Star Formation: Theory

The average density of the Galaxy is around 10^{-24} g cm^{-3}. In contrast, the average density of a star is ~ 1 g cm^{-3}, so obviously a rather substantial density increase needs to happen to form a star! On large scales, gravity dominates, so we need to think about gravitational collapse to go from the interstellar medium to a star. For collapse to happen, gravity must, locally, dominate all opposing mechanisms. What kinds of things might support gas against gravitational collapse? Examples include thermal energy/pressure, rotation, magnetic fields, and turbulence. But at any given time, most of the gas in a galaxy isn't collapsing to form stars. We therefore need to find a condition for when gas collapses and when it doesn't.

To simplify our lives we can imagine that we have set up a nonrotating, nonmagnetic, nonturbulent cloud and ask about when gravity will beat thermal pressure. This leads to a minimum mass called the *Jeans mass*, after Sir James Jeans. For a uniform-density spherical cloud of mass M and radius R, the gravitational binding energy is $E_g = \frac{3}{5}\frac{GM^2}{R}$. If there are N particles in thermal equilibrium at temperature T, then the thermal energy for a monatomic gas is $E_t = \frac{3}{2}NkT$ (for the interested: it's $(1/2)kT$ per degree of freedom, and for a monatomic gas the degrees of freedom are movement in each of the three perpendicular directions, giving $(3/2)kT$ per particle). The condition for gravity to win is $E_g > E_t$ (i.e., a negative total energy), so

$$\frac{3}{5}\frac{GM^2}{R} > \frac{3}{2}\frac{M}{\mu m_p}kT \ , \tag{1}$$

where $N = M/\mu m_p$ and μ is the mean molecular weight. Therefore, for collapse to be possible we need

$$M > \frac{5}{2}\frac{kT}{G\mu}R = \frac{5}{2}\frac{kT}{G\mu}\left(\frac{M}{\frac{4}{3}\pi\rho}\right)^{1/3} \Rightarrow M > \left(\frac{5}{2}\frac{kT}{G\mu m_p}\right)^{3/2}\left(\frac{4\pi}{3}\rho\right)^{-1/2} \equiv M_J \ . \tag{2}$$

The scaling of the Jeans mass M_J is $M_J \sim T^{1.5}n^{-0.5}$, where T is in Kelvin and n is the number density in cm^{-3}. Therefore, if $M > M_J$ then the cloud will start to collapse, whereas if $M < M_J$ the cloud will not collapse. Note that this is the absolute minimum mass for a bound cloud; any smaller, and the cloud will simply drift apart. For $\rho = 10^{-23}$ g cm^{-3} and $T = 100$ K, typical of the ISM, neutral hydrogen ($\mu = 1$) has $M_J \approx 10^4\,M_\odot$.

Suppose first that there is no support against collapse. In that case it would collapse on a free-fall timescale

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 3.4\times 10^7 n^{-1/2}\,\mathrm{yr} \ , \tag{3}$$

where again n is in cm^{-3}. Typical densities in sterile (non-star-forming) regions are $\sim 50-100$ cm^{-3}, implying $t_{ff} \approx 5\times 10^6$ yr. This is $\sim 0.1\times$ the inferred lifetime of clouds, so something must hold up the collapse.

As the gas cloud collapses, its thermal energy goes up and can in principle halt (or at least slow) the collapse. We have

$$M_J \sim T^{3/2}\rho^{-1/2} \ . \tag{4}$$

If the equation of state is polytropic, so that $P \propto \rho^\gamma$ and $T \propto \rho^{\gamma-1}$, then

$$M_J \sim \rho^{(3\gamma-4)/2} \ . \tag{5}$$

Ask class: what does this mean for whether collapse will stop or run away? If the Jeans mass decreases as the cloud collapses, there will be a runaway; if it increases, the collapse will stop. Therefore, for $\gamma < 4/3$ there is a runaway. That's the same power law that we derived earlier (in a problem set) from the equation of hydrostatic equilibrium.

