From Collapse to the Main Sequence

Now suppose that we have a portion of a molecular cloud that is unstable to contraction. This means that the mass is greater than the Jeans mass which, as we found earlier, is around $10^4\ M_\odot$ for typical molecular cloud properties. Thus we begin by considering a cloud with a mass somewhat larger than $10^4\ M_\odot$.

But stars are much less massive than this, so what's happening? To get some insight, let's go back to our understanding that the Jeans mass scales as $M_J \propto T^{3/2}\rho^{-1/2}$ for temperature T and density ρ. If the gas is an ideal gas, then $P = nkT$, and as a polytrope, $P \propto \rho^{5/3}$, so this means that $T \propto \rho^{2/3}$ for an ideal gas, and therefore $M_J \propto (\rho^{2/3})^{3/2}\rho^{-1/2} = \rho^{2/2}\rho^{-1/2} = \rho^{1/2}$. As a result, without cooling, the Jeans mass goes up and the contraction would be stabilized.

But cooling *does* happen. In fact, although details obviously play some role, it's not a bad approximation to say that until the cloud gets so dense that radiation can't easily escape, the cloud is maintained at a roughly constant temperature as it contracts. The argument is that detailed cooling processes involve atomic and molecular transitions; these can't go below a given temperature (because there aren't transitions below that temperature) and don't go much above, either. If T is constant, then $M_J \propto T^{3/2}\rho^{-1/2} \propto \rho^{-1/2}$. Thus in the isothermal case the Jeans mass decreases as the cloud contracts.

What this implies is that if we started with a cloud that was just above the Jeans mass, say $10^4\ M_\odot$, as it contracts isothermally then subclumps, of lower than the original mass, can also become unstable to contraction. This process is called *fragmentation*, which means what it says: an initial cloud "fragments" into multiple sub-clumps. The fragmentation stops when the clouds become so optically thick that cooling is no longer efficient. This, in very broad strokes, is why stars have masses of tenths of a solar mass to tens of solar masses, give or take.

It will not surprise you to know that there are other details that matter and that are being studied observationally and theoretically. For example, when integrated over a galaxy, the number of newly-formed stars per mass interval, above roughly 1 $M_\odot$, seems to follow an approximate power law. An old, but still pretty good, version of this is the Salpeter initial mass distribution

$$\frac{dN}{dM} \propto M^{-2.35}\ . \tag{1}$$

To get the *mass* in stars per stellar mass interval we multiply by M to get the strange-looking relation $dM/dM \propto M^{-1.35}$. To get the cumulative stellar mass above some given mass our integral results in another factor of M, so that $M(> M_0) \propto M_0^{-0.35}$. This diverges at low masses, which tells us that this power law has to turn over below some mass. Estimates are that this mass might be around 0.5 $M_\odot$, but it's not trivial to determine because low-mass stars are dim and thus tough to see. Please also note that all of this is for the *initial* mass function. Low-mass stars live a lot longer than high-mass stars, which means that even in a situation with steady-state star formation, the ratio of low-mass stars to high-mass stars is a lot higher than you would find using the initial mass distribution.

When we see a power law, it means that at least in the range where the power law applies there is no scale for the problem. The process that appears to take hold here is turbulence: when a system is turbulent for a while, it can establish a power law set of sizes of turbulent eddies, and this appears to be a good explanation for the initial mass function, at least at high-ish masses. Of course, it's more than that. For example, high-mass stars form more rapidly than low-mass stars (everything goes faster at high masses...), and they tend to form near the centers of molecular clouds where the density is larger. This can lead to "competitive accretion", in which protostellar cores continue accreting mass from the surrounding gas, even after the star forms. These scenarios need to be studied numerically.

Now let's shift gears a bit. Suppose that a core has produced a protostar. As this is moving toward hydrogen ignition, disks and outflows are important. So let's dive in!

