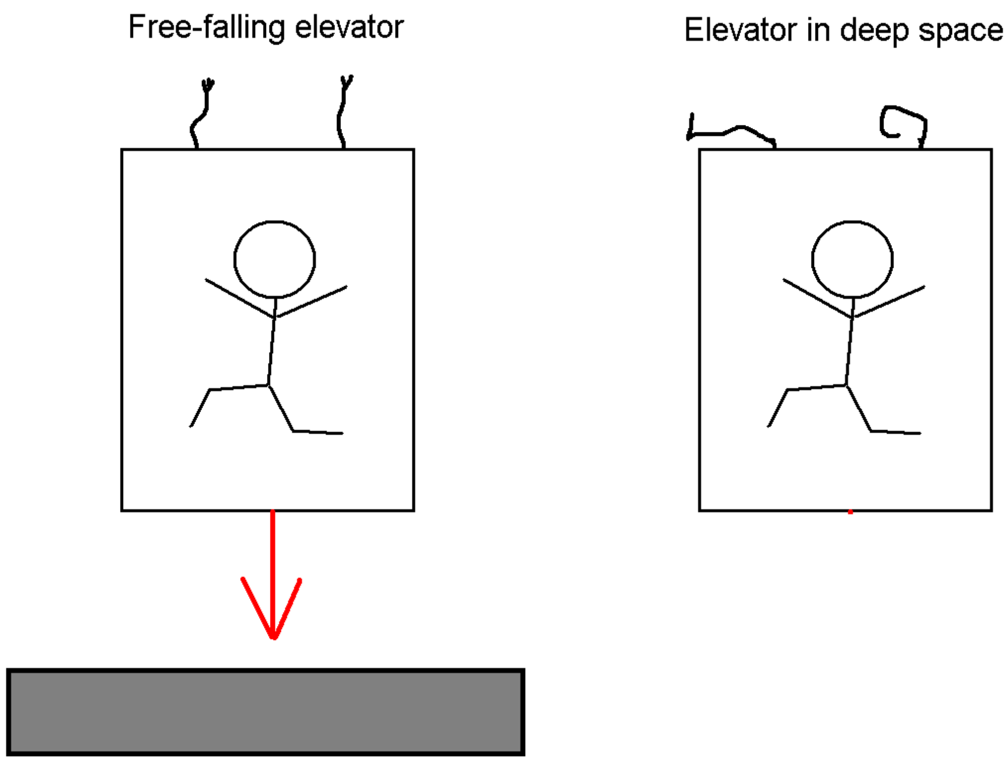
- Matter distorts spacetime in a manner analogous to how heavy weights distort a rubber sheet.

Key Ideas of General Relativity

- Equivalence principle: a freely falling observer in a gravitational field sees the same results of local, short-duration experiments as an observer not in a gravitational field
- Gravity arises from distortions of spacetime.
  "Matter tells spacetime how to curve, and spacetime tells matter how to move" --- John Archibald Wheeler
- One consequence: time runs slowly in gravitational fields *as measured by distant observers*
  But not as measured by the observer ***in*** the gravitational field

