Class 6 : General Relativity

ASTR350 Black Holes (Spring 2022)

Cole Miller

This class

- Finally talk about gravity
- Affect of gravity on light
- Affect of gravity on time
- Strong equivalence principle

First homework is due at the beginning of the next class (Tuesday, Feb 15); electronic submission on ELMS page.

Muddiest points

Any astro questions?

RECAP

- Einstein's postulates
  - Laws of physics look the same in any inertial frame of reference.
  - The speed of light is the same in any inertial frame of reference
- Strange consequences
  - Time dilation and length contraction
  - Relativity of simultaneity and ordering of events… need to have ultimate speed limit
  - Equivalence and conversion of mass and energy… reason for ultimate speed limit
- **We have been carefully avoiding gravity**

Reminder: Consequences of Special Relativity

There is no absolute time or absolute space.

It is impossible for two events to be simultaneous for all possible observers.

There are pairs of events which will happen in one order for some observers and in the other order for other observers.

The kinetic energy of massive moving bodies increases without bound as the velocity of the body approaches the speed of light.

The same holds for the momentum of massive moving bodies: it increases without bounds as the velocity approaches the speed of light

Object cannot travel at or faster than the speed of light.

General Relativity

- General relativity adds gravity and what happens in non-inertial frames
- Like special relativity, the general theory predicts phenomena which differ significantly from those of classical physics, and from everyday experience
  - the passage of time
  - the geometry of space
  - the motion of bodies in free fall, and the *propagation* of light.
- Examples of such differences include gravitational time dilation, gravitational lensing, the gravitational redshift of light, and gravitational time delay

First Newtonian mechanics (special relativity), now his law of gravity (general relativity)

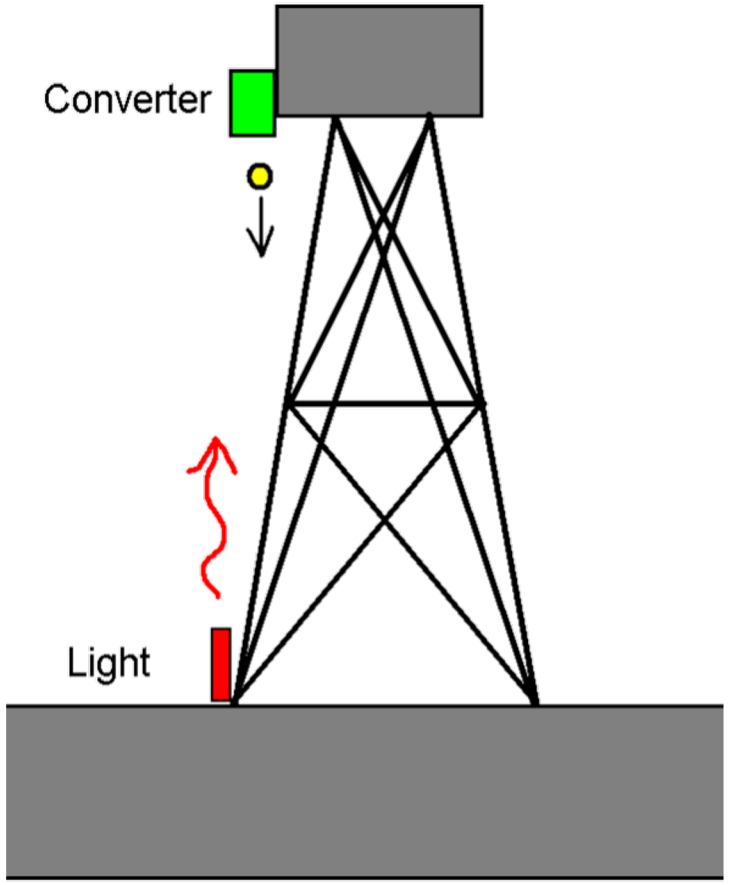
- As we have just learned we have to understand
  - In whose frame do we measure?
  - Does the force depend based on your reference frame?
  - Can gravity travel (communicate) faster than *c*?

General Relativity

- General relativity is crucial for interpreting many 'new' phenomena discovered in astrophysics in the last few decades
  - Black holes- active galaxies (quasars)
  - Pulsars
  - Accreting Neutron stars (x-ray sources)
  - Microwave background from the big bang
  - Gamma-ray bursts
  - Gravitational lenses
  - Hubble expansion of the universe
  - Gravitational waves
- Unfortunately the mathematics is a lot more complex than special relativity and calculations are difficult

I: Einstein's Tower

- Another thought experiment… **suppose that light is not affected by gravity.**
- Consider a tower on Earth
  - Shine a light ray from bottom to top
  - When light gets to top, turn its energy into mass.
  - Then drop mass to bottom of tower.
  - Then turn it back into energy



Perpetual motion?

- So…
  - Original photon energy E
  - *By assumption*, photons are is not affected by gravity so it has energy **E** once it reaches top
  - Convert this energy into mass, mass created at top is **$m=E/c^2$**
  - Then drop mass… at bottom of tower, it has picked up speed due to the conversion of gravitational potential energy (**$E_{grav}=mgh$**)*

$$E_{new} = E + mgh = E\left(1 + \frac{gh}{c^2}\right)$$

  - We have made energy! We're rich!!!! Infinite amount of energy! (do it over and over again)

  * A little physics.. the energy due to falling in a gravitational field is $E_{fall}=mgh$ (h=height you fall, m is the mass falling, and g is the local acceleration due to gravity, on surface of earth $9.8 m/s^2$)

- the new energy of the photon= the old energy+the energy gained by falling

Something is wrong with our assumptions…

- Only way we can conserve energy is to suppose that light **is** affected by gravity…
- We need the photon to lose energy as it climbs upwards… at top of tower, we must have

$$E_{top} = E\left(1 + \frac{gh}{c^2}\right)^{-1}$$

- This is known as **gravitational redshift**

Gravitational Redshift

- For light the energy of a photon is related to its frequency,ν, (alternatively its wavlength,λ)

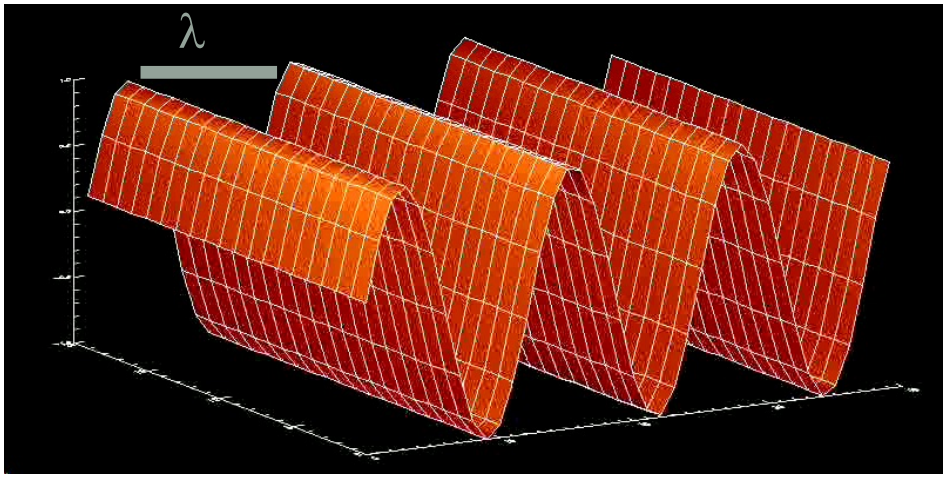
$E=h*\nu=hc/\lambda \rightarrow$ lower energy, longer wavelength– redder

- The profound nature of gravitational redshift…
  - Imagine a clock based on the frequency of light
  - Place the clock at the base of the tower… observe it from the top.
  - Photons lose energy… so their frequency decreases
  - Thus, we see the clock running slowly!
  - **Time passes at a slower rate in a gravitational field! (units of ν is cycles per sec, λ is wavelength of light)**

***h= Planck's constant** (6.62606957 × 10^{-34} m^2 kg/s)

Properties of Waves

- **Recall properties of waves:**
- Waves characterized by
  - Wavelength (λ) = distance between crests- (units length)
  - Frequency (f or ν) = number of crests passing a given point per second (units cycles/time)
- Speed of a crest; $s_c = \lambda\nu$ (this is generic to waves, for light $s_c=c$)
- **Energy** of a wave is proportional to frequency ν, **$E = h\nu$.**



To repeat....

- The profound nature of gravitational redshift…
  - Imagine a clock **based on the frequency of light**
  - Place the clock at the base of the tower…

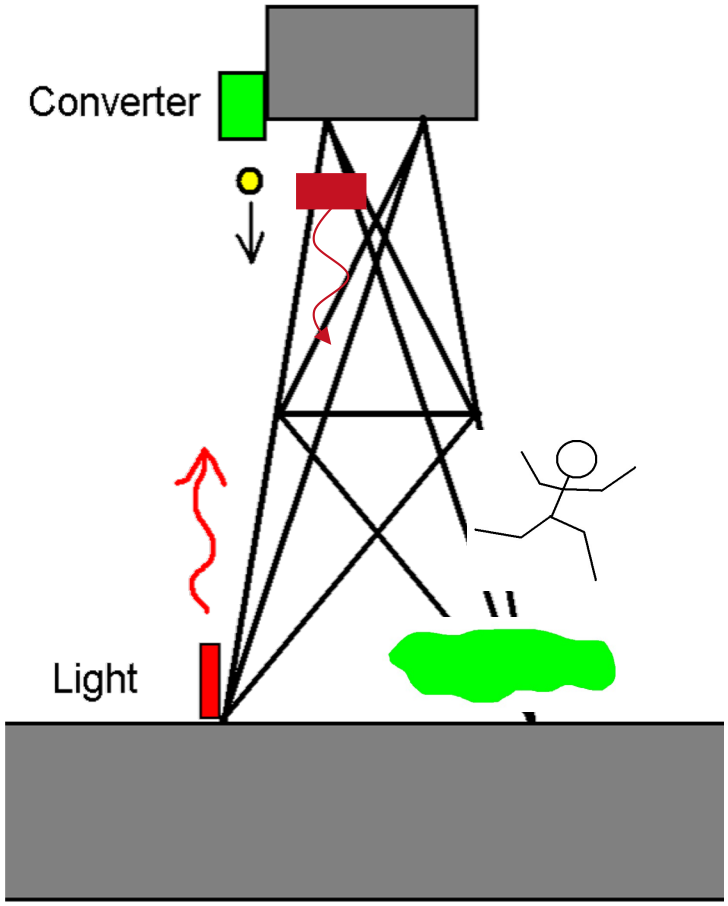
$$E_{top} = E\left(1 + \frac{gh}{c^2}\right)^{-1}$$