Halting collapse by rotation

Here, we need to compare the rotational energy with the gravitational energy:

$$\beta = \frac{E_{\rm rot}}{|E_{\rm grav}|} \ll 1 \tag{6}$$

is the usual initial condition. Now, $E_{\rm rot} \sim (R\Omega)^2$ and $E_{\rm grav} \sim 1/R$, so $\beta \propto R^3\Omega^2$. If angular momentum is conserved during the collapse, then L=const=$R^2\Omega$, so $\Omega \sim 1/R^2$ and $\beta \propto 1/R$. Since the collapse is over many orders of magnitude (say, 10^{18} cm for a solar mass cloud to 10^{11} cm for a star), this means that rotation can halt the collapse even if it is unimportant initially.

Let's work this out. The Galaxy rotates with a period of about 200 million years, so let's say that the initial molecular cloud shares at least that rotation. If a 1 $M_\odot$ portion of the gas has a radius of 10^{18} cm initially, then to collapse to something the size of the Sun ($\sim 10^{11}$ cm) and conserve its angular momentum it needs to spin 10^{14} times faster than its initial 200 million year period, or in about 1 minute(!) compared with $\sim$3 hr for breakup and $\sim$30 days for the actual rotation period of the Sun. This is a serious problem! As a sidelight, let's think for a second about what other angular momentum exists in the solar system. **Ask class:** do they know what fraction of the mass of the solar system is in the Sun? About 99.8%. What fraction of the angular momentum? The Sun rotates at about 1/300 of the Keplerian orbital frequency at its radius. Jupiter orbits at the Keplerian frequency, of course. In addition, Jupiter is at a radius of 5 AU=7×10^{13} cm, or 1000 times the radius of the Sun, so it has a specific angular momentum (recall that here "specific" means "per unit mass") about $1000^{1/2}$ times greater than a particle orbiting at the limb of the Sun. Therefore, the specific angular momentum of Jupiter is about $1000^{1/2}\times 300 = 10^4$ times that of the Sun. The Sun's mass is 10^3 times Jupiter's so Jupiter's angular momentum is 10 times that of the Sun. In reality, the Sun's mass is centrally concentrated, and $J_J/J_\odot \approx 100$. So, that helps, but not enough.

Magnetic support against collapse

Again, compare the magnetic energy to the gravitational energy. **Ask class:** what is the magnetic energy density for a field of strength B? $E_B = B^2/8\pi$. Then the condition for collapse against magnetic support is $\frac{3}{5}\frac{GM^2}{R} > \frac{B^2}{8\pi}\frac{4}{3}\pi R^3$, so

$$M^2 > \frac{B^2}{3.6G} R^4 \Rightarrow M > \frac{BR^2}{(3.6G)^{1/2}} \equiv M_J(\mathrm{mag}) \ , \tag{7}$$

the magnetic Jeans mass. If the gas is perfectly conducting, so that there is flux freezing and BR^2=constant during the collapse, then magnetic field cannot halt collapse once it starts. Typical values of the magnetic field strength are:

(1) ISM, $n \sim 10$ cm^{-3}, $B \sim 10^{-6}$ G, $T \sim 100$ K

$$M_J(\mathrm{therm}) \sim 10^4 \, M_\odot \sim M_J(\mathrm{mag}) \ . \tag{8}$$

(2) Giant molecular clouds, $n \sim 10^2 - 10^3$ cm^{-3}, $B \sim 3 \times 10^{-5}$ G, $T \sim 10$ K

$$M_J(\mathrm{therm}) \sim 10 \, M_\odot \, ; M_J(\mathrm{mag}) \sim 10^3 \, M_\odot \ . \tag{9}$$

(3) Dense cores, $n > 10^4$ cm^{-3}, $B \sim 10^{-5}$ G(?), $T \sim 10$ K

$$M_J(\mathrm{therm}) \sim \mathrm{few} \, M_\odot \sim M_J(\mathrm{mag}) \ . \tag{10}$$

At those densities, though, the magnetic field may be decoupled from the gas.

We have therefore identified three potential problems with star formation by gravitational collapse: (1) if the polytropic index exceeds 4/3 (and recall that the index is 5/3 for an ideal gas), then the cloud can heat up fast enough to stop the collapse with thermal pressure gradients, (2) rotation and a centrifugal barrier will generally set in if the cloud conserves its angular momentum, (3) in some cases the magnetic field may halt collapse. We will now consider ways out of these problems.