The Equivalence Principle

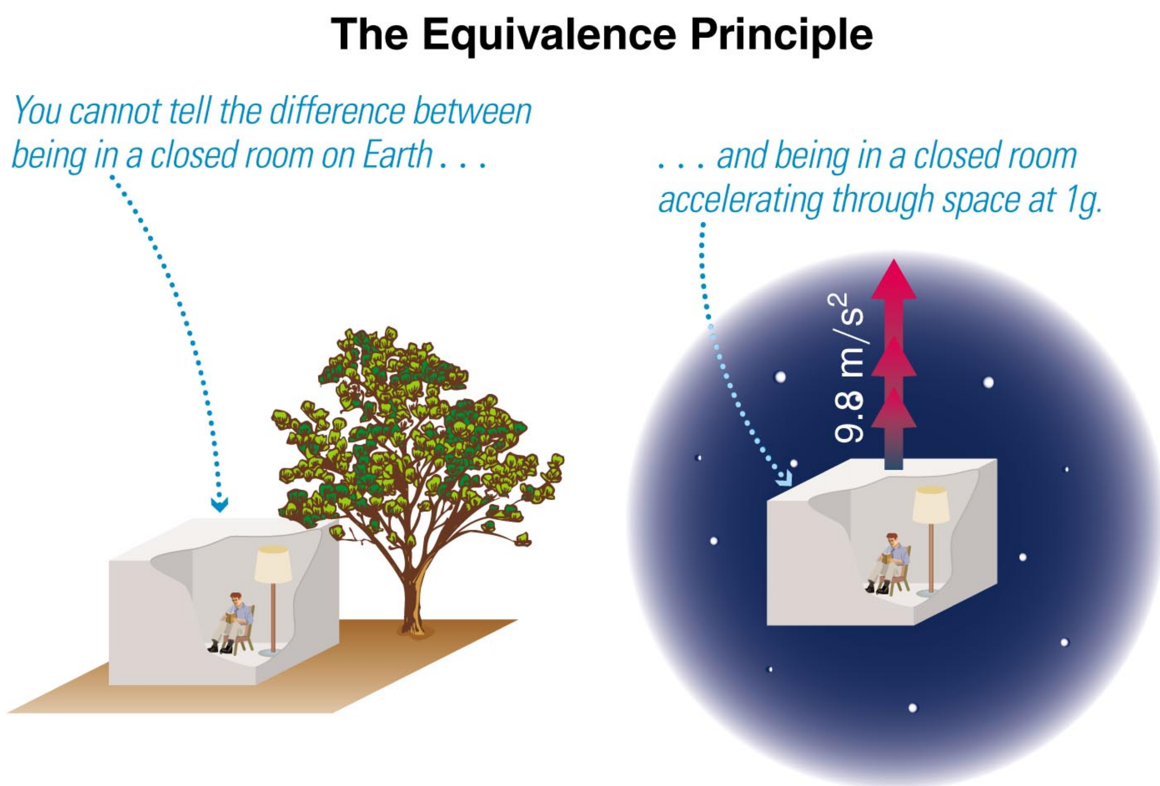
- There is no way to tell the difference between a free-falling frame in a gravitational field and an inertial frame in no gravitational field, over short times and small volumes… the two are equivalent.



Group Q: The Equivalence Principle

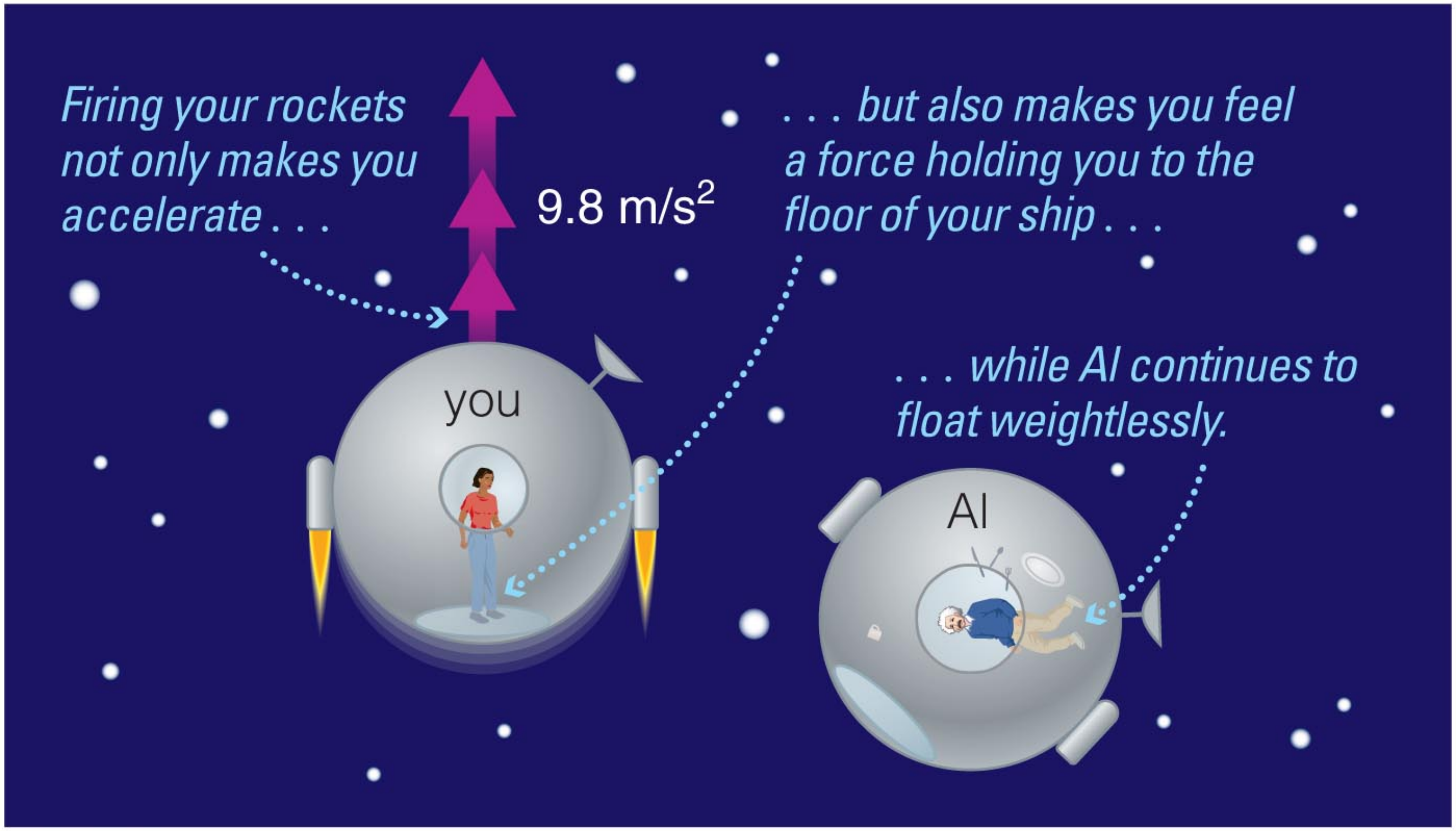
- To repeat: "There is no way to tell the difference between a free-falling frame in a gravitational field and an inertial frame in no gravitational field, over short times and small volumes"
- But note that we say "short times and small volumes"
- Give at least one example of how you ***could*** tell the difference if you wait enough time, or over a long enough distance
- Note: if in a grav field, free fall is maintained (e.g., you won't hit the ground!), so the questions is free-fall versus inertial frame in no gravitational field