  - Observe it from the top.
  - Photons lose energy… **so their frequency decreases**
  - Thus, we see the clock running slowly!
  - **Time passes at a slower rate in a gravitational field!**

Resolving the tower problem

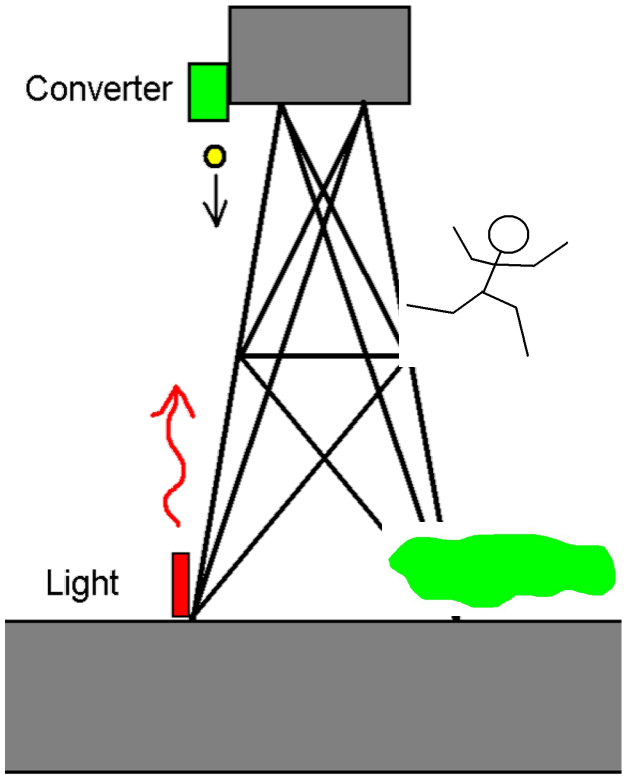
- Now consider light ray aimed from top to bottom of tower
- Free-falling(FF) observer sees light ray travel **unaffected** by gravity, since free fall is an inertial frame

- From"Earth's"frame…
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity (Doppler effect)
  - ***If FF observer is supposed to see a constant frequency light beam, then light must get relatively blueshifted as it falls in a gravitational field, to compensate***



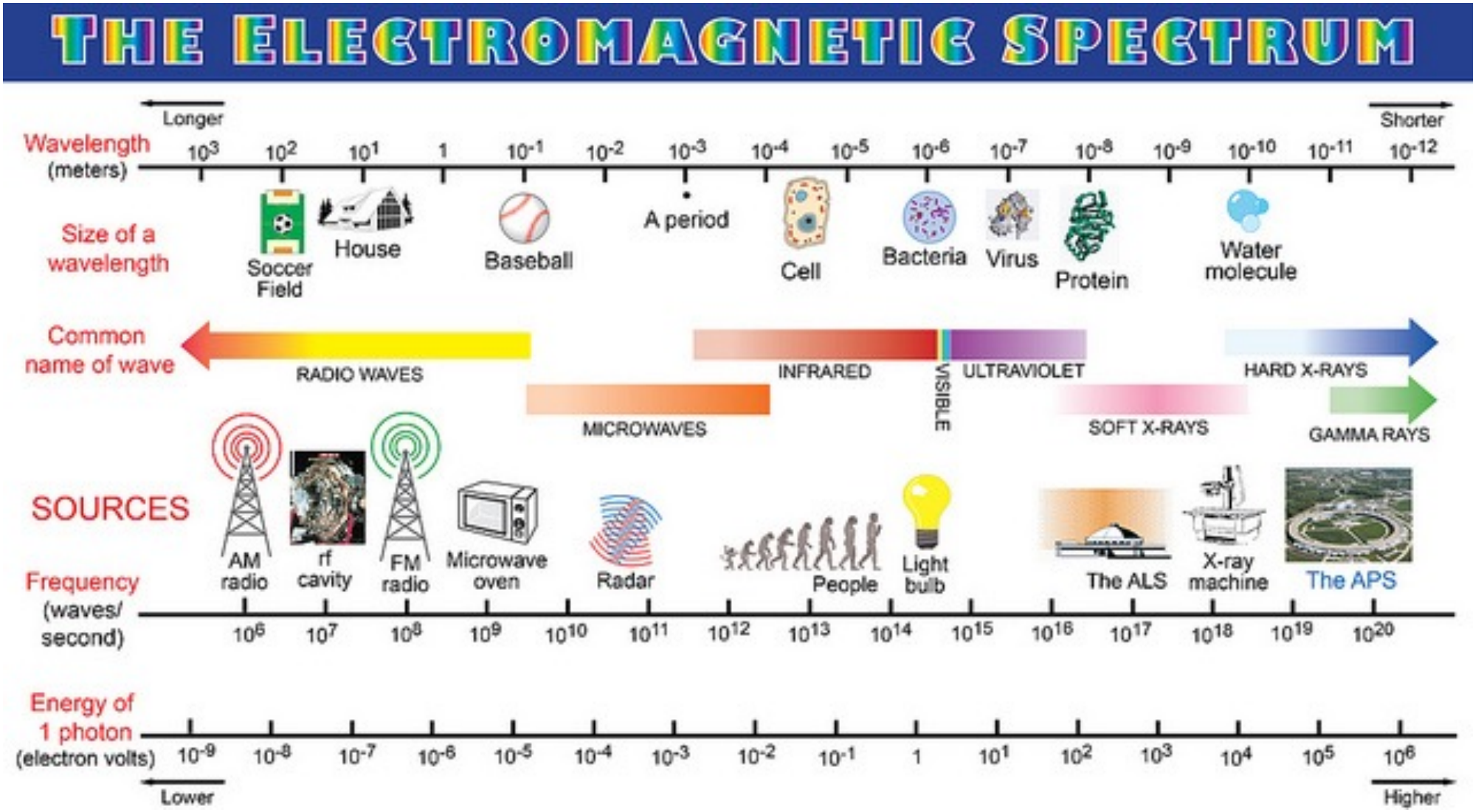
Gravitational Redshift

- Free-falling (FF) observer sees light ray travel unaffected by gravity.
- From “Earth’s” frame…
  - Free-falling (FF) observer traveling faster and faster
  - FF observer would an increasing *blueshift*
  - Since FF observer sees an unaffected (I.e. constant frequency) light beam, light must get progressively *redshifted* as it climbs up.
  - **Redshifting removes just the right amount of energy to solve tower paradox.**



Maxwell and gravity

- Clearly, our assumption was wrong…
  - ***light must be affected by gravity.***
  - But gravity does not appear in Maxwell's equations, which govern light
  - Thus, Maxwell's equations are not complete* and are not exactly valid in the reference frame of Earth's surface, where there **is** gravity.
  - The Earth's surface must not be an inertial frame of reference!


THE ELECTROMAGNETIC SPECTRUM
Longer
Shorter
Wavelength (meters)
10^3 10^2 10^1 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12
Size of a wavelength
Soccer Field
House
Baseball
A period
Cell
Bacteria
Virus
Protein
Water molecule
Common name of wave
RADIO WAVES
MICROWAVES
INFRARED
VISIBLE
ULTRAVIOLET
SOFT X-RAYS
HARD X-RAYS
GAMMA RAYS
SOURCES
AM radio
rf cavity
FM radio
Microwave oven
Radar
People
Light bulb
The ALS
X-ray machine
The APS
Frequency (waves/ second)
10^5 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20
Energy of 1 photon (electron volts)
10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6
Lower
Higher

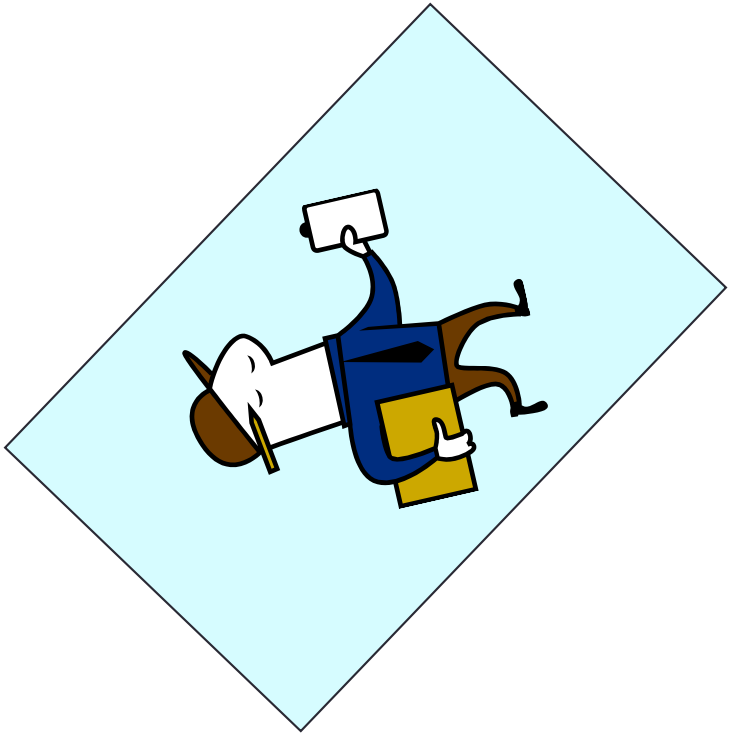
II: Strong Equivalence Principle

- Recap of the **weak equivalence principle**
  - All objects accelerate at the same rate in a given gravitational field.
  - In other words, inertial and gravitational masses are the same for any object.

