Specific fusion reactions

We will now continue from our previous general discussion of fusion to think about specific reactions that are important for stars. As we learned last time, the overwhelming majority of nuclear binding energy increase occurs from hydrogen to helium-4, so that's where we'll first focus our attention.

Helium-4 has two protons and two neutrons. Deuterium, aka heavy hydrogen, ^{2}H, has one proton and one neutron. Thus our initial thought might be that hydrogen fusion couldn't be easier:

$$^2\mathrm{H} +^2 \mathrm{H} \rightarrow^4 \mathrm{He}\ . \tag{1}$$

The problem, though, is that deuterium is relatively uncommon in the universe: its abundance is about 2.6×10^{-5} that of regular hydrogen (which has no neutron). Thus deuterium-deuterium fusion, although it can happen, is rare; instead, if a deuterium nucleus is in the center of a star, it is far more likely to fuse with ordinary hydrogen to produce helium-3: $^1\mathrm{H} +^2 \mathrm{H} \rightarrow^3 \mathrm{He}$. This, in fact, is what happens in the centers of brown dwarfs, which are too cool to fuse ordinary hydrogen.

But if we want to tap into the majority of available fusion energy, we need to fuse ordinary hydrogen into helium-4. Since helium-4 has four nucleons and ordinary hydrogen has one, our initial thought might be that the net reaction would be

$$4^1\mathrm{H} \rightarrow^4 \mathrm{He}\ . \tag{2}$$

But this equation isn't balanced: the electric charge on the left is +4 and on the right is +2. So we can add a couple of electrons to the left side:

$$4^1\mathrm{H} + 2e^- \rightarrow^4 \mathrm{He}\ . \tag{3}$$

Now we have balanced the electric charge on both sides, and both sides have a baryon number of 4, but the left hand side has a lepton number of 2 whereas the right hand side has a lepton number of 0. So our final balanced equation in terms of the particles is

$$4^1\mathrm{H} + 2e^- \rightarrow^4 \mathrm{He} + 2\nu_e\ . \tag{4}$$

The total energy released in this reaction is 26.73 MeV. This total is shared between the kinetic energy of the helium nucleus, the neutrinos, and the photons that are produced. For the purpose of supporting the star against gravitational collapse, the energy in the neutrinos is simply lost because they have such a tiny cross section (typically $\sim 10^{-44}$ cm^2 $(E_\nu/1\ \mathrm{MeV})^2$) that they leave the star without interacting. For the purposes of scientists studying the Sun, however, the neutrinos can provide a lot of information (and because there are so many produced by the nearby Sun, we can detect them occasionally).

Even this simple equation gives us significant insights:

1. This reaction cannot possibly happen in one step. It would require the simultaneous interaction of six(!) particles, at least four of which (the protons) would need to be within $\sim 10^{-13}$ cm of each other for the strong force to take effect. This is not possible. As a result, there has to be a multistage process to fuse hydrogen into helium.
2. Unlike deuterium+deuterium=helium, where you start and end with two protons and two neutrons, when you start with four protons you need to convert two of them into neutrons to make helium-4. This involves the weak force (which is why we end up with neutrinos), and that means that there is a significant delay built into such fusion reactions. We'll see later that this contrasts with helium fusion to carbon, which does not require the conversion of protons to neutrons and is thus not weak decay limited.

With that background, we will go into some details of hydrogen fusion. We will begin by assuming that we start with pure, normal, hydrogen: ^{1}H. We will then learn that the presence of carbon, nitrogen, and oxygen leads to another hydrogen fusion possibility that turns out to be more important for stars more massive than the Sun and thus hotter in their cores.

Consider first the case in which only protons are around. Two protons can't form a nucleus, so one has to have an inverse beta decay to a neutron:

$$^1\mathrm{H} +^1\mathrm{H} \rightarrow^2\mathrm{H} + e^+ + \nu_e\ . \qquad (5)$$

This isn't very probable, but it can happen: the rest mass-energy of two protons is 1876.544 MeV, whereas the sum of the rest mass-energies of a deuteron and a positron is 1876.124 MeV, so there is 0.42 MeV of energy released in this interaction.

