Coding in advance of the Feb 25, 2019 class

For this coding assignment we just want you to write a code to calculate the posterior probability (prior times likelihood of the data given the model) given the prior, the model, and the data. For both of the cases below, assume that you have flipped a coin 100 times, and that 70 heads and 30 tails have showed up.

a. In the first model, your prior is that all values of the fraction $f_{\rm Heads}$ of heads, from 0 to 1, are equally probable. Determine the posterior probability density at $f_{\rm Heads} = 0.7$.

b. In the second model, your prior for $f_{\rm Heads}$ is a Gaussian centered at 0.5:

$$\text{Prob}(f_{\rm Heads}) = \frac{1}{\sqrt{2\pi\sigma^2_{\rm Heads}}} e^{-(f_{\rm Heads}-0.5)^2/2\sigma^2_{\rm Heads}} \ , \tag{1}$$

where $\sigma_{\rm Heads} = 0.05$. Determine the posterior probability density at $f_{\rm Heads} = 0.5$.

Note: the posterior probability density must be normalized correctly. That is, it should be the case that $\int_0^1 \text{Posterior}(f_{\rm Heads}) df_{\rm Heads} = 1$. Also note that the *density* in this case is $\text{Posterior}(f_{\rm Heads})$, so that the probability that the fraction of heads is between $f_{\rm Heads}$ and $f_{\rm Heads} + df_{\rm Heads}$ is $\text{Posterior}(f_{\rm Heads}) df_{\rm Heads}$.