- Einstein introduced the **strong equivalence principle** – when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- What does this mean???
- **The complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system**

Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space

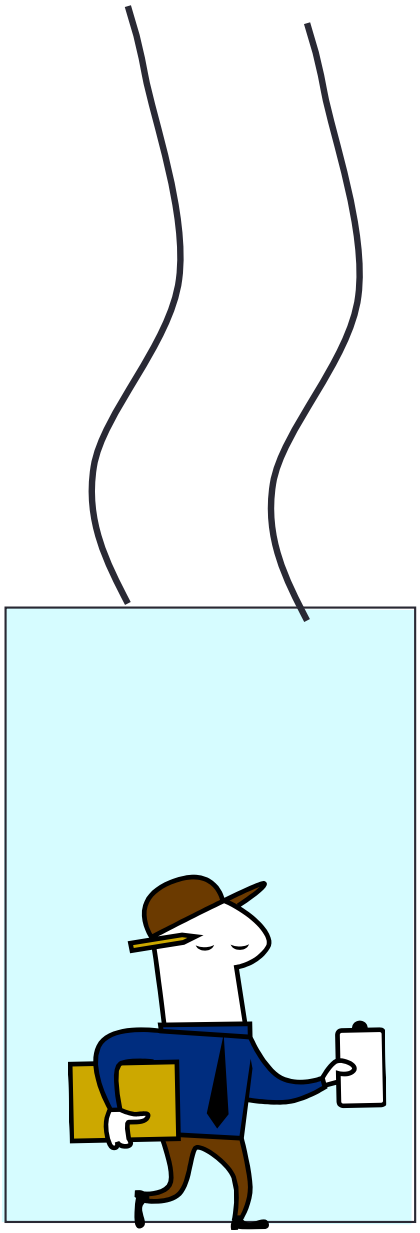
$a=9.8\ m/s^2$

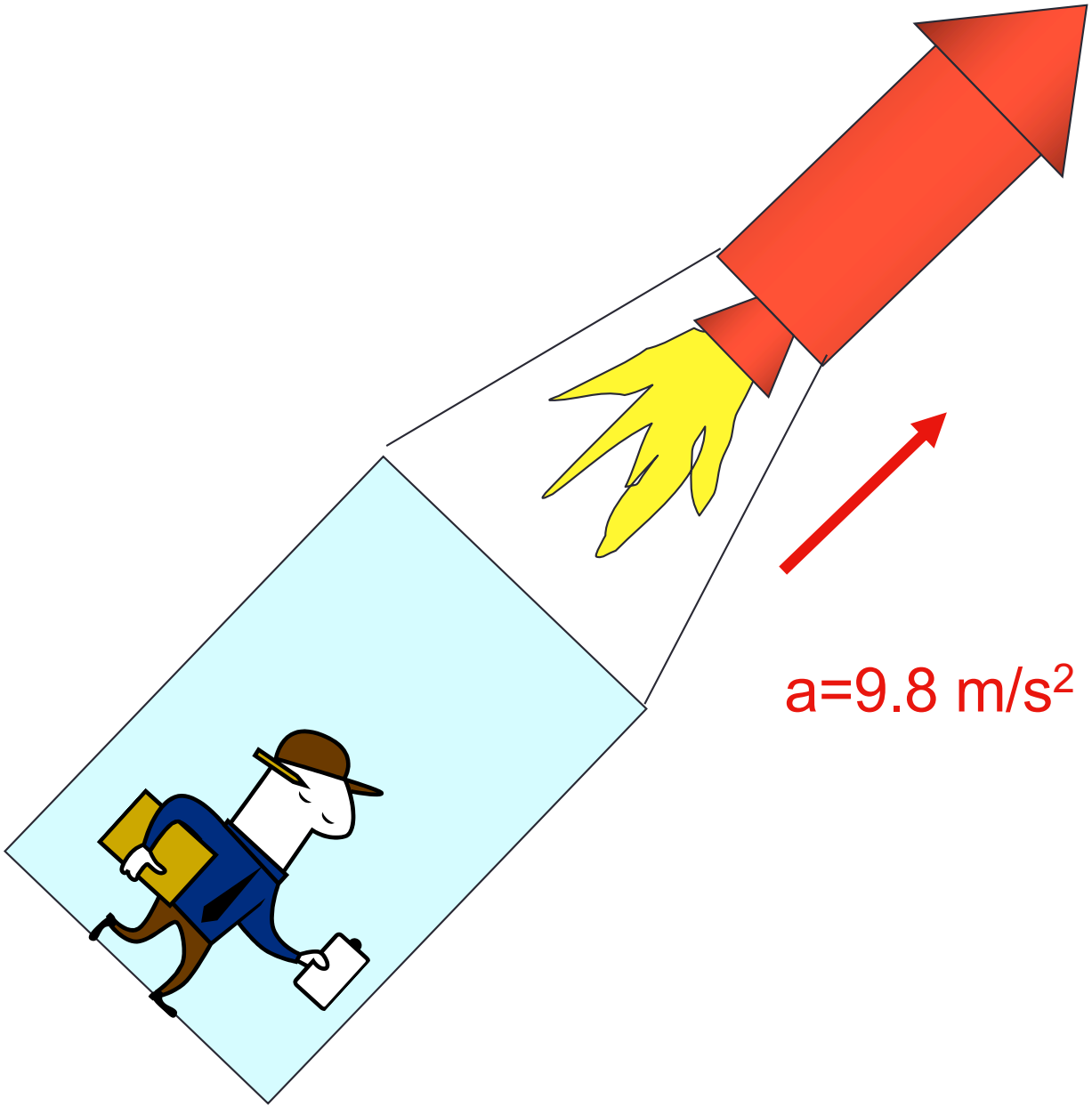


What about gravity?

- Suppose that you decide that your frame of reference is not inertial…
  - Freely moving bodies change velocity
  - Is it because of gravity or is the frame accelerating?
  - **Einstein says that you cannot tell the difference!**
  - **Gravity is a "fictitious force"** – i.e., a force which appears to exist because we are living in a non-inertial frame of reference.

Elevator at rest on Earth is equivalent to elevator being pulled by accelerating rocket in deep space

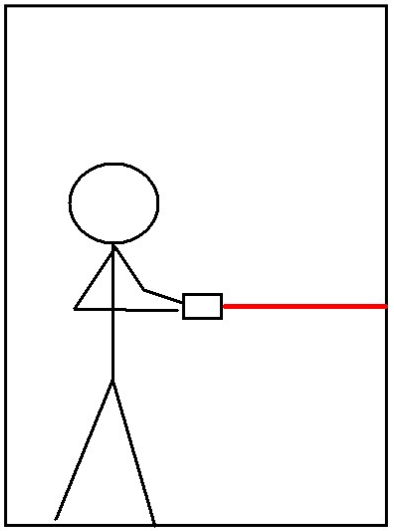




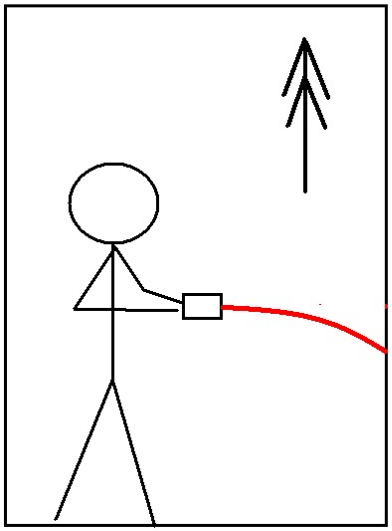
What about light? It "falls", too!

- Astronaut in inertial frame with flashlight
  - Inertial frame, so light goes in straight lines
  - It doesn't matter whether this is free fall or far from masses
- What if we now put flashlight in a gravitational field (accelerated frame)?
  - Light beam will bend: it must accelerate at the same rate and direction as the elevator
  - Strong equivalence principle ⇨ frame with gravity acts the same
  - Important conclusion – light "falls" due to gravity!- **how can we test this idea?**