The rate for this reaction is

$$r_{p-p} = \frac{1.15\times 10^9}{T_9^{2/3}} X^2(\rho/1\ \mathrm{g\ cm^{-3}})^2 e^{-3.38/T_9^{1/3}}\ \mathrm{cm^{-3}\ s^{-1}}\ . \qquad (6)$$

Here X is the mass fraction of hydrogen, Y is the mass fraction of helium, and Z is the mass fraction of everything else combined (obviously $X+Y+Z=1$; typically $X\approx 0.7$, $Y\approx 0.3$, and $Z\approx 0.03$). Here $T_9\equiv T/10^9$ K. As always, we want to check that this equation is reasonable. For example, should the rate be proportional to X^2? How about ρ^2? Yes to both; you've got two things reacting, so the rate per volume should be proportional to the product of their number densities, which means $(X\rho)(X\rho)$ in this case. Is the sign of the exponent correct, i.e., should the exponent depend inversely on T? To answer this, we need to ask whether the reaction is more common at low or at high T; since it is more common at high T, the sign in the exponent is right. I don't know of an obvious way to get the $T^{-1/3}$ dependence specifically, but that's what you get if you just consider the Gamow peak.

When we compute the rate of energy generation, we note that in addition to the 0.42 MeV emitted due to the formation of a deuteron, the positron will annihilate quickly with one of the many electrons around, and this will produce another 2×0.511 MeV $= 1.022$ MeV of energy, for a total of 1.442 MeV emitted in the reaction. As discussed above, some of that will be lost to the neutrino. This is a very rare reaction; the typical proton in the core of the Sun waits 9 billion years(!!!) until it fuses in this way with another proton. The rarity of the reaction means that it has not yet been measured directly in a laboratory on Earth; instead, reactions are measured at higher energies and then extrapolated to the stellar domain using theory.

Let's compute the energy generation rate per mass for this step. The rate given above is per volume, so since density is mass per volume if we divide by the density we get energy per mass per time. If we put in $\rho = 100$ g cm^{-3} (roughly the value for the Sun's core), $X = 0.7$, and $T = 1.5 \times 10^7$ K, we get about 10^6 reactions per gram per second. At 1.442 MeV per reaction, this is an energy generation rate of a little over 2 erg g^{-1} s^{-1}. This is *tiny*! For comparison, normal combustion (e.g., burning of paper) gives about 4×10^{11} erg g^{-1} s^{-1}. What do you think about that?

The temperature exponent ($d \ln \mathrm{rate}/d \ln T$) is

$$\nu_{p-p} = \frac{11.3}{T_6^{1/3}} - \frac{2}{3} \; , \tag{7}$$

which is much less sensitive to temperature than some of the reactions we will see coming up. For example, at the solar center, where $T_6 \approx 15$, $\nu \approx 4$. **Lesson from this:** when weak decays are required (as in this case, p$\longrightarrow$n), the reactions are slow.

Now that we have deuterium, what's the next step? We might think that it would be to combine two deuterium nuclei to make ^{4}He, which would be perfectly legal. But as we discussed briefly above, the number of protons is huge, so once deuterium is formed it almost immediately captures a proton and becomes more bound as ^{3}He:

$$^1\mathrm{H} +^2 \mathrm{H} \rightarrow^3 \mathrm{He} + 5.493 \ \mathrm{MeV} \; . \tag{8}$$

Note that this does not require the conversion of a proton into a neutron and thus the weak force is not involved. As a result, this is extremely fast; one estimate is that under Solar core conditions once a deuterium nucleus is formed, it only lasts for about one second before it is converted into helium-3.

From this point, there are three different ways to convert the helium-3 into helium-4. These are called the proton-proton (or p-p) "chains", i.e., p-p I, pp-II, and pp-III (and in principle there is a pp-IV although it has never been observed). You can find the details in many places including Wikipedia and our textbook, where you will learn that different branches are important at different temperatures, but unless you end up specializing in precisely this type of astrophysics there is no reason to memorize the specifics. The overall energy release by these three chains depends on their relative weighting, but to leading order it is

$$\epsilon_{\rm eff} \approx \frac{2.4\times 10^4 \rho X^2}{T_9^{2/3}} e^{-3.38/T_9^{1/3}}\ {\rm erg\ g^{-1} s^{-1}}\ . \tag{9}$$

Note that we now have a ρ^1 dependence, in contrast to the earlier ρ^2 dependence, because we are now quoting the energy generation rate per mass rather than the energy generation rate per volume.

