Coding in advance of the March 11, 2019 class

The task here is to do parameter estimation based on an effectively continuous data set. That set, which is in the file data7_1.txt on the website, has the visual luminosities in solar units, L_V, of the 20 closest main sequence stars to us including the Sun; the data are derived from the absolute magnitudes listed in https://en.wikipedia.org/wiki/List_of_nearest_stars_and_brown_dwarfs. The basic principles will be the same as are given in the Gaussian example in Lecture 7.

Your task will be to fit the following model to the data:

$$\frac{dN}{dL_V} = aL_V^b \ . \tag{1}$$

That is, in this model the expected number of stars between luminosities L_V and $L_V + dL_V$ is $aL_V^b dL_V$.

We will normalize by saying that the total number of stars between $L_V = 2.0 \times 10^{-5}$ and $L_V = 25.0$ (remember that both are in units of the Sun's luminosity) equals 20, which is the actual number of stars. Thus, as in the Gaussian example in the notes, there is actually only one parameter in this model. That parameter is b, which is the power law slope. Your prior on the slope will be that b is equally probable anywhere in the full range of -2 to 0. **Warning!** $b = -1$ is a special case (it involves logarithms), so please skip the $b = -1$ case by searching on a grid of values of b that avoids -1 (e.g., -1.015, -1.005, -0.995, -0.985, etc.).

Your tasks:

1. Determine a as a function of b, given the normalization condition.
2. Determine the value of b, $b_{\rm max}$, which maximizes the log posterior density (i.e., the log of the prior times the likelihood) given the data set, normalization condition, prior, and model.
3. Determine the values of b below and above $b_{\rm max}$ that produce a log posterior density 0.5 less than the maximum.

Of course, we always want to think about the results. Something that is *not* part of the assignment, but that I encourage you to do, is to ask yourself: is an analysis of the *visual* luminosities an apples-to-apples comparison? That is, are all stars being compared on equal footing when we do this analysis?

Good luck!