Inertial Frame



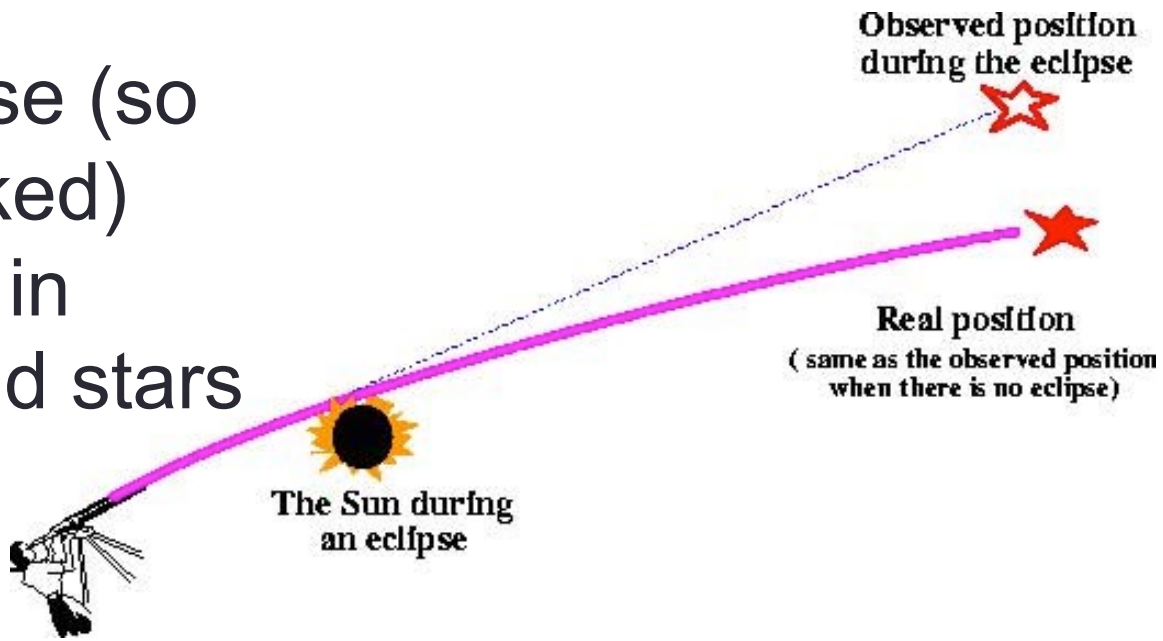
Accelerating frame or gravity



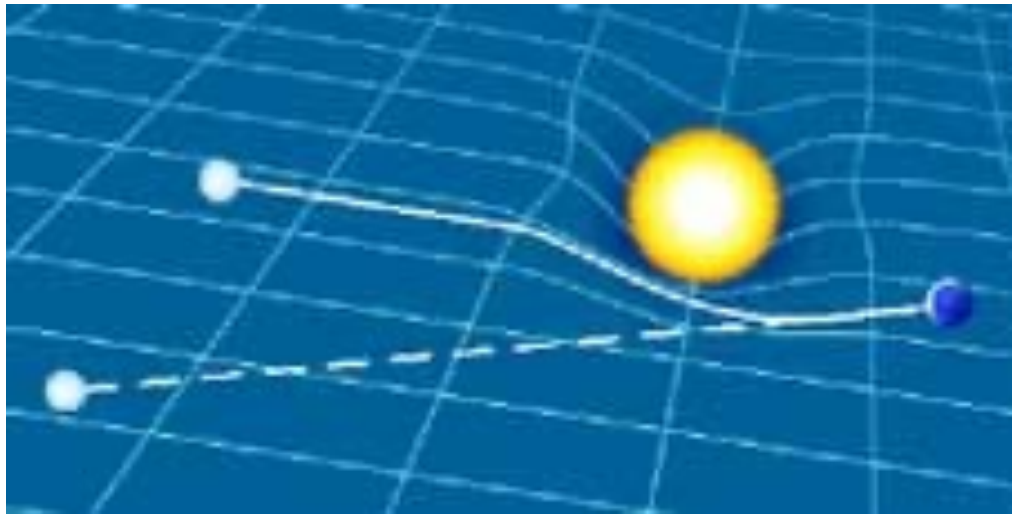
How to Test if Light is influenced by gravity

Find a situation where a background light source is (in projection) near a massive object
(the effect is small so need a big mass)

Answer: a solar eclipse (so light from sun is blocked) and look for changes in position of background stars

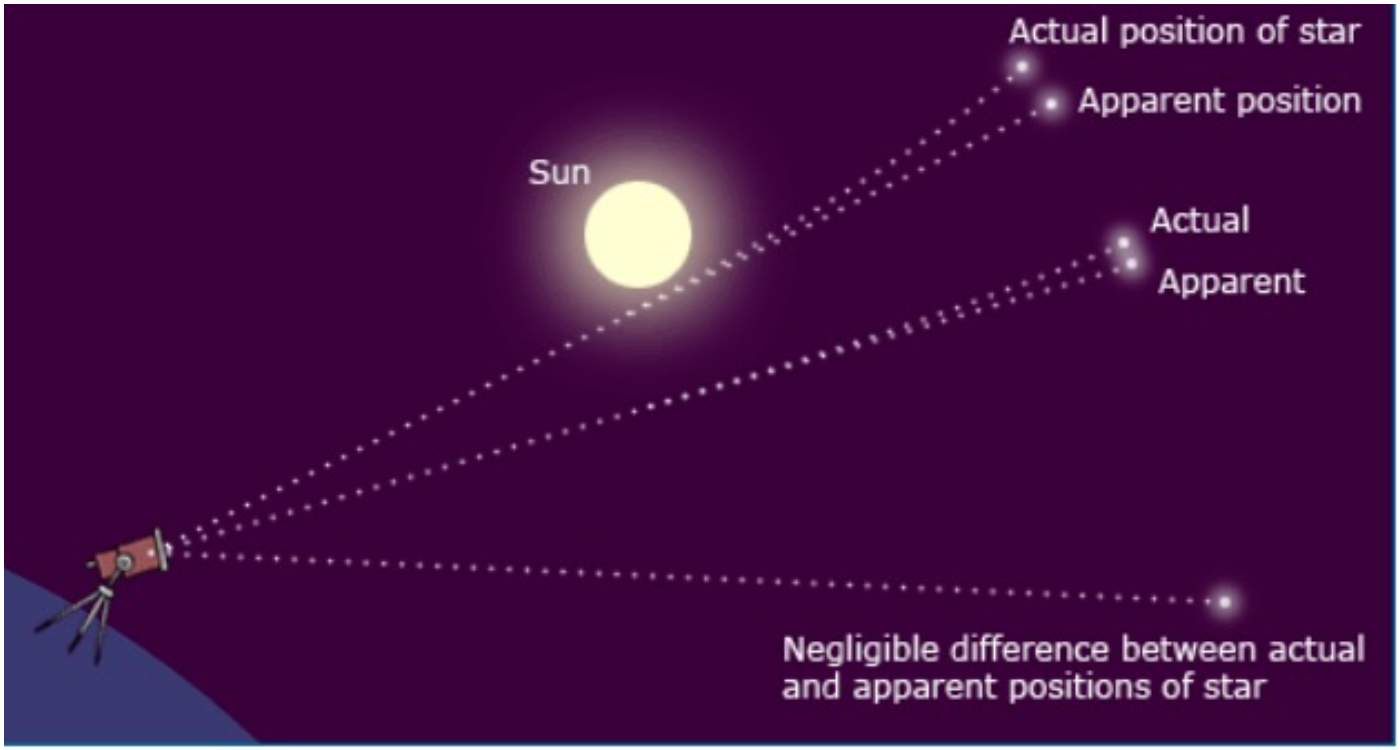


Light Bending



- Einstein predicted that gravity distorts space-time

- The gravity from the sun bends the light from a background star- need to block light from sun to see the effect



The Dyson-Eddington Test

- 1919 – the first "accessible" total Solar eclipse since Einstein postulated the Strong Equivalence Principle (SEP)
- Arthur Eddington and Frank Dyson
  - Famous British Astronomers
  - Led expedition to Brazil (Sobral) and Principe, to observe eclipse
  - Were looking for effects of gravitational light bending by searching for shifts in positions of stars just next to the Sun*.
  - The shift was consistent with Einstein's calculation! (1.75 arc seconds)

Extremely important test of GR

*Einstein predicted that the magnitude of the shift depended on how large the angle was between the sun and the background star and the mass of the sun

- After the Dyson-Eddington test, Einstein became a world famous figure

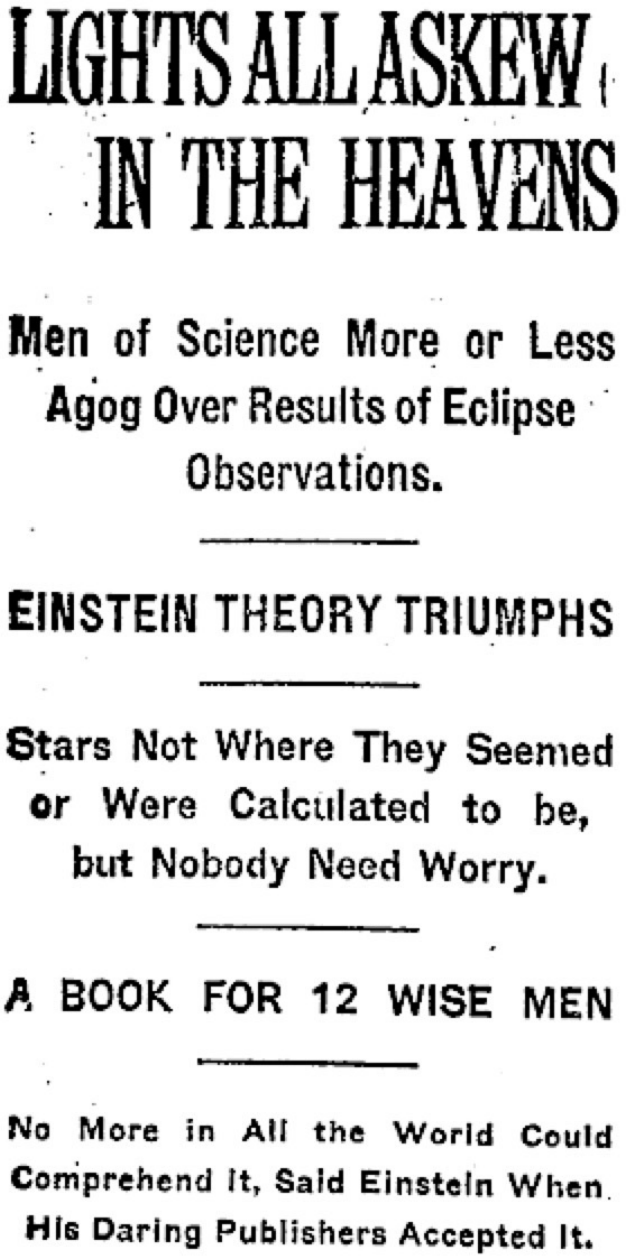
✦Can Gravity Bend Light?

✦If light, does not have mass, how could it possibly be affected by gravity?

✦After all, the force of gravity – – is directly dependent upon the mass of two objects.