The Equivalence Principle



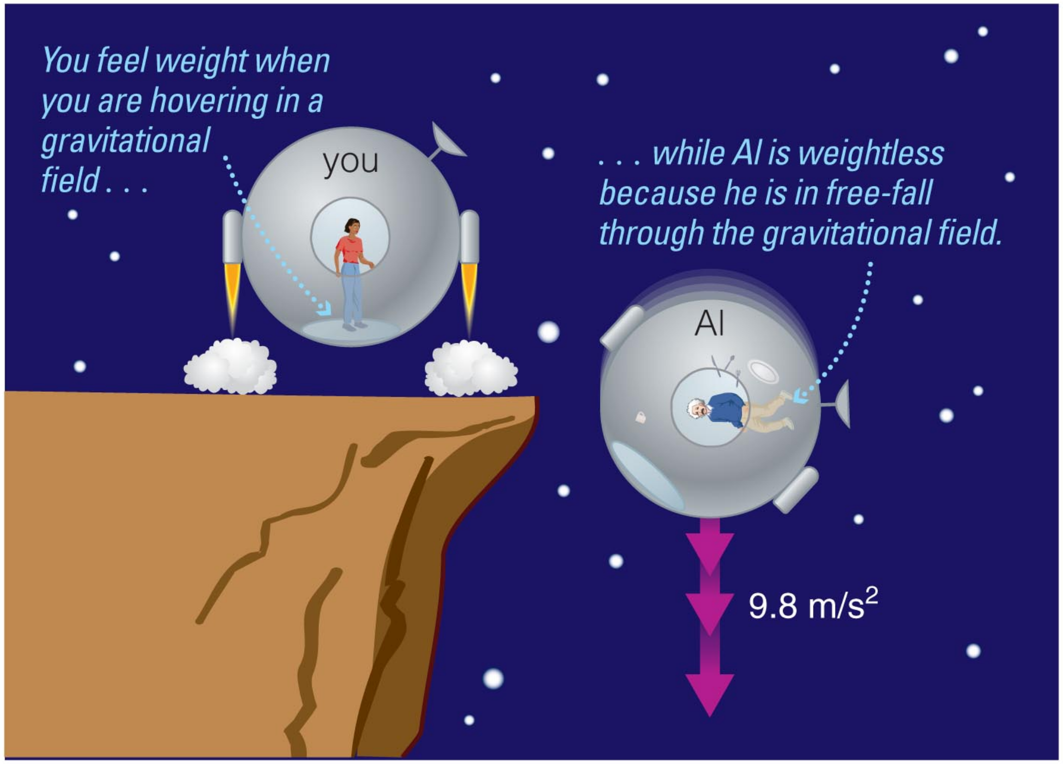
- Einstein preserved the idea that all motion is relative by pointing out that the effects of acceleration are exactly equivalent to those of gravity, over short times and small volumes

Acceleration and Relative Motion



- How can your motion be relative if you're feeling a force causing acceleration?