As an aside: you probably know that there are efforts to have controlled nuclear fusion on Earth because of the lure that this would be an enormous source of energy and would not have any carbon footprint. In contrast to the core of a star, which has high density and relatively low temperature, laboratory fusion efforts have low density and much higher temperature. The plasma has to be confined, lest it melt the laboratory, and that confinement uses magnetic fields. There are, however, large numbers of instabilities which have so far prevented the plasma from being confined long enough for economically viable fusion, and we are decades away from this being a useable energy source. Another approach that has been tried is to use lasers to blast a deuterium pellet to collapse it and induce fusion, but again there are troublesome instabilities. It's a tough problem.

The p-p chains aren't the only way to burn (i.e., fuse; "burn" is commonly used) protons to helium. In fact, at higher temperature the "CNO cycles" are more important, because they have a stronger temperature dependence than the p-p chains (as you would expect given that the heavier nuclei C, N, and O have greater electric charge than protons). There are lots of details and sub-branches (e.g., see the Wikipedia page on "CNO cycle"), but these are genuine catalytic reactions in which you start with a nucleus and go through various steps including reactions with hydrogen which leave you with the original nucleus plus a ^{4}He nucleus. To give you an idea of how that works, here's the so-called "cold CNO-I" cycle (don't bother memorizing this), which is relevant for stars with core temperatures slightly greater than that of the Sun. Remember that C has 6 protons, N has 7 protons, and O has 8 protons.

$$\begin{aligned}
{}^{12}{\rm C} + {}^{1}{\rm H} &\rightarrow {}^{13}{\rm N} \\
{}^{13}{\rm N} &\rightarrow {}^{13}{\rm C} + e^+ + \nu_e \\
{}^{13}{\rm C} + {}^{1}{\rm H} &\rightarrow {}^{14}{\rm N} \\
{}^{14}{\rm N} + {}^{1}{\rm H} &\rightarrow {}^{15}{\rm O} \\
{}^{15}{\rm O} &\rightarrow {}^{15}{\rm N} + e^+ + \nu_e \\
{}^{15}{\rm N} + {}^{1}{\rm H} &\rightarrow {}^{12}{\rm C} + {}^{4}{\rm He}\ .
\end{aligned} \tag{10}$$

Here we're not indicating specifically when a photon is emitted. The total energy released is 26.73 MeV (after including the inevitable annihilation of the positrons with electrons), as it is in any reaction that takes four protons and eventually becomes helium-4. It is good practice to go through the lines of these reactions to see how baryon number and lepton number are the same on both sides in every step. As promised, we start with ^{12}C, add four protons, and end with $^{12}\mathrm{C} + ^4\mathrm{He}$. Then we can use the carbon again, which means that this is a catalytic reaction. Other variants of the CNO cycle have different specifics, but they are all catalytic. It is possible to break out of this cycle, e.g., if one of the intermediate nuclei fuses with a helium nucleus.

The importance of this process is partially responsible for the abundance of these elements. In fact, the mass fraction of the three combined is about 3/4 of the total mass fraction of "metals" (Z>2). The energy generation rate for the CNO cycles is (depending on the relative weighting of the cycles)

$$\epsilon_{\rm CNO} \approx \frac{4.4\times 10^{25}\rho X Z}{T_9^{2/3}} e^{-15.2/T_9^{1/3}}\ {\rm erg\,g^{-1}\,s^{-1}}\ . \tag{11}$$

Note that there is an X and Z dependence here, compared with the X^2 dependence for p-p, because we need heavier elements for the CNO cycle. One consequence of this is that the *very* first stars in the universe wouldn't have been able to tap into the CNO cycle because those heavier elements had not been generated. The CNO energy generation rate is much higher than it is for p-p, and the temperature exponent is

$$\nu({\rm CNO}) = \frac{50.8}{T_6^{1/3}} - \frac{2}{3}\ , \tag{12}$$

or about 18 for $T = 2\times 10^7$ K. Note that this is *much* steeper than the $\nu \sim 4$ exponent for the p-p chain.

Now let's take a step up: once we run out of hydrogen to fuse in the core, how can we take advantage of the available nuclear binding energy by going to yet heavier elements?

The first step is helium burning. You might think that this is straightforward: just have $^4\mathrm{He} + ^4\mathrm{He} \rightarrow ^8\mathrm{Be}$. But there is no stable element of mass 8, and indeed to go from two helium nuclei to beryllium-8 you need an *input* of 92 keV (i.e., the reaction is endothermic by 92 keV). Instead, it is necessary to have *three* helium-4 nuclei come together to form a carbon-12 nucleus: $3^4\mathrm{He} \rightarrow ^{12}\mathrm{C}$. In (moderate) detail, this needs to happen by two helium nuclei very temporarily ($\sim 10^{-16}$ seconds!) forming an unstable beryllium-8 nucleus, followed by another helium nucleus approach and forming carbon. For this to occur the Gamow peak needs to be around the ~ 90 keV energy, which turns out to imply a temperature of at least $T \sim 10^8$ K. Note that there are no weak decays involved, which means that this reaction can happen very quickly.