Amazing Result Made the Newspapers

The New York Times of November 10, 1919,



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

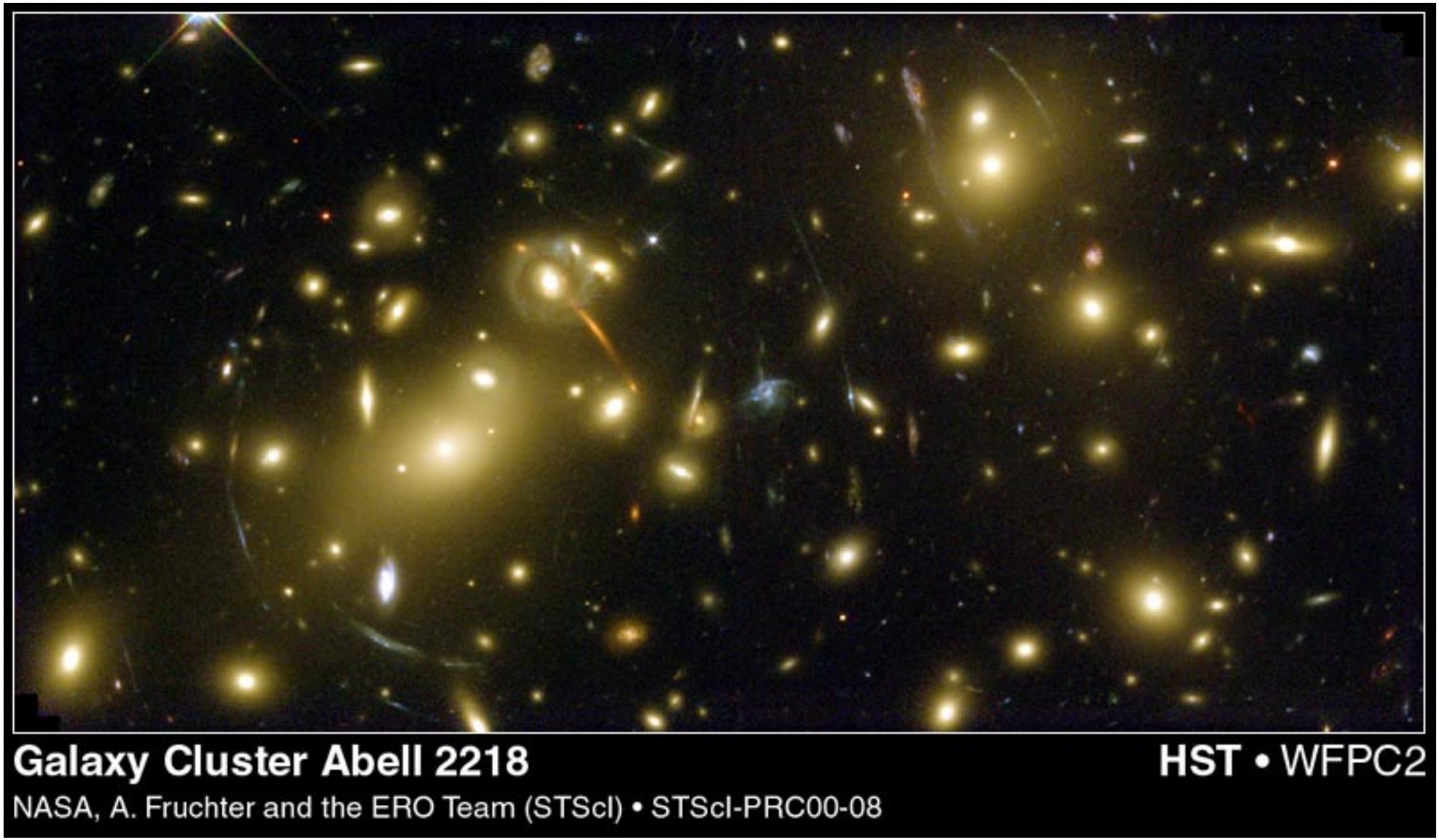
EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

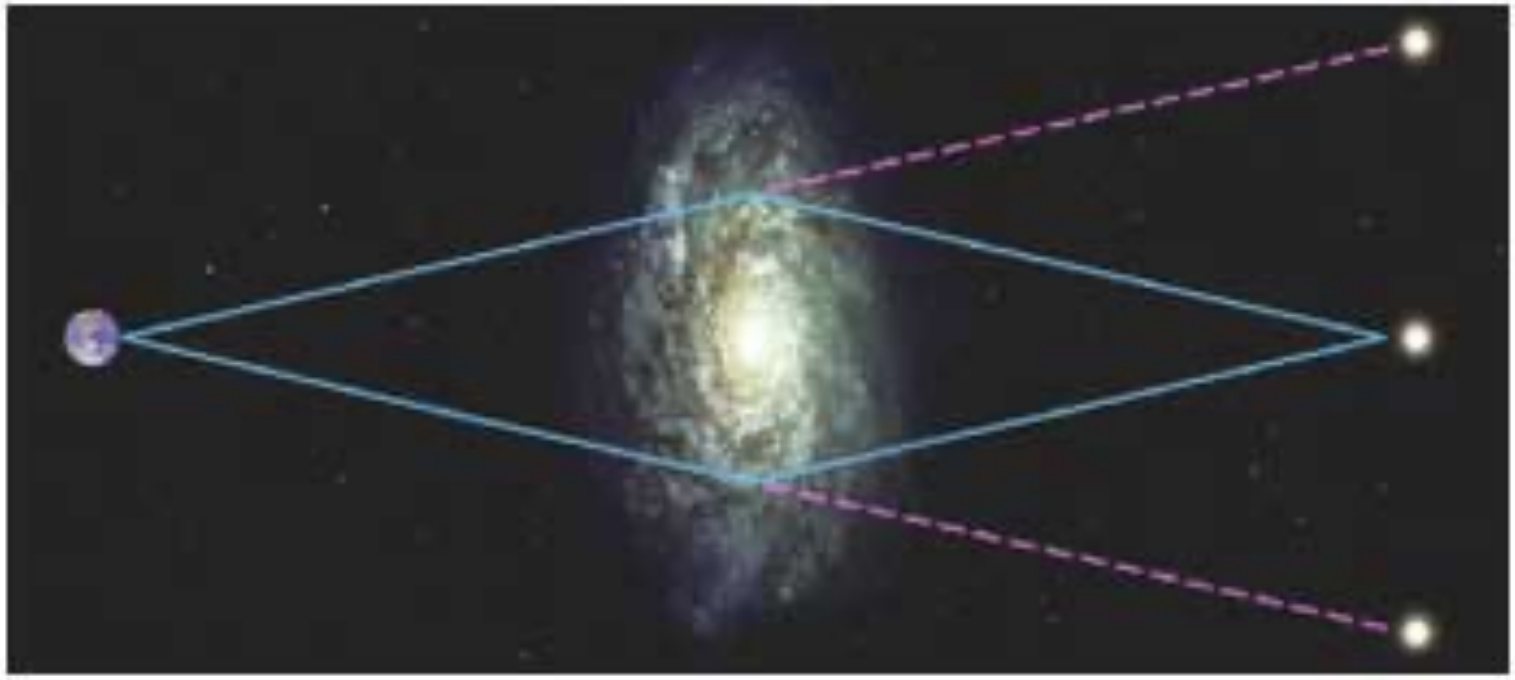
No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

Giant lenses in the sky

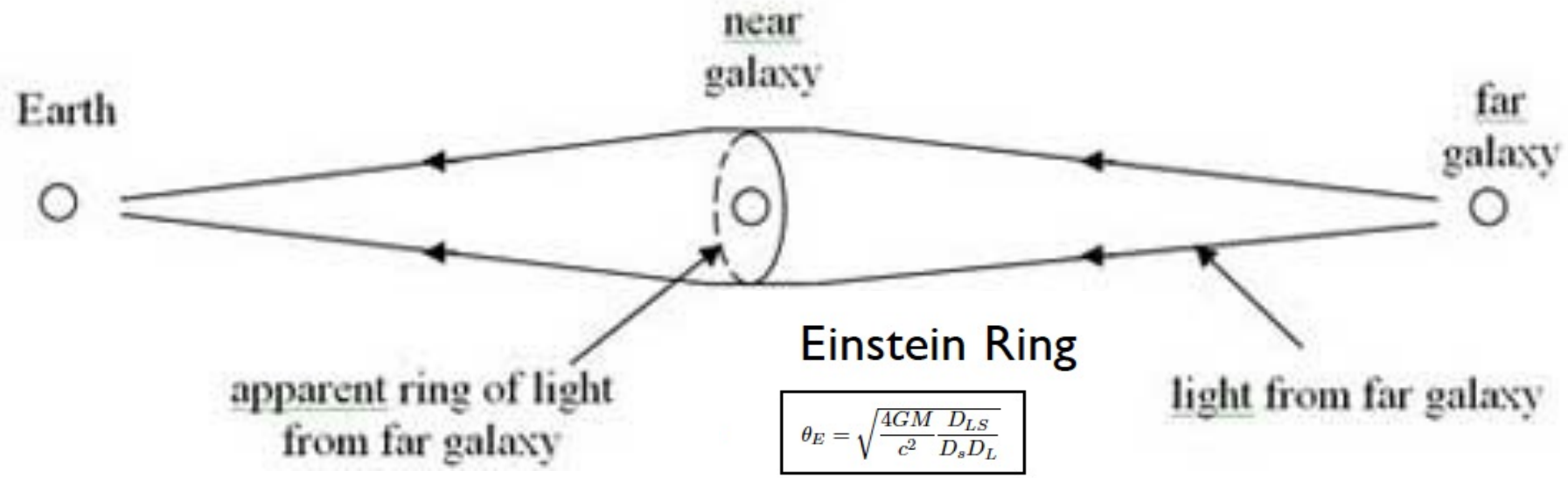


Gravitational Lensing

- The gravity from a foreground galaxy bends/images the light from a background galaxy or quasar.