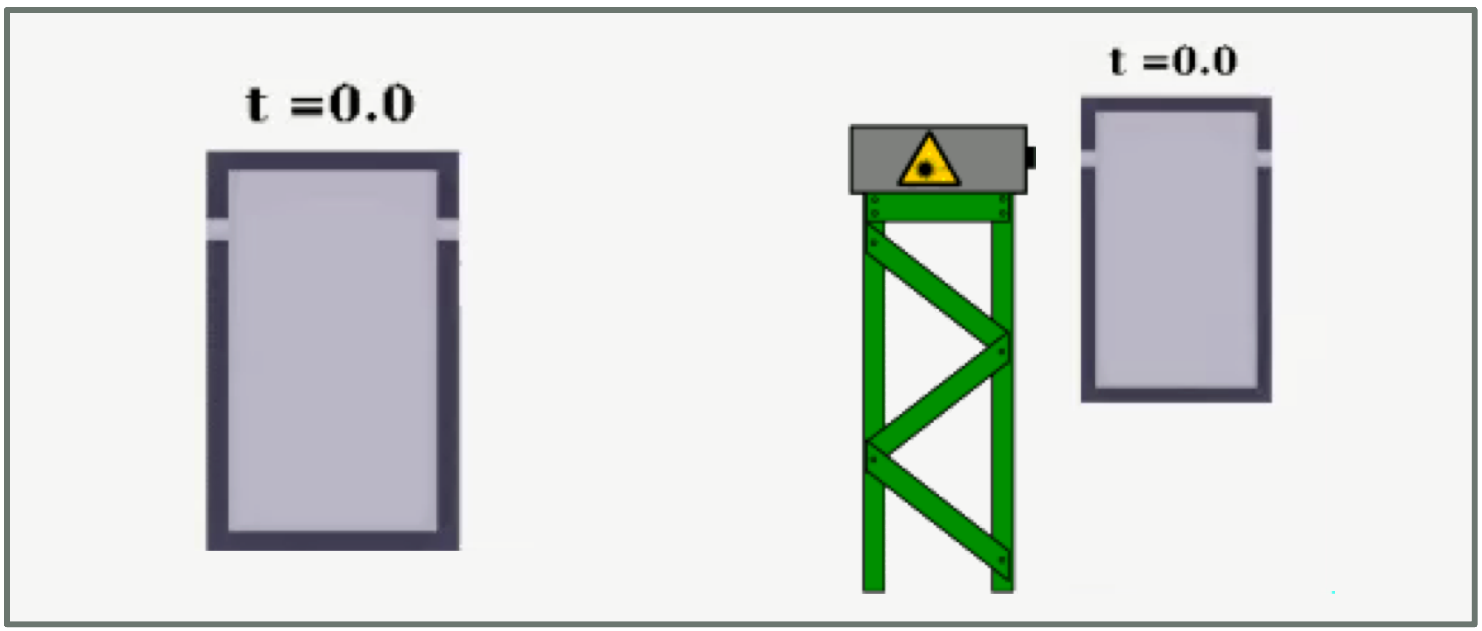
Gravity and Relative Motion



- Someone who feels a force may be hovering in a gravitational field.
- Someone who feels weightless may be in free-fall.