Heating and Cooling

First, the thermal problem. A molecular cloud is heated by external radiation (X-rays, gamma-rays, UV) when it is low-density, and by cosmic rays more generally. At low densities and high temperatures, cooling is relatively inefficient. It tends to proceed via molecular radiation, such as from H_2 and CO. At low temperatures and high densities, cooling from dust grains dominates. This radiation occurs in the IR because that's where it is able to escape. This is why IR mapping tends to track dust.

The net result is that when a cloud becomes optically thick (say, $\tau > 1/2$), then it is self-shielded from external radiation and the interior portions of the gas can cool in peace.

This happens by formation of molecules, for example, and the equilibrium temperature drops to about 10 K. Therefore, the inner part of the gas can radiate away the energy it gets from gravitational collapse ("settling" might be more appropriate), and continue to contract.

Angular momentum

As we indicated before, the specific angular momentum of giant molecular clouds is vastly greater than that of stars, so you have to get rid of most of it. For a $\sim$1 pc GMC, $L/M > 10^{23}$ (here L is the angular momentum, M is the mass, and the number is in cgs units). For a dense cloud core, $\sim$0.1 pc, $L/M \sim 10^{21}$. For the Sun, $L/M \sim 10^{15}$. Where does the excess go?

You might think it could go to binaries, or planets, or that young stars might have a lot of angular momentum, but it isn't so. A 3-day binary has $L/M \sim 10^{19}$. **Ask class:** how would the angular momentum go with orbital period? Like $P^{1/3}$, so even at a 10^4 yr binary, $L/M \sim 10^{21}$. We already found that Jupiter doesn't have enough angular momentum either. Young stars such as T-Tauris have $L/M \sim 10^{17}$, which is more than the Sun but nothing close to that of the initial GMC.

Therefore, specific angular momentum must be transported away from the system entirely. **Ask class:** what are some ways that this can happen? Winds (magnetic, especially), jets, disks. There is still a lot of discussion about how this happens. In somewhat more detail, angular momentum can be removed by:

(1) Magnetic braking. If a magnetic field threads a cloud, it will try to enforce uniform rotation. This moves angular momentum outward. This can happen before the collapse of the cloud. It is also effective in slowing down the rotation of stars.

(2) Collapse to a disk. Stresses within the disk transport angular momentum outward and mass inward. An especially important source of such stress is the "magnetorotational instability", or MRI. If the disk has enough ionization, even a weak magnetic field is amplified if fluid at smaller radii has a smaller angular momentum than fluid at larger radii. If the ionization fraction is really low (or more properly if the magnetic Reynolds number is high enough), this mechanism is ineffective. This may lead to "dead zones" in some protoplanetary disks.

(3) Star-disk coupling. If the star has a significant magnetic field, it can get slowed down by interaction with the disk (or spun up, for that matter).

Magnetic fields: ambipolar diffusion and reconnection

Remember that magnetic fields are only an important mechanism to stave off collapse if the mass is subcritical relative to the magnetic Jeans mass. Masses in excess of this can collapse without significant hindrance from the field.

The gas contains both charged and neutral matter. The ions can be tied to the field, but the neutrals move freely through it. The two components interact by mutual collisions. By balancing the gravitational force on the neutral component with the drag produced by the ions (assumed tied to the field), people have estimated a time for this process, which is called ambipolar diffusion:

$$t_{\rm AD} \approx 5 \times 10^5 (X/10^{-8})\,{\rm yr}\ , \tag{11}$$

where X is the ionized fraction. This probably plays a role in many clouds, but there is dispute about whether this is the final word: the time scale for formation may be too large, and since the magnetic Jeans mass is $\sim 10^3\,M_\odot$, lower-mass cores could be tough to form.

Another possibility is reconnection. As the magnetic field is compressed by the matter (which wants to collect in the gravitational well), the field can change its topology by forming small closed loops. If this happens, the field does not provide as much support against large-scale motions of the matter. In addition, field may self-annihilate and reduce ratio of magnetic flux to mass. But this process is not well-studied.