The energy generation rate is

$$\epsilon_{3\alpha} = \frac{5.1\times 10^{8}\rho^2 Y^3}{T_9^{3}} e^{-4.4/T_9}\ {\rm erg\,g^{-1}\,s^{-1}}\ . \tag{13}$$

Here we have ρ^2 and Y^3 because we have three helium nuclei in the interaction. The density and temperature exponents are

$$\lambda_{3\alpha} = 2, \qquad \nu_{3\alpha} = \frac{4.4}{T_9} - 3 \; . \tag{14}$$

This is very temperature sensitive ($\nu \approx 40$ when $T = 10^8$ K!).

Fusion of Carbon, Neon, and Oxygen: As we move toward fusion of heavier elements, the reactions tend to get more diverse. For example, carbon can capture an alpha (recall that an alpha particle is a ^{4}He nucleus) and become oxygen: $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$, and oxygen can become neon in the same way: $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$. We can also have nuclear reactions between two heavy nuclei, such as $^{12}\text{C}+^{12}\text{C} \rightarrow^{20}\text{Ne}+\alpha$ or $\rightarrow^{23}\text{Na}+p$, or $^{16}\text{O}+^{16}\text{O} \rightarrow^{28}\text{Si}+\alpha$. We expect, and find, that these reactions happen at higher temperatures than what we discussed previously (because the electric charges are larger), and for the same reason these reactions are more temperature sensitive than our previous reactions. For example, the exponential part of the $^{12}\text{C}+^{12}\text{C}$ reaction (summed over channels) is like $e^{-84T_9^{-1/3}}$, and the leading factor for $^{16}\text{O}+^{16}\text{O}$ is $e^{-136T_9^{-1/3}}$!

Photodisintegration: At such high temperatures (around 10^9 K), photons in the tail of the Planck (blackbody) distribution are energetic enough to photodissociate some nuclei. For example, we can have $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$ as well as the reverse process, and since neon is more bound than oxygen, this takes net energy rather than generating energy. Reactions such as this therefore reduce the total energy generation from the system. There can also be several reactions in succession, such as $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$. In general, things therefore get more complicated when we consider the fusion of heavier elements..

Even heavier elements: Above 3×10^9 K, things get *really* involved. For example, one reaction is $^{28}\text{Si}(\gamma,\alpha)^{24}\text{Mg}(\alpha,p)^{27}\text{Al}(\alpha,p)^{30}\text{Si}$, which effectively has added two neutrons to the original silicon nucleus. This kind of stuff, taken upwards to the "iron peak" elements which have the highest binding energy per nucleus, constitutes silicon burning but is awfully complex. The most-bound nucleus is ^{56}Fe, but in some cases (e.g., supernovae) the time scale is so short that instead ^{56}Ni is produced (because it has an equal number of protons and neutrons and can therefore be built up from things such as ^{28}Si without waiting for weak interactions or electron captures, which are slow). A big difference is that for light element fusion, one has a slow transition from one nucleus to another. For heavy element fusion, temperatures are so high and reactions are so varied that one reaches a state close to nuclear statistical equilibrium, where one can figure out relative proportions of different nuclei as a function of temperature.

Another thing that happens at these extremely high temperatures is that electron neutrinos can be generated. We saw this before when weak decays happened in the burning of hydrogen, but at high energies electron-positron pairs can be produced, and when they annihilate they can produce neutrino-antineutrino pairs which then escape. This speeds the doom of massive stars.

Finally, if a large iron core is built up, no net energy can be generated in the core, and a supernova results. But that's for later.

Summary: Proton burning, which supplies most of the energy for most of the time because of the binding energy curve, can happen in two basic ways, which each have multiple subvariants:: the pp chain (more important for lower temperatures and thus lower-mass stars) and the CNO cycles. Helium burning happens via the triple-α process. Burning of heavier elements can happen by alpha capture or by a number of other processes, which multiply as the temperature increases. Neutrino production at high temperatures takes energy away from the system in a very efficient manner.