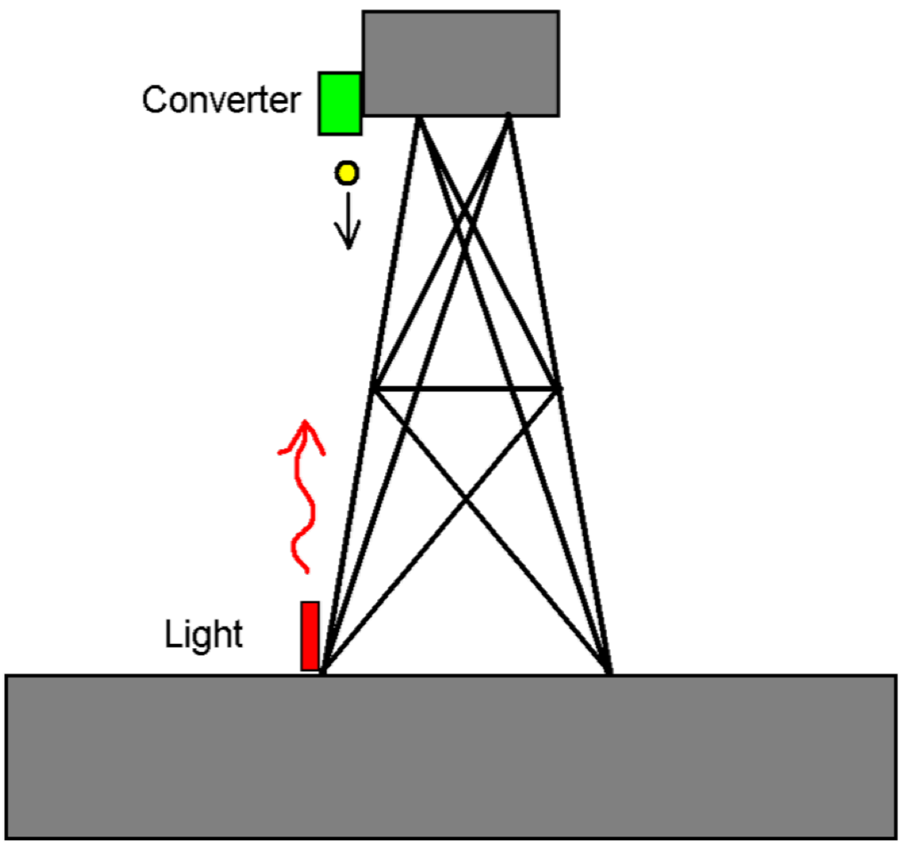
More implications: Light falls!

- Laser light enters a hole on one side of a free-floating elevator. The beam travels horizontally and exits the hole on the opposite side.
- What will this look like to an outside observer if the elevator is in free fall above a planet? (Remember, this is an equivalent frame!)
- The light must follow a curved path from the observer's perspective to exit the hole on the opposite side: light "falls" due to gravity!



Einstein’s Tower Experiment

- Another classic thought experiment: suppose that light is *not* affected by gravity.
- Consider a tower on Earth:
  1. Send a photon from bottom to top.
  2. When photon gets to top, turn its energy into mass.
  3. Then drop mass to bottom of tower.
  4. Then turn it back into a photon.
- Let’s think about this carefully…



If light is not affected by gravity…

- Suppose original photon has energy E.
- By assumption, it is not affected by gravity so it has energy E once it reaches top.
- Thus, mass created at top is $m = E/c^2$.
- Then drop mass. At bottom of tower, it has picked up speed due to the conversion of gravitational potential energy ($E_{\text{grav}} = mgh$).
- When we convert it back into energy, we have

$$E_{\text{new}} = E + mgh = E\left(1 + \frac{gh}{c^2}\right).$$

- We have made energy! We're rich!!!!

Implication of Einstein’s Tower

- Clearly something is wrong with our assumptions…
  - Only way we can conserve energy is to suppose that light *is* affected by gravity…
  - We need the photon to lose energy as it climbs upwards. At the top of the tower, we must have

$$E_{\text{top}} = E\left(1+\frac{gh}{c^2}\right)^{-1}.$$

  - (Check: at the bottom, $E_{\text{new}} = E_{\text{top}} + (E_{\text{top}}/c^2)gh = E_{\text{top}}(1+gh/c^2) = E.\checkmark$)
  - This effect is known as **gravitational redshift**. (Why?)

Gravitational Redshift & Time Dilation

- Gravitational redshift has profound consequences…
  - Imagine a clock based on the frequency of light.
  - Place the clock at the base of the tower. Observe it from the top.
  - Photons lose energy, so they decrease in frequency.
  - Thus, we see the clock running slowly!
  - Time passes at a slower rate in a gravitational field, as seen by a distant observer

The gravitational redshift of light emitted a distance r from the center of a spherical body, mass M, is:

$$z = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} - 1$$

For a clock based on the frequency of light, time interval $= 1/f$, so since $f_{\rm obs} = f_{\rm emit}/(z+1)$, $t_{\rm obs} = (z+1)t_{\rm proper}$, or:

$$t = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} t_{\rm proper}$$

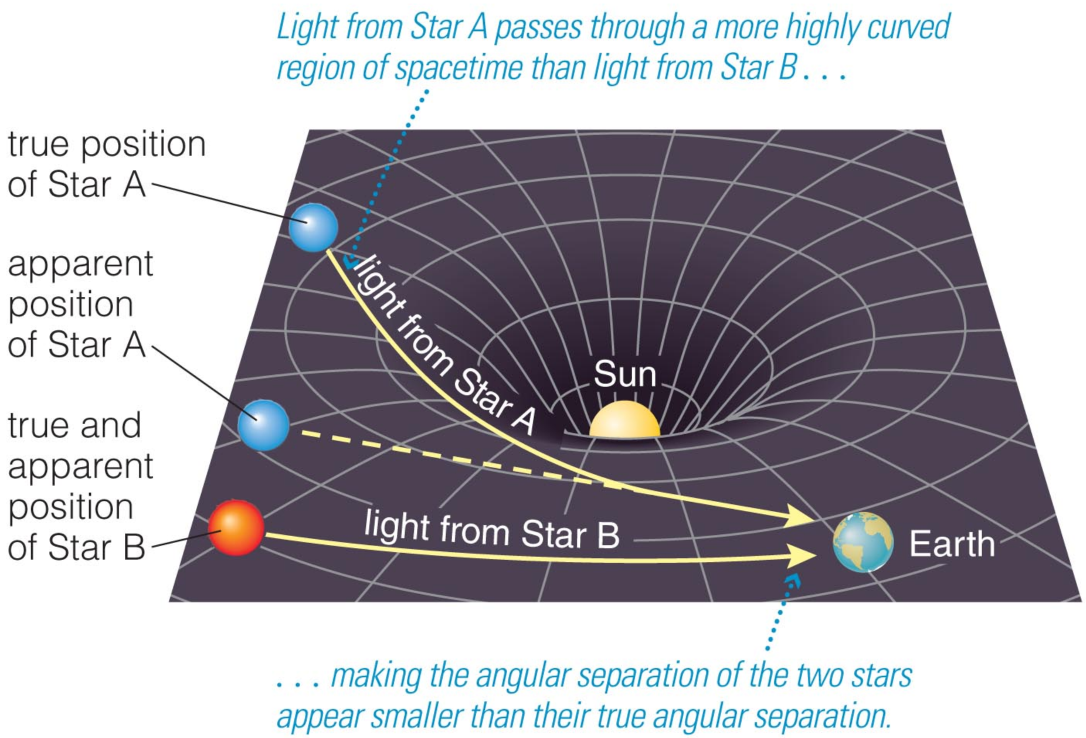
The General Theory of Relativity

- After a monumental effort, Einstein figured out a new theory of gravity that…
  1. Reduces to special relativity in the absence of gravity.
  2. Reduces to Newton's theory for slow-moving objects with weak and slowly varying gravitational fields.
  3. Includes the equivalence principle (free fall is inertial motion).
- This is a mathematically complicated theory, but the essentials are…
  1. Tidal forces are related to the curvature of "spacetime."
  2. The curvature of spacetime tells matter how to move.
  3. The distribution of matter/energy tells spacetime how to curve.

Gravity is an Illusion

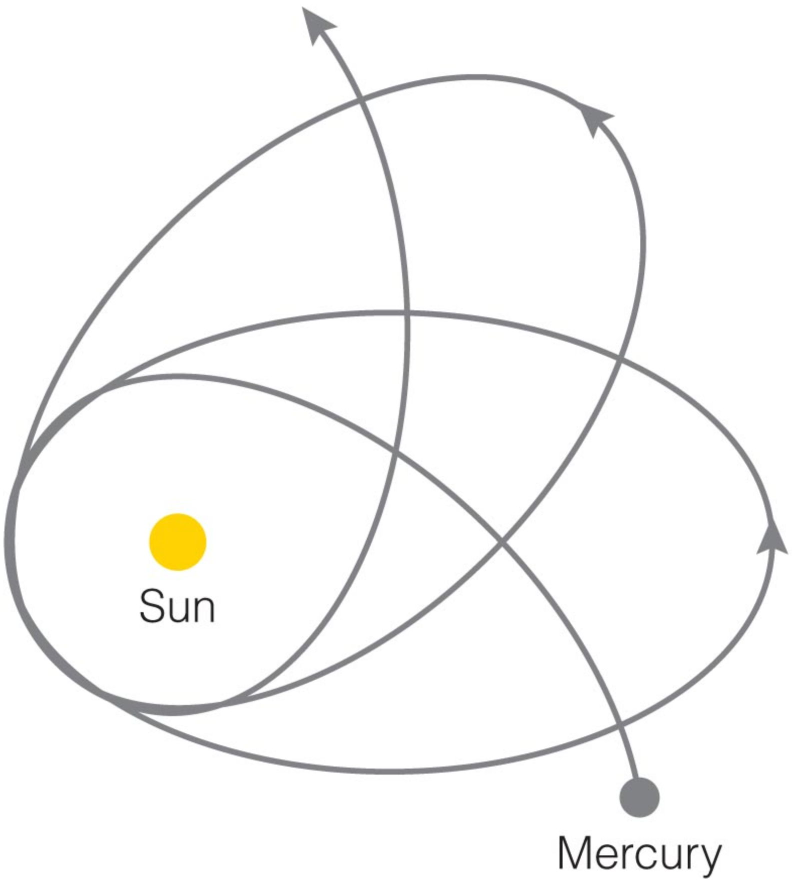
- Objects’ motions are bent (and their clocks slow down) because matter and energy curve spacetime.
- Paths are not curved due to a “force of gravity.”