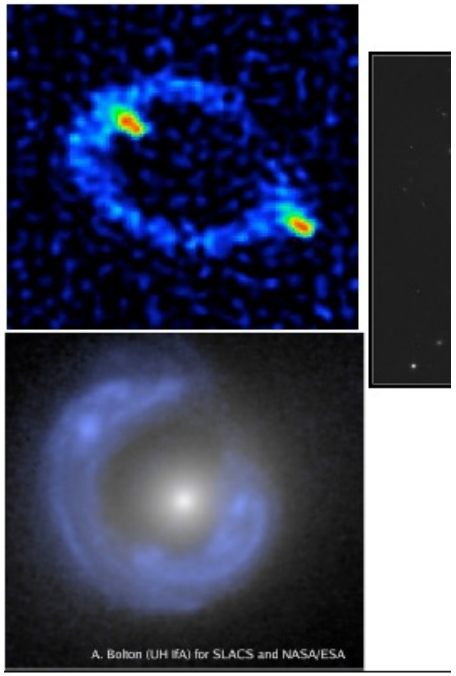


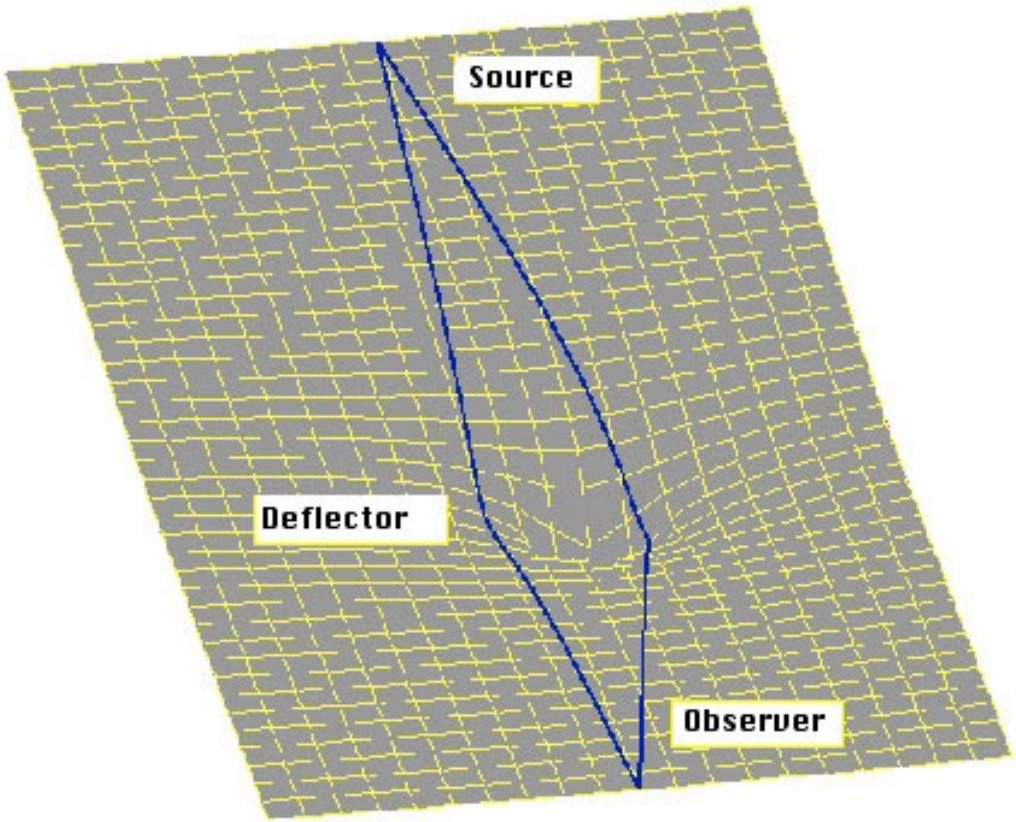
Light Is Bent On Cosmic Scales

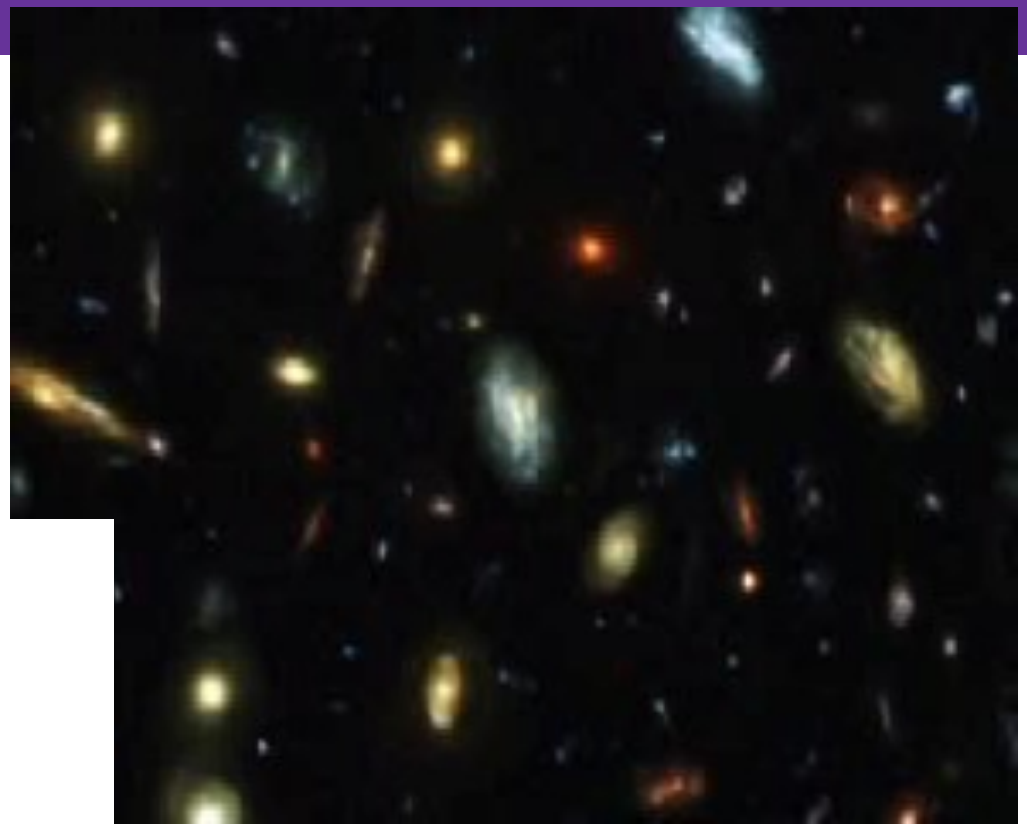


Einstein Ring

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$





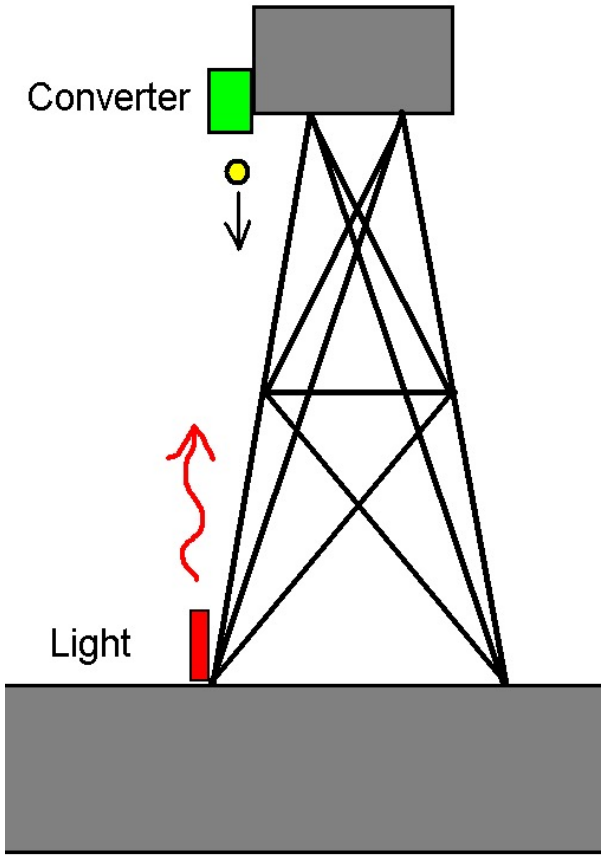


light 'bending' is a consequence of the bending of space itself

Remember the tower…

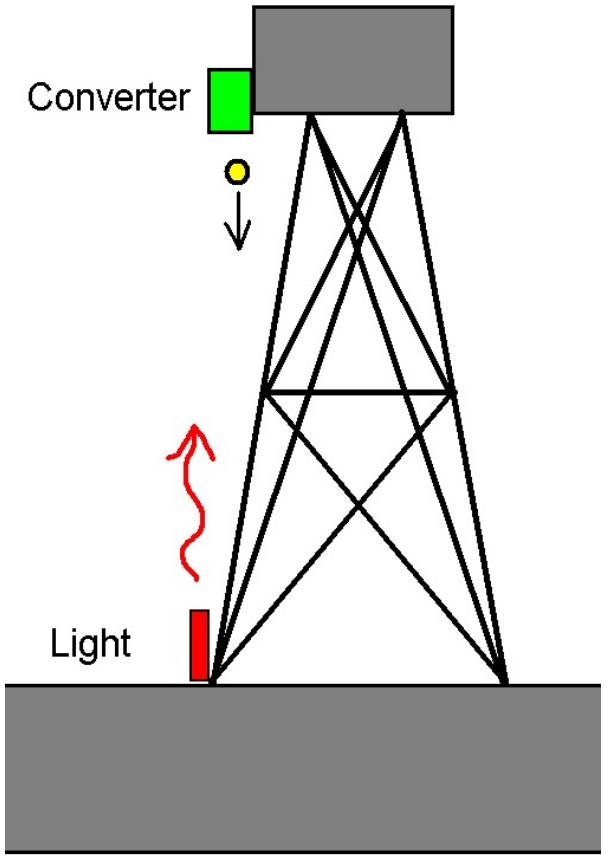
Light beam must lose energy as it climbs up

- So…frequency must decrease-wavelength increase
- i.e., light is redshifted.
- Gravitational redshifting



Remember the tower…

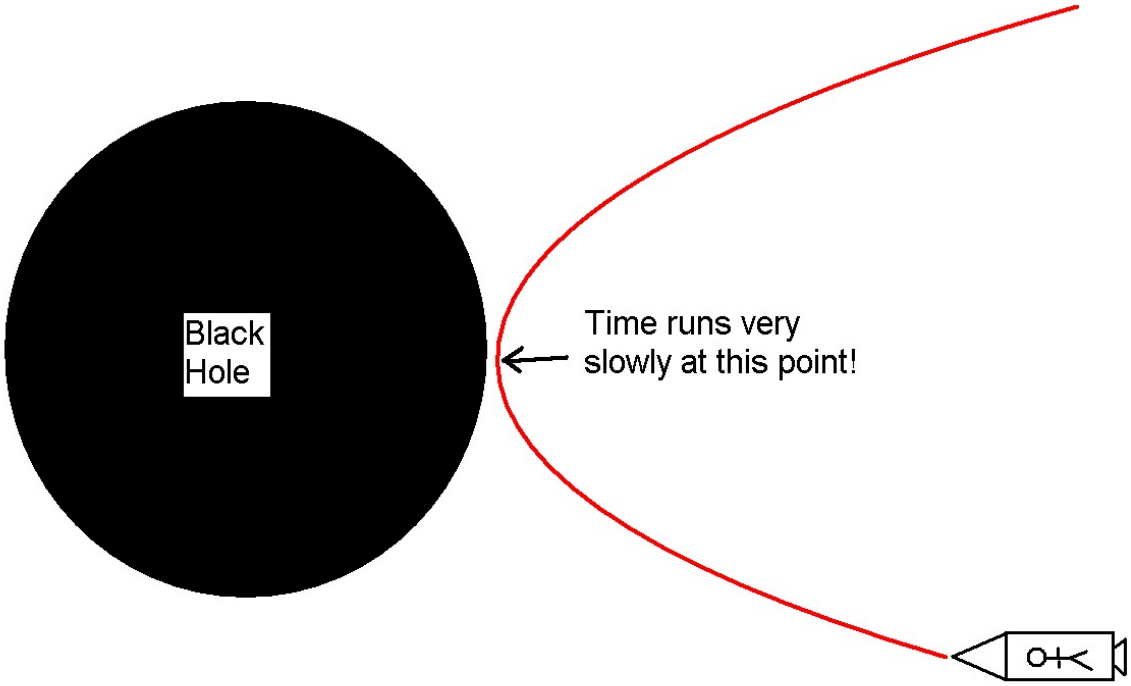
- Imagine a clock based on frequency of laser light…
  - 1 "tick" = time taken for fixed number of crests to pass
  - Gravitational redshifting slows down the clock.
  - **Clocks in gravitational fields must run slowly**



$$t_{grav} \approx \left(1 - \frac{GM}{c^2 r}\right) t_{space}$$

if gravitational field is "weak"

How to live for a 1000 years!-Interstellar the movie



- Go where gravity is very strong!
- Observer on Earth would see astronaut's clock running **very** slowly when close to black hole – astronaut would age very slowly. But not from the astronaut's perspective!
- (In fact, there are other discomforts from of being near a black hole!)