Disks and Outflows

As we found last time, getting rid of angular momentum is one of the major problems of star formation. The high angular momentum means that as a cloud collapses it tends to form a disk. Matter flows through the disk and accretes on the central object, so it is called an accretion disk. Accretion disks are common in many astronomical settings, including mass transfer systems (some binaries) and active galactic nuclei.

Disks are observed directly by imaging (cf Hubble), and indirectly because of their spectral energy distribution (IR excess). Their presence can also be inferred indirectly by asymmetric line profiles: if there is a wind from the young star it should be bipolar, so we should see both the blueshifted and redshifted components, but instead we see only the blueshifted because the disk blocks the part going away from us.

Disks usually rotate such that each fluid element is moving almost (but not exactly!) in a circular orbit. If there were no interactions between fluid elements, the angular velocity as a function of radius would be $\Omega \propto R^{-3/2}$, so there is a shearing flow. This means that coupling between adjacent radii exerts a force. **Ask class:** given that the outer parts rotate more slowly, in which direction will the force be and what will be the effect on the angular momentum and on the movement of mass? The inner annulus tries to speed up the outer annulus, giving it a higher velocity. This increases the angular momentum of the outer annulus and decreases the angular momentum of the inner annulus, so the net result is that angular momentum is transferred outward and mass flows inward (there are some subtleties, of course). The disk spreads as a result. This has similarities to the effect of "shepherd moons" except that there the coupling is purely gravitational.

The amount of coupling is often handled phenomenologically. The types of disks considered in a protostellar context are usually geometrically thin disks, i.e., their half-thickness H is much less than the radius: $H \ll R$. For these disks, the coupling depends on the kinematic viscosity of the fluid, and is usually parameterized in terms of the "alpha parameter" of Shakura and Sunyaev (1973): $\nu = \alpha c_s H$, where c_s is the sound speed. In many circumstances $\alpha \sim 0.01 - 1$. For such a disk we have

$$c_s \approx (H/R) v_K \ , \tag{2}$$

where v_K is the local Keplerian orbital speed. Therefore, the orbital motion is highly supersonic. The radial velocity is much smaller still:

$$v_r \sim \nu/R \sim \alpha (H/R)^2 v_K \ . \tag{3}$$

But what is the actual source of this viscosity? Kinematic viscosity, i.e., normal interactions between particles, is way too small in pretty much any disk in astrophysics. You might consider convection, but (1) the disks can be convectively stable, (2) numerical simulations show the angular momentum is transported the wrong way, and (3) the magnitude is too small anyhow. Therefore, since around 1990, MHD turbulence has been the favored mechanism. There is an instability (called the magnetorotational instability or Balbus-Hawley mechanism) by which a seed magnetic field can be amplified rapidly to a magnitude such that it dominates intradisk interactions.

On a more global level, gravitational torques from a companion star might also play a role in some circumstances.

Outflows

Jets are highly collimated and have high speeds (typically around the escape speed of the central object, which could be around 400 km/s for a star). Around protostars, jets are thought to be mostly (95-99%) neutral gas, and are usually bipolar. Outflows are poorly collimated and have lower speeds (100 km/s), and are thought to be driven by the jets. The power in jets/outflows diminishes with time, and the opening angle is thought to increase with time. The total energy in a protostellar jet can be 10^{43-47} erg.

There is a strong theoretical link between accretion and outflow, in the sense that it is inferred that accretion powers outflows. This link is much more difficult to establish observationally. The mass loss from outflows is typically $\sim 10^{-6}\, M_\odot\ \mathrm{yr}^{-1}$. The mean accretion rate during the embedded phase is typically $\sim 10^{-5}\, M_\odot\ \mathrm{yr}^{-1}$, so the mass loss rate in winds is $\sim 0.1 M_{\mathrm{acc}}$.

Theories for Jets

Jets across astrophysics have certain things in common. The speed of the jet seems correlated with the escape speed at the surface of the star. This is true from protostellar jets up to jets from black hole sources such as AGN, where the motion is close to the speed of light. Anyway, this suggests that jets are formed deep in the potential well, meaning near the star or in the innermost portions of the disk.