Tests of General Relativity



- Deflection of light in a gravitational field (Eddington 1919).
  - Observed stars close to Sun during eclipse.
  - Found that they shifted position by the amout predicted in GR as light was bent by Sun.
- We'll return to the idea of "gravitational lensing" later in the course.

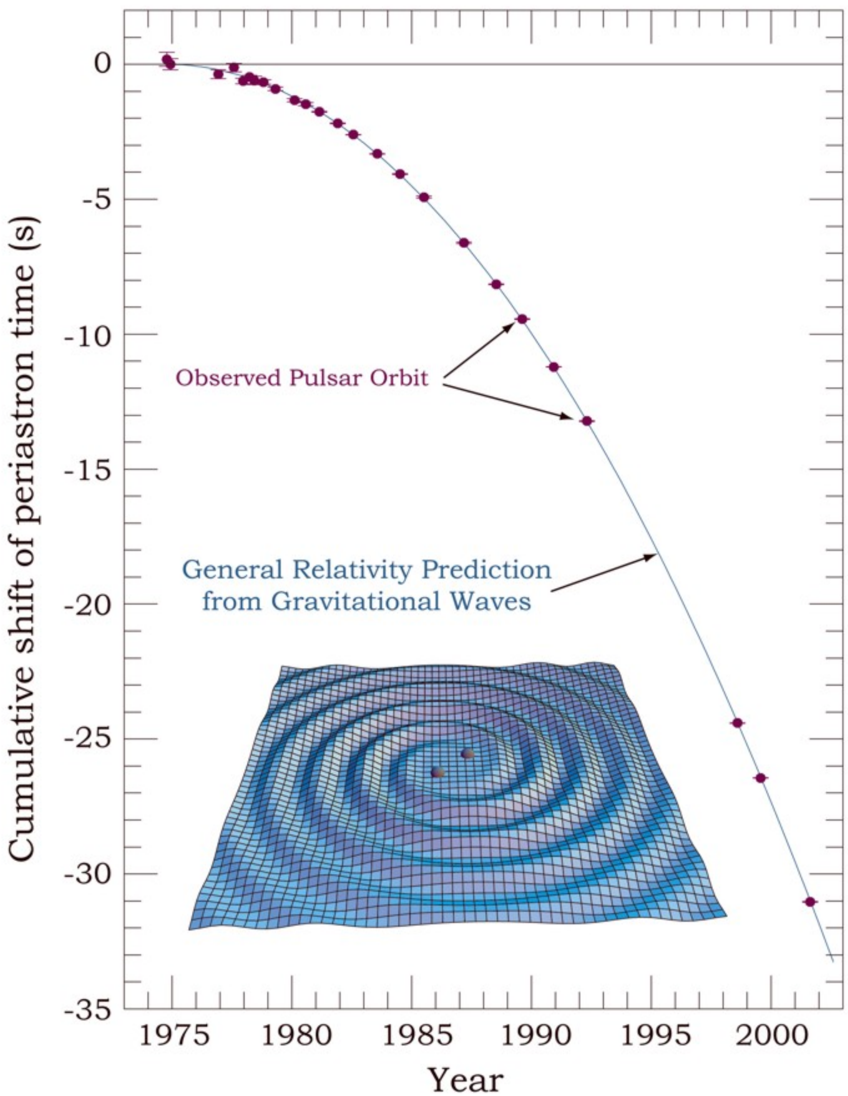
Tests of General Relativity



Note: The amount of precession with each orbit is highly exaggerated in this picture.

- Anomalous precession of Mercury's orbit.
  - Mercury's orbit precesses.
  - Even accounting for a bunch of Newtonian effects (oblateness of Sun, etc.), 43 arcsec/century of precession is unexplained.
  - GR precisely explains this as due to deviations in the force-law away from Newtonian gravity.