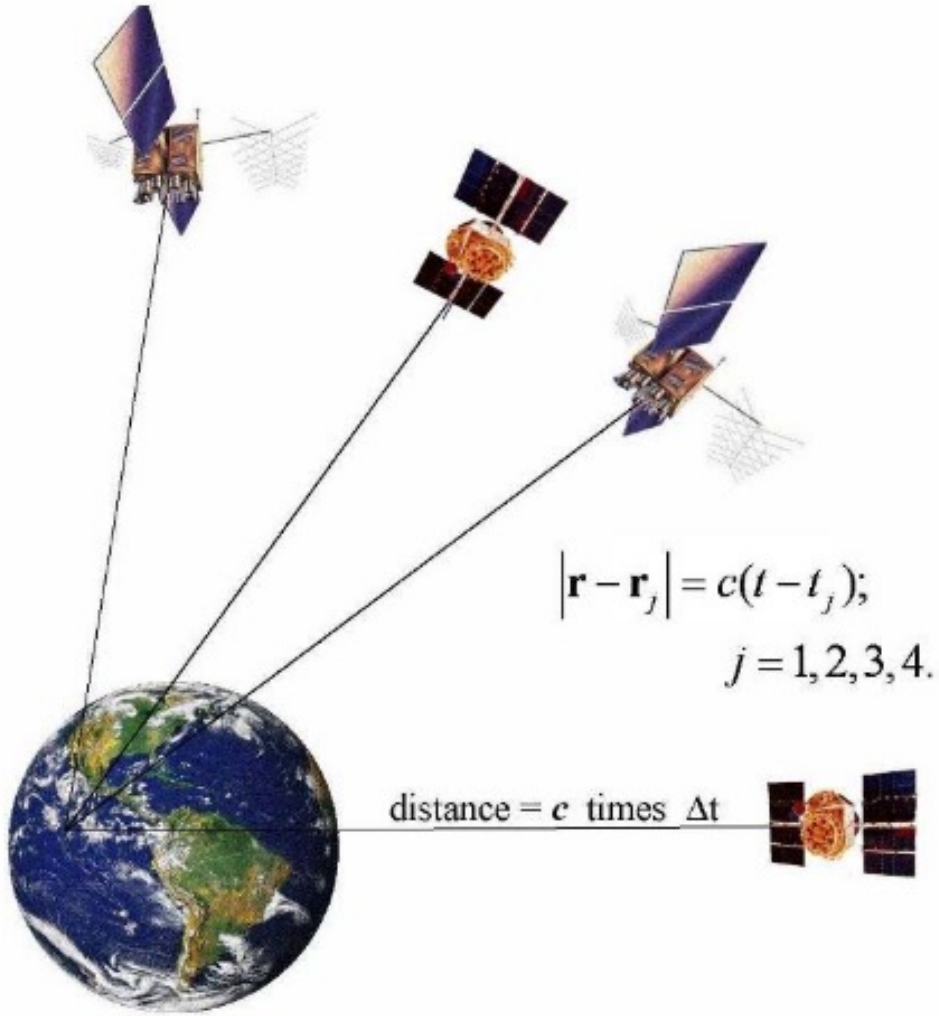
Gravitational time dilation has practical importance!

- Global Positioning System (GPS)
  - System of satellites that emit timing signals
  - Detector on Earth receives signals
  - Can figure out position on Earth's surface by measuring time delay between signals from different satellite (light travel time gives distance to satellite)
  - Need to measure time of signal from satellite very well!
    - 10m positioning requires ~30ns time accuracy
  - Satellites are at varying heights; clocks run at varying rates
    - Satellite clocks drift by ~38us per day wrt Earth clocks!
- If GR effects were not included, computed GPS positions would drift from true position by **kilometers per day**!

GPS Geometry

Have both special relativity (velocity of satellites) and GR effects (gravity) effects

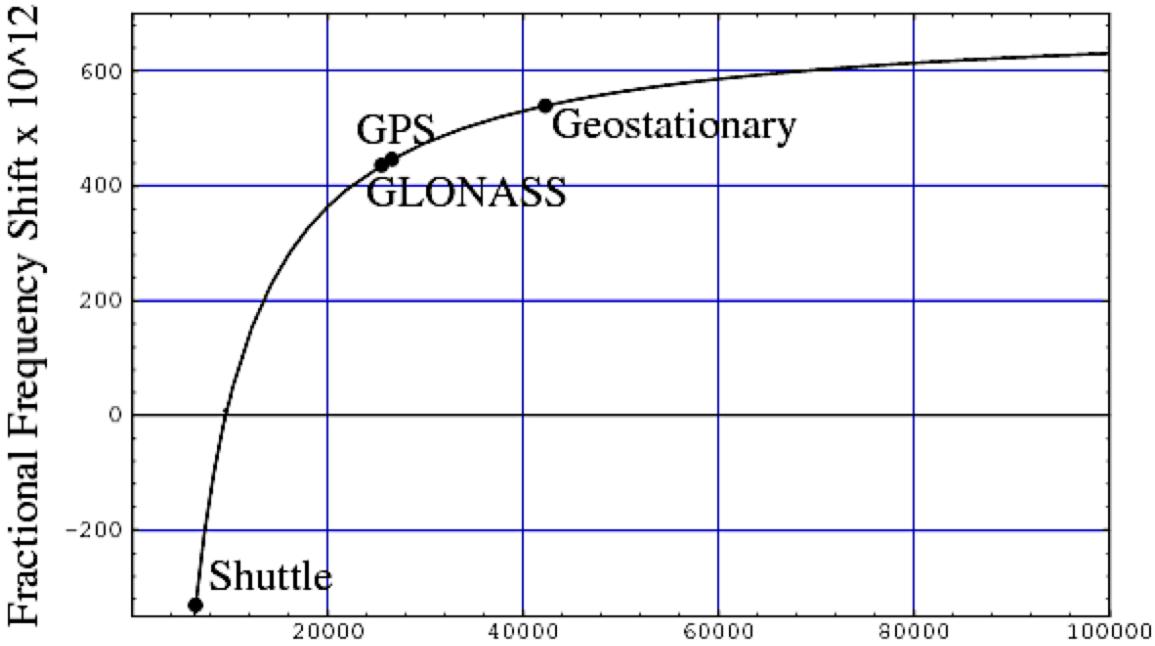
Without relativity corrections GPS would not work!



igure 11: The positions of receivers are shown. Transmitted at t_j, and at positions r_j.
alues of GvT is found by solving four simultaneous equations. Courtesy: (Ashby, 20

Relativistic Effects on GPS

- At height of GPS satellites GR effect ~6×10^{-10} sec/sec- ~15km/day
- For the shuttle time dilation due to its velocity is the dominant effect
- while for a GPS satellite clock, the gravitational blueshift is greater. The effects cancel at a ≈ 9545 km.



Consequences of Strong Equivalence Principle

Gravity affects space-time itself !

- Gravity can bend light
- Gravitational redshift
- Gravity can slow down time

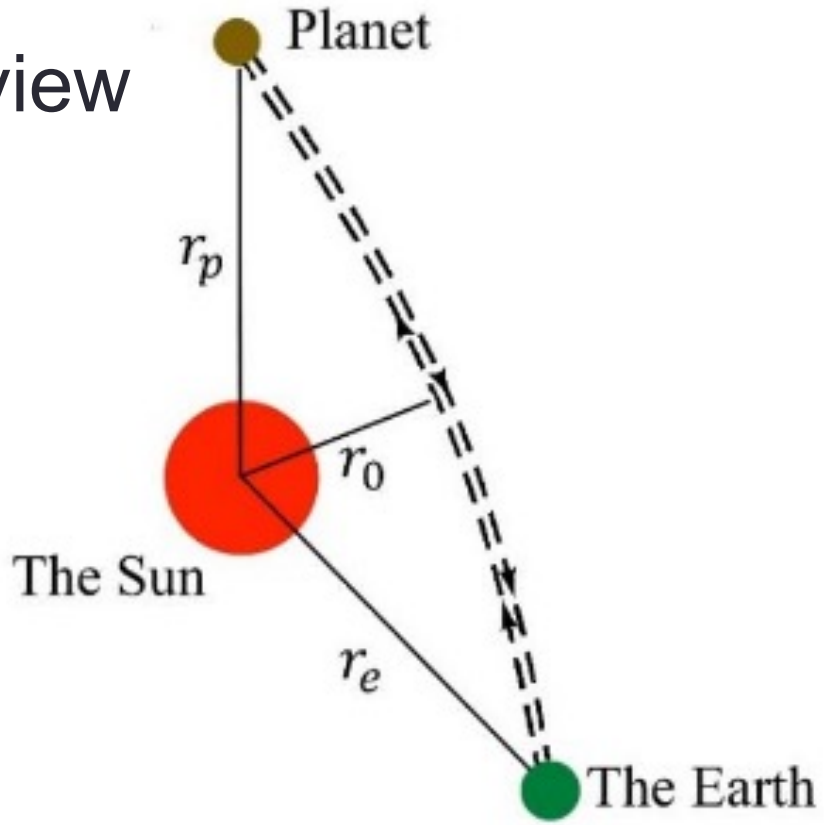
The **Shapiro time delay** effect, or **gravitational time delay**

