In this lecture we will complete our discussion of the pre-main-sequence phase of stellar evolution, and then talk about the evolution of single low-mass stars.

The Hayashi track

We'll begin by wrapping up our discussion of pre-main-sequence stars. In 1961, Chushiro Hayashi recognized that these stars have to be essentially fully convective, rather than radiative; Donald Osterbrock also suggested this a few years earlier, but did not follow up (as Hayashi did) by constructing full stellar models based on this assumption. Under this assumption, pre-main-sequence stars have temperatures that are essentially independent of their luminosities. The fundamental reason for this is that the $H-$ opacity, which dominates for the relatively low temperatures of protostars, depends very strongly on temperature: $\kappa_{H-} \propto T^9$. Thus if the photosphere becomes hotter, it is pushed out, becomes cooler, and settles back to its original temperature (and vice versa if the photosphere becomes cooler).

Given some reasonable assumptions, the effective temperature on this branch is

$$T_{\rm eff} \approx 4000\mu^{13/51} \left(\frac{M}{M_\odot}\right)^{7/51} (L/L_\odot)^{1/102} \ . \qquad (1)$$

This means that on this track, the effective temperature is virtually independent of the luminosity. But what physical effects can get a star off of this track? Accretion will boost the temperature, which will destroy H^-. Also, thermonuclear fusion will increase the temperature.

The Hayashi track is mainly a theoretical construct instead of an observational fact. This is because most of the track, if it exists, is not visible because of dust obscuration and the deuterium burning sequence. If there is accretion, the star stays at about 10,000 K because cooling is very efficient down to that temperature and not so efficient at lower temperatures. This is enough to destroy $H-$.

Evolution of Solitary Stars

We now move to the evolution of single stars, i.e., stars not in close binaries or multiple systems (we'll get to those later). The evolution of such a star depends primarily on its mass, but this is because of the microphysics involved. Here are some framing questions to determine the course of evolution:

(1) When the star fuses hydrogen on the main sequence, does it do so mainly via the pp chain or the CNO cycle? We expect CNO to be more important for high-mass stars, because those stars have higher temperatures in their centers, and the CNO energy rate is highly temperature-sensitive.

(2) When the star starts to burn helium in the core, is the helium degenerate or nonde-

generate? Recall that if helium ignites when it is degenerate, then we get a helium flash and rapid burning. This is because degenerate matter does not expand much when it is heated, and thus does not cool in the same way as nondegenerate matter.

(3) When/if the star starts to burn CO in the core, is the CO degenerate or nondegenerate?

(4) Is the metallicity that of solar-type stars (Pop I) or is it much lower (Pop II or even Pop III)?

(5) At a given stage of evolution, is mass loss important for the structure of the star? Here we are thinking about mass lost because radiation from the star can produce a significant wind. This will only be important for stars with much higher luminosity than the Sun, or for high-luminosity parts of the star's life (such as the red giant phase). Line driving is important, which means that winds are much stronger when the metallicity is higher.

These questions determine the pace and nature of the evolution of stars, which is especially well-represented in the Hertzsprung-Russell diagram. We'll start with low-mass stars.

Evolution of low-mass stars

Types of hydrogen fusion and their consequences.—As we know, the two main ways to fuse four hydrogen nuclei (i.e., protons!) into one ^{4}He nucleus (two protons and two neutrons) are through the p-p chain and the CNO cycle. As we also know, the rate of energy production via the CNO cycle is much more temperature sensitive than the rate via the p-p chain. This suggests, correctly, that for higher-mass stars (which have higher temperatures in their cores), CNO will provide a larger fraction of the energy. For example, it is estimated that in the Sun (core temperature 1.5×10^7 K), only about 1.5% of the energy is generated in the CNO cycle, whereas in the center of a 1.5 $M_\odot$ star (core temperature 1.8×10^7 K), half the energy comes from CNO(!).

A less obvious consequence also follows from the very different temperature sensitivities. The core of a star, of course, does not have constant temperature; it's hottest at the center and somewhat cooler farther out. Because the p-p chain isn't very sensitive to temperature, the energy generation per mass isn't much larger at the center than it is farther out. But since the CNO cycle *is* very temperature-sensitive, the rate is a lot higher at the center. This leads to temperature and energy generation gradients, and when those are large enough we get convection. As a result, main sequence stars with masses greater than 1.2 $M_\odot$ have convective cores.

Consequences of loss of particles due to fusion.—When we fuse hydrogen into helium, we turn four protons plus the four electrons that balance their charge, into a single helium nucleus with two electrons. That is, this turns eight particles into three. In thermal equilibrium, all particles have the same temperature and contribute equally to the pressure. For an ideal gas, $P = nkT$, and the pressure gradients are set by the weight of the overlying matter. This means that as fusion progresses, to keep holding off gravity the density and temperature of the core must increase. Since the rate of fusion increases with increasing temperature and density, this means that the luminosity goes up. Among other things, this leads to the "faint young Sun" paradox, which is that the early Sun might have had only 70% of its current luminosity, yet the early Earth was comparably warm to today (possibly due to different reflectivity of its clouds at that time). The increased luminosity also leads to an increased photospheric radius.