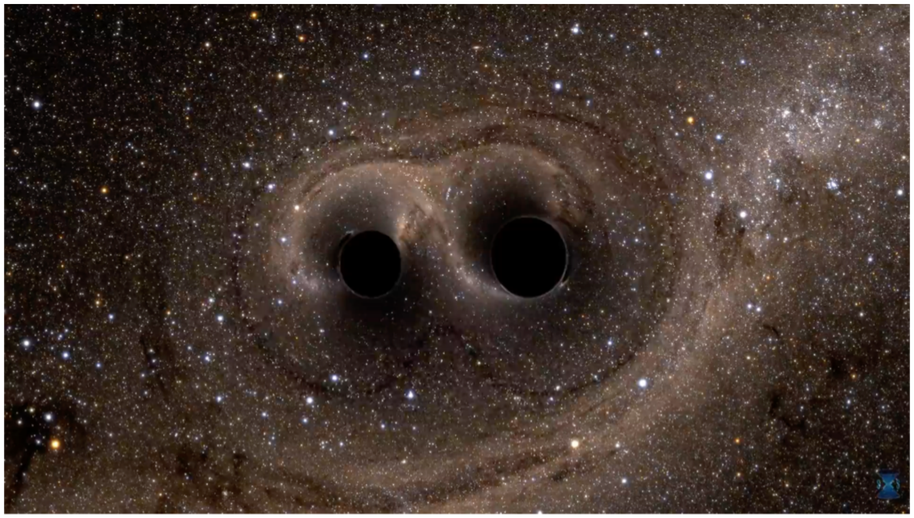
Testing GR with Binary Pulsars

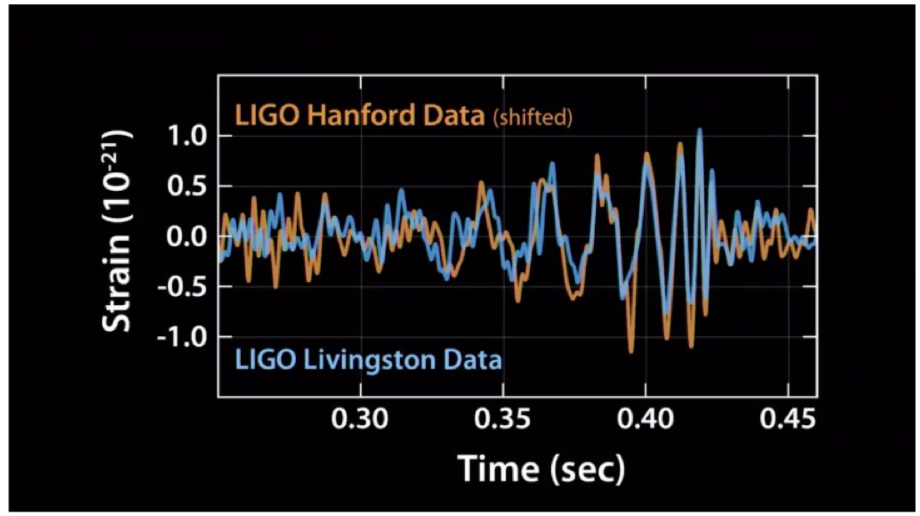


- GR predicts existence of gravitational waves (GWs).
- Hulse-Taylor binary pulsar: a 59 ms pulsar in an eccentric orbit around another neutron star (7.75 hour period).
- Can determine orbits accurately by timing pulsar…
  - Find that orbit is decaying; period is decreasing.
  - Fits prediction of GR perfectly… GWs are carrying away orbital energy.
  - This, and other binary pulsars, was the only unambiguous detection of the effects of gravitational radiation, until…

- Laser Interferometer Gravitational-wave Observatory (LIGO).
- 4 km, 2-armed interferometer. First detection announced 2/11/16!

The LIGO Detections





- The first LIGO detection was of two very distant (1.3 Gly) and very massive (~30 $M_{\odot}$) black holes merging.
- The interferometers can measure a change in the arm length to a precision of 1/10,000th the width of a proton…
- The detection opens a new window onto the universe for astronomy! ("Multi-messenger astronomy.")

Related Astrophysical Phenomena

- Relativity plays a role in the following astrophysical phenomena, most of which we discuss in this course:
  1. Anomalous precession.
  2. Decay of pulsar orbits.
  3. Black holes.
  4. “Superluminal” jets.
  5. Gravitational lensing.
  6. Expansion of the universe.
  7. Gravitational waves.