ASTR 320
Problem Set 1
Due Thursday, February 13

1. Order of magnitude problems.

(a) It is sometimes said that the Great Wall of China is the only human construct that can be seen with the naked eyes from the moon. Evaluate this claim.

(b) Could all the calories you've ever consumed eject you from the Earth? Assume you start from the surface of the Earth, but ignore air resistance.

2. Skyhook, Inc. has set up shop around a spherical planet of mass M, radius R, and angular velocity ω radians per second. They put a satellite in a circular synchronous orbit above the equator, at a radius $R_{\rm synch}$ such that the orbital angular velocity equals ω. They want to have a cable dangling from the satellite to the surface, to facilitate transport of materials from the surface to the satellite. If the mass per unit length of the cable is μ, what is the effective force that must be exerted on the cable to support it?

A) $F = GM\mu/R$.
B) $F = GM\mu\left[1/R - 3/(2R_{\rm synch}) + R^2/(2R_{\rm synch}^3)\right]$.
C) $F = GM\mu(R_{\rm synch}/R^2 - 5/(2R))$.
D) $F = GM\mu\left[1/R + 3/(2R_{\rm synch}) + R^2/(2R_{\rm synch}^3)\right]$.

In order to receive full credit for this problem, you must consider each one of the four possibilities in turn and either show that they cannot be correct or show that they could be correct because they satisfy the constraints of units, limits, or symmetry. You should *not* derive the answer; the point is to determine which of these expressions could be correct.

3. Show that the polar equation

$$r = r_0 \frac{1+e}{1+e\cos\theta} \tag{1}$$

is equivalent to the Cartesian equation for an ellipse $(x-x_0)^2/a^2 + y^2/b^2 = 1$, at the points $\theta = 0, \pi/2, \pi$, and $3\pi/2$. What are the expressions for x_0, a, and b?

4. You have done some observations of stars in order to discover extrasolar planets. You find a particular star, of mass 1.0 solar masses, which is moving in a circle at a total speed of 13.5 meters per second, with a period of 4.947 days. Assuming that the motion of the star is caused by the gravity of a planet, derive the mass of the planet and the semimajor axis of its orbit. **Hint:** the planet's mass is much less than the mass of the star.