Running out of hydrogen in the core.—At some point, the core runs out of hydrogen to fuse. If the core is convective ($M > 1.2\ M_\odot$) then this happens fairly rapidly because fuel has been mixed in throughout the burning process rather than just having an inert helium core build up gradually. When the convective core exhausts its hydrogen, the whole star falls short of energy and contracts. Therefore, the radius decreases slightly, the surface temperature increases, and as a result the star moves to the left in the H-R diagram. This motion stops only when all hydrogen in the core is depleted.

Hydrogen shell burning.—At this stage the central temperature is not yet high enough to burn helium, but the temperature outside the core is enough to burn hydrogen in a shell. When there is burning in a shell, it tends to be at a node of the movement of matter (that is, it doesn't move in or out radially much even if other parts of the star are expanding or contracting). That's because if it moves out it cools and stops burning, causing it to fall inward to where it is hot enough to fuse; if it moves in it heats up so much that it expands. Therefore, if matter is contracting inside a shell it is expanding outside, and vice versa. In a star with two shells (as can happen with a helium shell close to the core, and a hydrogen shell farther away), if the core contracts then so does the outer envelope, and if the core expands then so does the outer envelope.

Growth of the inert core.—The inert helium core therefore grows, the burning shell moves out, and the star moves towards the giant branch. For $M < 1.2\,M_\odot$ this happens gradually. But for $M > 1.2\,M_\odot$, the central helium core is larger, the core contraction happens faster, and lots of gravitational energy is released. This happens so fast that the expanding layers cool, and the opacities of the outer layers increase and become opaque. Thus the radius has to increase even more, which produces stronger cooling and increases opacities. This is a runaway and produces a very rapid expansion. It is halted when the envelope becomes fully convective. At that point, the star ascends the giant branch (like the Hayashi track in reverse).

Contraction of the helium core.—Meanwhile, the inert (not fusing) helium core cools and thus contracts. This increases the density of the core without increasing its temperature much, and for low-mass stars the helium core becomes degenerate. When the central temperature becomes $T > 10^8$ K, the triple-alpha process (which fuses helium to carbon) operates, releasing energy. This runs away if the core is degenerate because the core cannot cool by expansion). This helium flash continues until the temperature is high enough to lift degeneracy, at which point $\sim 20\%$ of the helium has probably been consumed. The helium flash is thought to last a few minutes, but as we learned in a homework set we do not expect to see an observational signature of the flash because the time necessary to transport the energy is millions of years.

Helium fusion.—After the flash, $\sim 10^6$ yr are required to stabilize the star, after which helium burning happens quietly, on the "helium main sequence". This phase is much shorter than the hydrogen (aka, "normal") main sequence, because less energy is available and because the energy generation rate is higher. We then get a recap, with an outer H burning shell, inert He, a He burning shell, and degenerate inert C and O in the core.

Thermal pulses.—At this stage, the luminosity in the He burning shell increases, which expands and cools the envelope, which shuts off the H shell burning, decreasing the luminosity, contracting and heating the envelope, reigniting the H shell, and so on. These pulses occur with a $\sim 10^5$ yr interval between them, and the luminosity can change by a factor of 2. After one full cycle, the luminosity has gone up a little, but the high temperature sensitivity of the opacity means that the temperature of the surface is about constant, so the star ascends a roughly vertical track (again similar to the protostellar Hayashi track). At this stage, the star is on the asymptotic giant branch, or AGB.

Dynamical instability.—Near the end of the AGB, the luminosity can increase by a factor 10 before rapidly decreasing, and the size can change by 50% (as in Mira variables, named after Mira Ceti. Some 20–30% of the stellar mass is lost in this phase. Finally, it reaches a point where it is no longer unstable (when enough H and He have been consumed), and it shrinks into a white dwarf. The ejected mass is illuminated by the hot central star, and appears as a planetary nebula. This, again, is for stars slightly more massive than the Sun (around 1.2 $M_\odot$ or larger), so sadly the Sun will not leave behind a beautiful planetary nebula.

Effect of metals.—Metals (elements heavier than helium) are produced in stars and distributed throughout the interstellar and intergalactic medium. This has happened over the age of the universe, which means that stars have formed with a wide range of metallicities. Stars with few metals have lower opacities, which means that for all else equal a star with smaller metallicity is more compact and its surface temperature is higher. This led to one of the big revisions in the distance scale from Cepheids: the period-luminosity relation differs from Pop I (high metallicity) to Pop II (low metallicity), and as Pop II are brighter for a given period, they are farther away for a given flux.

The final fate.—All of the stars we have discussed, and indeed all solitary stars that begin their lives with less than about 8 times the mass of the Sun, slip gracefully into their retirement as white dwarfs. These can have different masses, with a typical mass of about 0.6 $M_\odot$ and a maximum of the Chandrasekhar mass of $\sim 1.4\ M_\odot$.