- Gravitational time delay effect, one of the four classic solar-system tests of general relativity.
- Radar signals passing near a massive object take slightly longer to travel to a target and longer to return than they would if the mass of the object were not present due to bending of space-time
  - bounce radar beams off the surface of Venus and Mercury and measure the round-trip travel time as a function of orbit.
- Results agree with GR to 20 parts per million

Shapiro Delay Geometry

- Estelle Asmodelle (UCLAN Honors) thesis)

GR view



Newtonian view

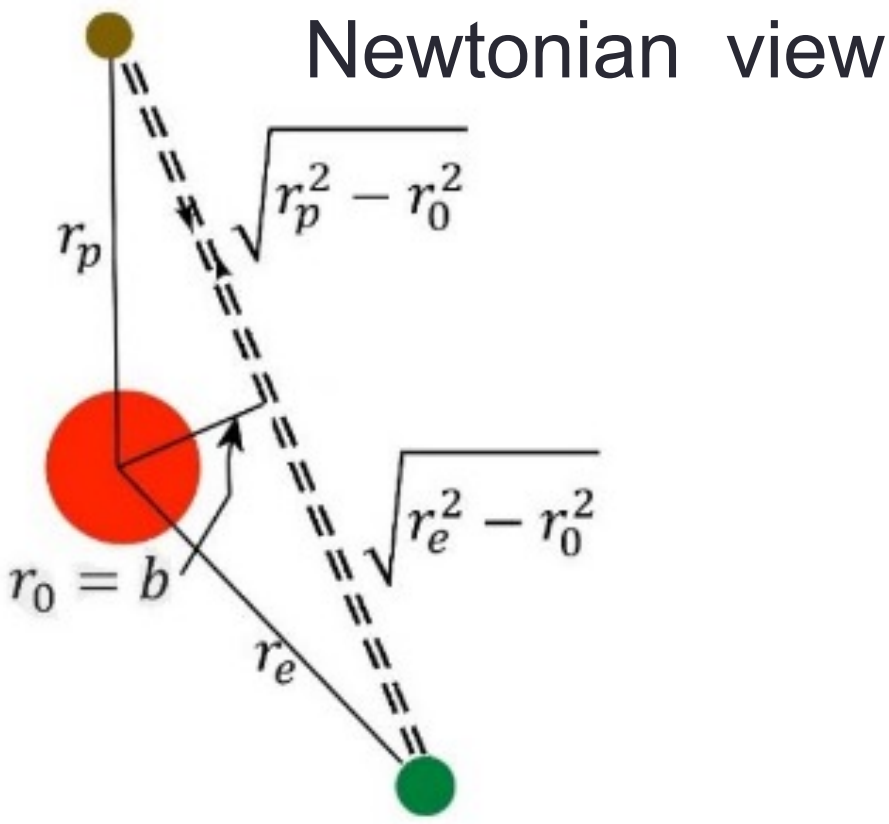


Figure 8 The radar reflection of photons from the Earth to a planet and back. The left image is the actual path, exaggerated. The right image is the Euclidean form. Illustration E. Asmodelle.

Hafele–Keating experiment- in previous lecture

- Hafele & Keating (1971) flew around world with atomic clocks…
- Clock on plane gained time relative to one on ground by…
  - 273±7ns (Westbound)
  - -59±10 (Eastbound)
  - But TWO terms
    - **special relativity**
    - **general relativity**

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

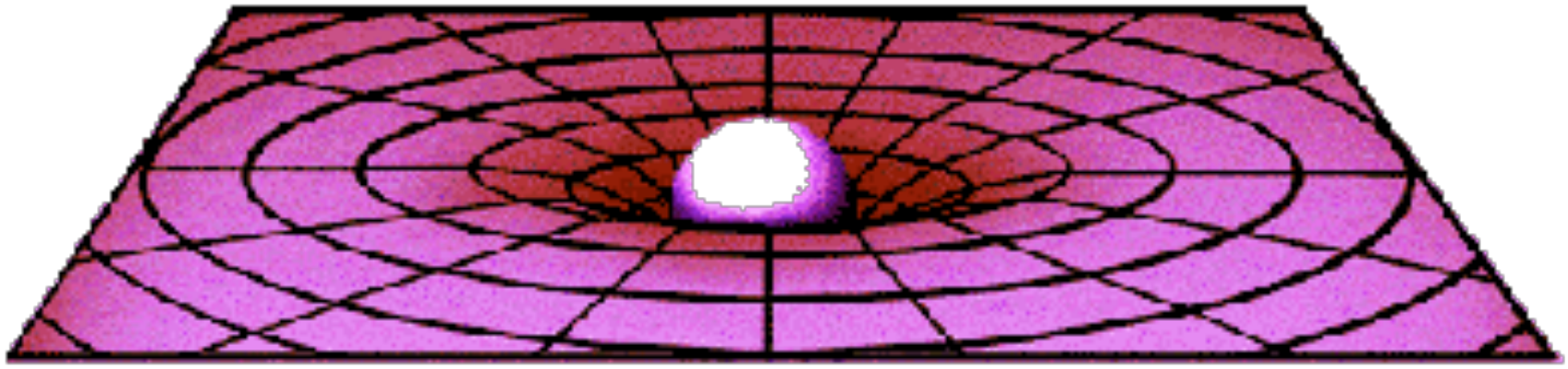
Two terms are need- special and general relativity

Another Effect of Gravity on Light

- Gravitational time delay- time it takes for light to travel to a source is affected by gravity (paths longer in *curved* space)

Curved Space (more in next lecture)

- Curved space around the Earth looks something like this…

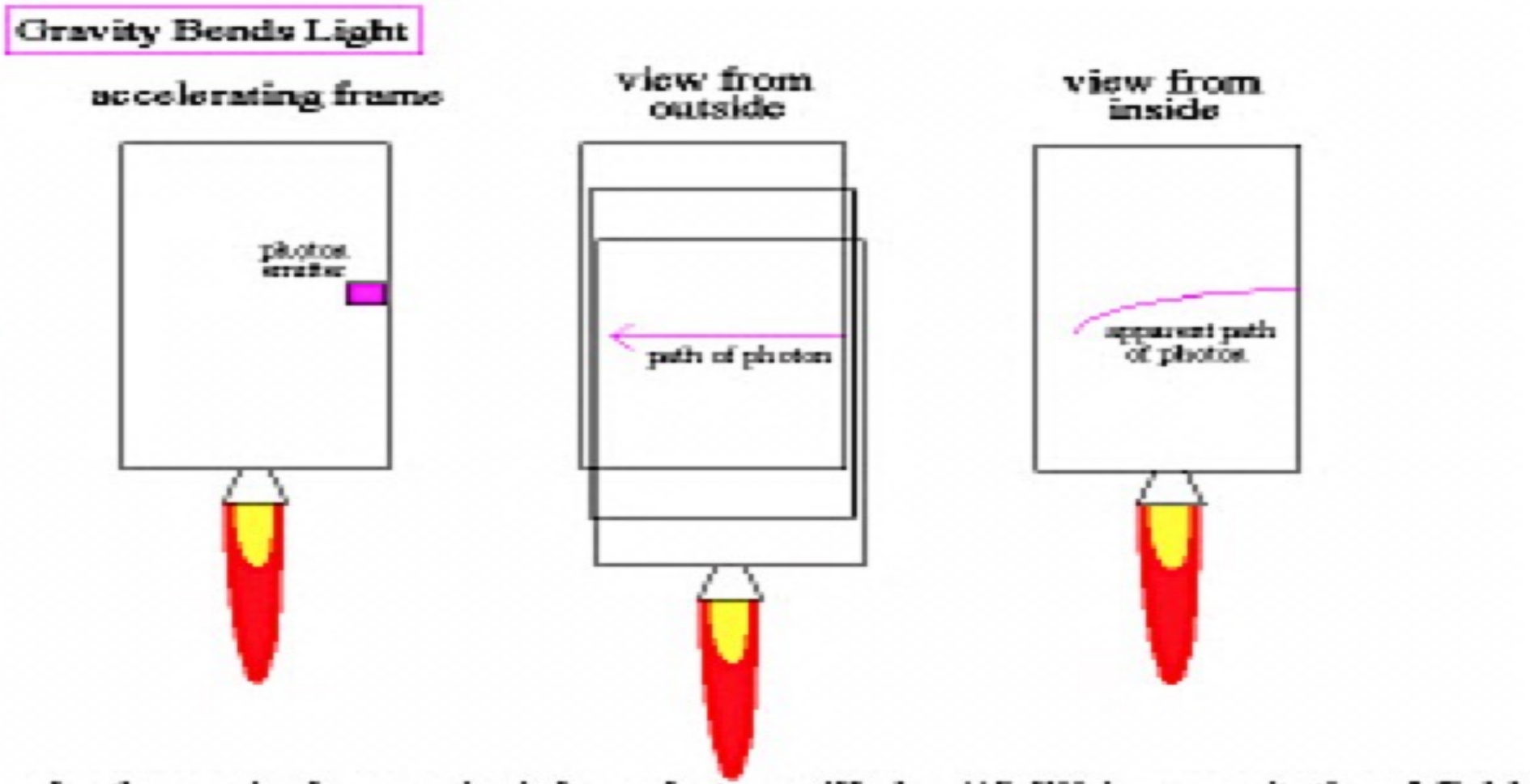


From web site of UCSD

EQUIVALENCE PRINCIPLES

- Recall the "**weak" equivalence principle:**
  - All objects are observed to accelerate at the same rate in a given gravitational field (Galileo tower of Pisa experiment).
  - Therefore, the inertial and gravitational masses must be the same for any object.
  - This has been verified experimentally, with fractional difference in masses $<10^{-11}$
- **As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot, locally, be distinguished**

Bending of light by Gravity



The principle of equivalence renders the gravitational field fundamentally different from all other force fields encountered in nature.

http://abyss.uoregon.edu/~js/cosmo/lectures/lec06.html