The problem of the formation of jets is twofold: how do you accelerate jets, and how do you collimate them. There are three general ways that have been proposed to acclerate jets:

(1) Hydrodynamic acceleration. If the jet material starts flowing outward in a region with sufficient external pressure, its cross-sectional area decreases and its velocity increases. Then, when the speed reaches the speed of sound, Bernoulli's equations show that the cross-sectional area increases (but not faster than the radius, so the collimation angle is small) and the speed also increases. A converging-diverging channel like this is called a de Laval nozzle, and is an essential principle behind rockets. This is therefore an efficient way to produce fast, collimated jets. In some cases (e.g., extragalactic jets) the external gas pressure cannot be high enough to provide the required initial collimation, because if it were then the gas would radiate a higher X-ray flux than observed. It might, however, play some role in protostellar jets.

(2) Radiative acceleration. Another possibility is that radiation from the central object can accelerate matter, which is separately collimated by a surrounding torus of gas. **Ask class:** suppose we had a gas of fully ionized hydrogen; what approximate luminosity would be required for such acceleration? About the Eddington luminosity, or around 10^{38} erg s^{-1} for a solar mass. This can be decreased if higher cross section absorption processes dominate. It is thought that interactions with the radiation may explain why the maximum Lorentz factor is around 20 in AGN: higher than that, and radiation *drag* slows down the jet.

(3) Hydromagnetic acceleration. Power comes from, and symmetry is provided by, the rotation of the accretion disk. In AGN, the rotational energy of the black hole itself could be important.

Protostar to the main sequence

High-mass stars are believed to join the main sequence while they are still in the accretion phase. They therefore do not convey much information about their origins when they become visible as optical objects. The study of pre-main-sequence stars is therefore confined primarily to low-mass stars.

The initial stellar core has $M \ll M_\odot$. It accretes gas directly from the envelope, and also from the circumstellar disk. The star is not in thermal equilibrium: instead, energy is dumped into the outer portions by accretion. The star is fully convective, and it adjusts towards thermal equilibrium on the Kelvin-Helmholtz (gravitational contraction) timescale:

$$t_{\rm KH} = \frac{GM_*^2}{R_* L_*} \sim 3 \times 10^7\,{\rm yr} \tag{4}$$

for a solar-type star. The timescale is less for higher-mass stars, because their much higher luminosity compensates for their greater amount of gravitational energy.

The temperature increases as the star contracts, until it gets to the point where it can fuse deuterium ($T \approx 10^6$ K). At that point, contraction stops because of the energy input. It takes $\sim 1 t_{\rm KH}$ to burn all the deuterium, then contraction starts again. When the center becomes hot enough for fusion of hydrogen (few$\times 10^6$ K), contraction stops again and the star moves to the main sequence. On the main sequence, the star is in thermal equilibrium. Stars aren't observed before they reach the deuterium burning sequence. An explanation for this is that younger stars are obscured by dust envelopes, and that when D burning commences it blows away the envelope. But if so, why should the wind start just when the deuterium burning happens? It may also be that the pre-D stars evolve rapidly down the Hayashi track (more on this in the next lecture) and are obscured, so that they are difficult to see.

Classification of protostars

(1) Class 0. Energy is provided by accretion, and the full stellar mass has not yet been accumulated. Phenomenologically, it is a sub-mm source ($\lambda > 350\mu$m has more than 5×10^{-3} of the bolometric luminosity) and has a blackbody spectrum characteristic of $T \sim 15 - 30$ K. This is primarily a useful concept for low-mass stars: high-mass stars at such a young age may already be burning hydrogen, so things are more confusing.

(2) Class I. IR excess, spectral flux increases with longer wavelength into the infrared, associated with dense molecular cores.

(3) Class II. IR excess but flux decreases with longer wavelength into the infrared. Optically visible.

(4) Class III. Spectra similar to blackbody, seen in visible light, may have a small optically thin disk around them.